Fall 2004 Physics 3
Tu-Th Section

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Last time….

Sound:
- A longitudinal wave in a medium.

How do we describe such a wave?
- Mathematically, just like transverse wave on string:
  \[ y(x,t) = A \cos(kx - \omega t) \]

Careful: the meaning of \( y(x,t) \) is quite different!!
Displacement vs. pressure (last time...)

- Can describe sound wave in terms of displacement or in terms of pressure

\[ y(x, t) = A \cos(kx - \omega t) \]

or

\[ p(x, t) = (A \cdot B \cdot k) \sin(kx - \omega t) \]

Displacement and pressure out of phase by 90°.

Bulk Modulus

Pressure fluctuation
Bulk Modulus (last time…)  
A measure of how easy it is to compress a fluid.  

\[ B = -V \frac{dP}{dV} \]

Ideal gas:  
\[ B = \gamma P \]

\[ \gamma = \frac{C_P}{C_V} \]

\( \gamma \approx 1.7 \) monoatomic molecules (He, Ar,..)
\( \gamma \approx 1.4 \) diatomic molecules (O_2, N_2,..)
\( \gamma \approx 1.3 \) polyatomic molecules (CO_2,..)

Heat capacities at constant \( P \) or \( V \)
Speed of sound (last time...)

\[ v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \]

Molar mass

For air: \( v = 344 \text{ m/sec at } 20^\circ \text{C} \)
Intensity (last time...)

- The wave carries energy
- The intensity is the \textit{time average} of the power carried by the wave crossing unit area.
- Intensity is measured in W/m2

\[ I = \frac{\langle P \rangle}{S} = \frac{\langle dE/dt \rangle}{S} \]
Decibel (last time...)

• A more convenient sound intensity scale
  ➢ more convenient than W/m².
  
• The sound intensity level $\beta$ is defined as

$$\beta = (10 \text{ dB}) \log \frac{I}{I_0}$$

• Where $I_0 = 10^{-12} \text{ W/m}^2$
  ➢ Approximate hearing threshold at 1 kHz

• It's a log scale
  ➢ A change of 10 dB corresponds to a factor of 10
Standing sound waves

Recall standing waves on a string

• A standing wave on a string occurs when we have interference between wave and its reflection.
• The reflection occurs when the medium changes, e.g., at the string support.
• We can have sound standing waves too.
• For example, in a pipe.
• Two types of boundary conditions:
  1. Open pipe
  2. Closed pipe
• In an closed pipe the boundary condition is that the displacement is zero at the end
  ➢ Because the fluid is constrained by the wall, it can't move!
• In an open pipe the boundary condition is that the pressure fluctuation is zero at the end
  ➢ Because the pressure is the same as outside the pipe (atmospheric)
Remember:

- Displacement and pressure are out of phase by 90°.
- When the displacement is 0, the pressure is ± $p_{\text{max}}$.
- When the pressure is 0, the displacement is ± $y_{\text{max}}$.
- So the nodes of the pressure and displacement waves are at different positions

➢ It is still the same wave, just two different ways to describe it mathematically!!
More jargon: **nodes** and **antinodes**

- In a sound wave the pressure nodes are the displacement antinodes and vice versa
Example

- A directional loudspeaker bounces a sinusoidal sound wave off the wall. At what distance from the wall can you stand and hear no sound at all?
- A key thing to realize is that the ear is sensitive to pressure fluctuations.
- Want to be at pressure node.
- The wall is a displacement node $\rightarrow$ pressure antinode.
Organ pipes

• Sound from standing waves in the pipe

• Remember:

  ➢ Closed pipe:
    • Displacement node (no displacement possible)
      → Pressure Antinode

  ➢ Open pipe:
    • Pressure node (pressure is atmospheric)
      → Displacement Antinode
Open Pipe
(displacement picture)

(a) \( f_1 = \frac{v}{2L} \)

(b) \( f_2 = 2 \frac{v}{2L} = 2f_1 \)

(c) \( f_3 = 3 \frac{v}{2L} = 3f_1 \)

Closed Pipe
(displacement picture)

(a) \( f_1 = \frac{v}{4L} \)

(b) \( f_3 = 3 \frac{v}{4L} = 3f_1 \)

(c) \( f_5 = 5 \frac{v}{4L} = 5f_1 \)
Organ pipe frequencies

- Open pipe:
  \[ f = n \cdot \frac{v}{2L} \quad (n = 1, 2, 3\ldots) \]

- Closed (stopped) pipe:
  \[ f = n \cdot \frac{v}{4L} \quad (n = 1, 3, 5\ldots) \]
Sample Problem

• A pipe is filled with air and produces a fundamental frequency of 300 Hz.
  ➢ If the pipe is filled with He, what fundamental frequency does it produce?
  ➢ Does the answer depend on whether the pipe is open or stopped?

Open pipe: \[ f = n \cdot \frac{v}{2L} \quad (n = 1, 2, 3...) \]
Closed (stopped) pipe: \[ f = n \cdot \frac{v}{4L} \quad (n = 1, 3, 5...) \]

→ Fundamental frequency \( v/2L \) (open) or \( v/4L \) (stopped)

What happens when we substitute He for air?

The velocity of sound changes!
From last week, speed of sound:

\[ \nu = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \]

\[ \gamma = \frac{C_P}{C_V} \]

\[ \gamma \sim 1.7 \text{ monoatomic molecules (He, Ar,..)} \]
\[ \gamma \sim 1.4 \text{ diatomic molecules (O}_2, \text{ N}_2,\ldots) \]
\[ \gamma \sim 1.3 \text{ polyatomic molecules (CO}_2,\ldots) \]

\[ \nu_{\text{air}} = \nu_{\text{He}} \sqrt{\frac{M_{\text{He}}}{M_{\text{air}}}} \sqrt{\frac{\gamma_{\text{air}}}{\gamma_{\text{He}}}} \nu_{\text{He}} \]
We had:

Open pipe: \[ f = n \cdot \frac{v}{2L} \quad (n = 1, 2, 3..) \]

Closed (stopped) pipe: \[ f = n \cdot \frac{v}{4L} \quad (n = 1, 3, 5..) \]

i.e., the fundamental frequency is proportional to velocity for both open and stopped pipes

\[ f_{He} = \frac{v_{He}}{v_{air}} \cdot f_{air} \]

But: \[ v_{air} = \sqrt{\frac{\gamma_{air}}{\gamma_{He}}} \sqrt{\frac{M_{He}}{M_{air}}} \cdot v_{He} \]

So: \[ f_{He} = \sqrt{\frac{\gamma_{He}}{\gamma_{air}}} \sqrt{\frac{M_{air}}{M_{He}}} \cdot f_{air} \]

\[ f_{He} = \sqrt{\frac{1.7}{1.4}} \sqrt{\frac{29 \text{ g/mol}}{4 \text{ g/mol}}} \quad (300\text{Hz}) = 890\text{Hz} \]
Resonance

• Many mechanical systems have natural frequencies at which they oscillate.
  - a mass on a spring: $\omega^2 = k/m$
  - a pendulum: $\omega^2 = g/l$
  - a string fixed at both ends: $f = n\nu/(2L)$

• If they are driven by an external force with a frequency equal to the natural frequency, they go into resonance:
  - the amplitude of the oscillation grows
  - in the absence of friction, the amplitude would go to infinity
Interference

- Occur when two (or more) waves overlap.
- The resulting displacement is the sum of the displacements of the two (or more) waves.
  - Principle of superposition.
  - We already applied this principle to standing waves:
    - Sum of a wave moving to the right and the reflected wave moving to the left.
Interference (cont.)

• The displacements of the two waves can add to give
  ➢ a bigger displacement.
    • Constructive Interference.
  ➢ or they can even cancel out and give zero displacement.
    • Destructive interference.
    • Sometime, sound + sound = silence
    • Or, light + light = darkness
Interference Example

- Two loudspeakers are driven by the same amplifier and emit sinusoidal waves in phase. The speed of sound is $v=350$ m/sec. What are the frequencies for (maximal) constructive and destructive interference.
• Wave from speaker A at P \((x_1=AP)\)
  \[y_1(t) = A_1 \cos(\omega t - kx_1)\]

• Wave from speaker B at P \((x_2=BP)\)
  \[y_2(t) = A_2 \cos(\omega t - kx_2)\]

• Total amplitude
  \[y(t) = y_1(t) + y_2(t)\]
  \[y(t) = A_1 \cos \omega t \cos kx_1 + A_1 \sin \omega t \sin kx_1\]
  \[+ A_2 \cos \omega t \cos kx_2 + A_2 \sin \omega t \sin kx_2\]

• When \(kx_1 = kx_2 + 2n\pi\) the amplitude of the resulting wave is largest.
  ➢ In this case \(\cos kx_1 = \cos kx_2\) and \(\sin kx_1 = \sin kx_2\).

• Conversely, when \(kx_1 = kx_2 + n\pi\) (with \(n\) odd), the amplitude of the resulting wave is the smallest.
  ➢ Then \(\cos kx_1 = -\cos kx_2\) and \(\sin kx_1 = -\sin kx_2\).
Constructive Interference

\[ kx_1 = kx_2 + 2n\pi \]
\[ \frac{2\pi}{\lambda}x_1 = \frac{2\pi}{\lambda}x_2 + 2n\pi \]

\[ x_1 - x_2 = n\lambda \]

Destructive Interference

\[ kx_1 = kx_2 + (2n + 1)\pi \]
\[ \frac{2\pi}{\lambda}x_1 = \frac{2\pi}{\lambda}x_2 + (2n + 1)\pi \]

\[ x_1 - x_2 = \frac{2n+1}{2}\lambda \]
• Constructive interference occurs when the difference in path length between the two waves is equal to an integer number of wavelengths.

• Destructive interference when the difference in path length is equal to a half-integer number of wavelengths.

• CAREFUL: this applies if
  - The two waves have the same wavelength.
  - The two waves are emitted in phase.

• What would happen if they were emitted (say) 180° out of phase?
Back to our original problem:

The waves are generated in phase; \(v=350\) m/sec

Constructive interference:
- \(AP-BP=n\lambda\)
- \(\lambda = v/f\)
- \(f = n \times 350/0.35\) Hz
  - \(f = 1, 2, 3, \ldots\) kHz

Destructive interference:
- \(AP-BP=n\lambda/2\) (n odd)
- \(\lambda = v/f\)
- \(f = n \times 350/0.70\) Hz
  - \(f = 0.5, 1.5, 2.5\) kHz
Beats

• Consider interference between two sinusoidal waves with similar, but not identical, frequencies:

• The resulting wave looks like a single sinusoidal wave with a varying amplitude between some maximum and zero.

• The intensity variations are called beats, and the frequency with which these beats occur is called the beat frequency.
Beats, mathematical representation

- Consider two waves, equal amplitudes, different frequencies:
  - $y_1(x,t) = A \cos(2\pi f_1 t - k_1 x)$
  - $y_2(x,t) = A \cos(2\pi f_2 t - k_2 x)$

- Look at the total displacement at some point, say $x=0$.
  - $y(0,t) = y_1(0,t) + y_2(0,t) = A \cos(2\pi f_1 t) + A \cos(2\pi f_2 t)$

- Trig identity:
  - $\cos A + \cos B = 2 \cos[(A-B)/2] \cos[(A+B)/2]$

- This gives
  - $y(0,t) = 2A \cos[\frac{1}{2} (2\pi)(f_1-f_2)t] \cos[\frac{1}{2} (2\pi)(f_1+f_2)t]$
$y(0,t) = 2A \cos\left[\frac{1}{2} (2\pi)(f_1-f_2)t\right] \cos\left[\frac{1}{2} (2\pi)(f_1+f_2)t\right]$

An amplitude term which oscillates with frequency $\frac{1}{2} (f_1-f_2)$. If $f_1 \approx f_2$ then $f_1-f_2$ is small and the amplitude varies slowly.

A sinusoidal wave term with frequency $f = \frac{1}{2} (f_1 + f_2)$.

Beat frequency is $\frac{1}{2} |(f_1 - f_2)|$
Example problem

• While attempting to tune the note C at 523 Hz, a piano tuner hears 2 beats/sec.
  (a) What are the possible frequencies of the string?
  (b) When she tightens the string a little, she hears 3 beats/sec. What is the frequency of the string now?
  (c) By what percentage should the tuner now change the tension in the string to "fix" it?

(a) \( f_{\text{beat}} = \frac{1}{2} |f_C - f_{\text{piano}}| \)
\[ f_{\text{beat}} = 2 \text{ Hz} \text{ and } f_C = 523 \text{ Hz} \]
\[ \Rightarrow f_{\text{piano}} = 527 \text{ or } 519 \text{ Hz} \]

(b) \( f_{\text{beat}} = 3 \text{ Hz} \)
\[ \Rightarrow f_{\text{piano}} = 529 \text{ or } 517 \text{ Hz} \]

To decide which of the two, use the fact that the tension increased.
For string fixed at both end, we had \( f = \frac{nv}{2L} \), i.e., \( f \) proportional to \( v \).
But \( v^2 = \frac{F}{\mu} \) \( \Rightarrow \) higher \( F \) \( \Rightarrow \) higher \( v \) \( \Rightarrow \) higher \( f \) \( \Rightarrow \)
\[ f_{\text{piano}} = 529 \text{ Hz} \]
(c) The frequency is \( f_{\text{piano}} = 529 \) Hz, we want \( f_C = 523 \) Hz.

Frequency is proportional to \( v \) (\( f = nv/2L \))

Velocity is proportional to the square root of tension (\( v^2 = F/\mu \))

\( \rightarrow \) Frequency is proportional to the square root of tension

\[
\frac{f_{\text{piano}}}{f_C} = \sqrt{\frac{F_{\text{piano}}}{F_C}}
\]

\[
\frac{F_{\text{piano}}}{F_C} = \frac{f_{\text{piano}}^2}{f_C^2} = \frac{529^2}{523^2} = 1.023
\]

The tension must be changed (loosened) by 2.3%
Doppler Effect

• When a car goes past you, the pitch of the engine sound that you hear changes.
• Why is that?
• This must have something to do with the velocity of the cars with respect to you (towards you vs. away from you).

➤ Unless it is because the driver is doing something "funny" like accelerating to try to run you over 😊
Consider listener moving towards sound source:

- Sound from source: velocity $v$, frequency $f_s$, wavelength $\lambda$, and $v=\lambda f_s$.
- The listener sees the wave crests approaching with velocity $v+v_L$.
- Therefore the wave crests arrive at the listener with frequency:

$$f_L = \frac{v+v_L}{v} f_s = \left(1 + \frac{v_L}{v}\right)f_s$$

$\rightarrow$ The listener "perceives" a different frequency (Doppler shift).
Now imagine that the source is also moving:

• The wave speed relative to the air is still the same \(v\).
• The time between emissions of subsequent crests is the period \(T=1/f_s\).
• Consider the crests in the direction of motion of the source (to the right)
  - A crest emitted at time \(t=0\) will have travelled a distance \(vT\) at \(t=T\)
  - In the same time, the source has travelled a distance \(v_sT\).
  - At \(t=T\) the subsequent crest is emitted, and this crest is at the source.
  - So the distance between crests is \(vT-v_sT=(v-v_s)T\).
  - But the distance between crests is the wavelength
    - \(\lambda = (v-v_s)T\)
    - But \(T=1/f_s\) \(\Rightarrow \lambda = (v-v_s)/f_s\) (in front of the source)
• \( \lambda = \frac{(v-v_s)}{f_s} \) (in front of the source)
• Clearly, behind the source \( \lambda = \frac{(v+v_s)}{f_s} \)
• For the listener, \( f_L = \frac{(v+v_L)}{\lambda} \)
  ➢ Since he sees crests arriving with velocity \( v+v_L \)

\[
\rightarrow \quad f_L = \frac{v+v_L}{v+v_s} f_S
\]
Sample problem

- A train passes a station at a speed of 40 m/sec. The train horn sounds with f=320 Hz. The speed of sound is v=340 m/sec.

What is the change in frequency detected by a person on the platform as the train goes by.

Approaching train:

\[ f_L = \frac{v + v_L}{v + v_S} f_S \]
In our case $v_L=0$ (the listener is at rest) and the source (train) is mowing towards rather than away from the listener.

$I$ must switch the sign of $v_S$

$$f_L = \frac{v_v + v_L}{v + v_S} f_S \text{ becomes } f_{L1} = \frac{v}{v-v_{\text{train}}} f$$

When the train moves away:

Clearly $I$ need to switch the sign of $v_{\text{train}}$:  

$$f_{L2} = \frac{v}{v+v_{\text{train}}} f$$

$$\Delta f = f_{L1} - f_{L2} = .. (\text{algebra}) .. = -2 \frac{vv_{\text{train}}}{v^2 - v_{\text{train}}^2} f = 76 \text{ Hz}$$