PHYSICS 24
SET 2

(1) $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$ and $u_y' = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$

In our case $u_x = 0$ and $u_y = c$

$\Rightarrow u_x' = \dfrac{-v}{1-0} = -v$ and $u_y' = \dfrac{c}{\gamma(1-0)} = \dfrac{c}{\gamma} = \sqrt{1 - v^2/c^2}\, c$

Then $u' = \sqrt{u_x'^2 + u_y'^2} = \sqrt{v^2 + c^2(1 - v^2/c^2)} = c$ ✓

(2) No loss of generality in taking $\vec{u}$ in the xy plane, ie, no z-component (ie $u_z = 0 \Rightarrow u_z' = 0$)

$$u'^2 = u_x'^2 + u_y'^2 = \left(\frac{u_x - v}{1 - u_x v/c^2}\right)^2 + \frac{u_y^2}{\gamma^2 (1 - u_x v/c^2)^2}$$

$$u'^2 = \frac{(u_x - v)^2 + u_y^2(1 - v^2/c^2)}{(1 - u_x v/c^2)^2} = \frac{u_x^2 + v^2 - 2u_x v + u_y^2 - u_y^2 v^2/c^2}{(1 - u_x v/c^2)^2}$$

Use $u_x^2 + u_y^2 = u^2$. Also rewrite $u_y^2 = u^2 - u_x^2$

$$u'^2 = \frac{u^2 + v^2 - 2u_x v - u^2 v^2/c^2 + u_x^2 v^2/c^2}{\left(1 - \frac{u_x v}{c^2}\right)^2}$$

Now let's look at $c^2 - u'^2$

$$c^2 - u'^2 = \frac{c^2(1 - u_x v_x/c^2)^2 - u^2 - v^2 + 2u_x v + u^2 v^2/c^2 - u_x^2 v^2/c^2}{\left(1 - \frac{u_x v}{c^2}\right)^2}$$

$$c^2 - u'^2 = \frac{c^2 - \cancel{2u_x v} + u_x^2 v^2/c^2 - u^2 - v^2 + \cancel{2u_x v} + u^2 v^2/c^2 - \cancel{u_x^2 v^2/c^2}}{\left(1 - \frac{u_x v}{c^2}\right)^2}$$

$$c^2 - u'^2 = \frac{c^2 - u^2 - v^2 + u^2 v^2/c^2}{\left(1 - \frac{u_x v}{c^2}\right)^2} = \frac{(c^2 - u^2)(1 - v^2/c^2)}{\left(1 - \frac{u_x v}{c^2}\right)^2} \underline{\underline{> 0}}$$

Because
- the denominator is always positive
- $(c^2 - u^2) > 0$ if $u < c$
- $(1 - v^2/c^2) > 0$ if $v < c$

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3) $\lambda_0 = 656.3$ nm $\quad \nu_0 = \frac{c}{\lambda_0}$

Doppler shifted frequency observed on the earth
Use equation 12.6 in K&K but change the sign of v because the source is travelling away from the observer

$$\nu_D = \nu_0 \sqrt{\frac{1 - v/c}{1 + v/c}} \qquad \nu_D = \frac{c}{\lambda_0} \sqrt{\frac{1 - v/c}{1 + v/c}}$$

$$\lambda = \frac{c}{\nu_D} = \underline{\underline{\lambda_0 \sqrt{\frac{1 + v/c}{1 - v/c}}}} \checkmark$$

$$\sqrt{\frac{1 + v/c}{1 - v/c}} = \sqrt{\frac{1 + 4\,10^4/3\,10^8}{1 - 4\,10^4/3\,10^8}} = \sqrt{\frac{1.00013333}{0.99986666}} = 1.0001333$$

Also $\sqrt{\frac{1 + v/c}{1 - v/c}} = (1 + v/c)^{1/2} \left(1 - \frac{v}{c}\right)^{-1/2} \simeq \left(1 + \frac{1}{2}\frac{v}{c}\right)\left(1 + \frac{1}{2}\frac{v}{c}\right) \approx 1 + v/c$

$\lambda \cong 656.4$ nm, $\Delta\lambda \sim 0.1$ nm, longer wavelength

REDSHIFT

4 S = frame attached to earth

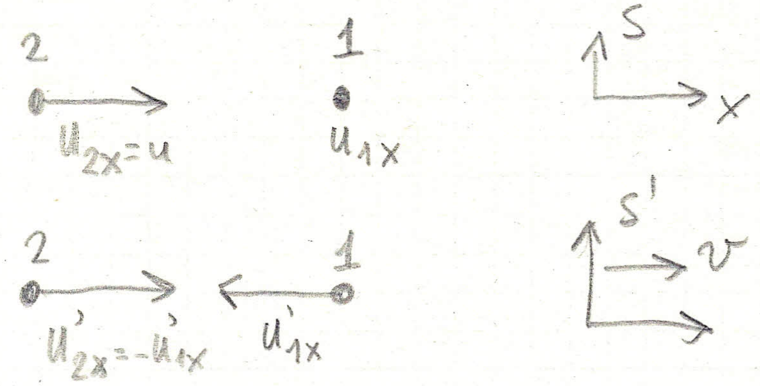
S' = frame attached to B, moving with $V = -0.8c$

Spaceship A has velocity $u_x = -0.5c$ in S

$$u'_x = \text{velocity of A in } S' = \frac{u_x - v}{1 - \frac{u_x V}{c^2}} = \frac{-0.5 + 0.8}{1 - 0.5 \cdot 0.8} c$$

$$u'_x = \frac{0.3}{0.6} c \qquad \boxed{u'_x = 0.5c} \checkmark$$

5



$$u'_{1x} = \frac{u_{1x} - v}{1 - \frac{v u_{1x}}{c^2}} = \frac{0 - v}{1 - 0} = -v$$

$$u'_{2x} = \frac{u_{2x} - v}{1 - \frac{v u_{2x}}{c^2}} = \frac{u - V}{1 - \frac{uV}{c^2}}$$

Want $u'_{2x} = +V$ $\qquad \dfrac{u - V}{1 - \frac{uV}{c^2}} = V$

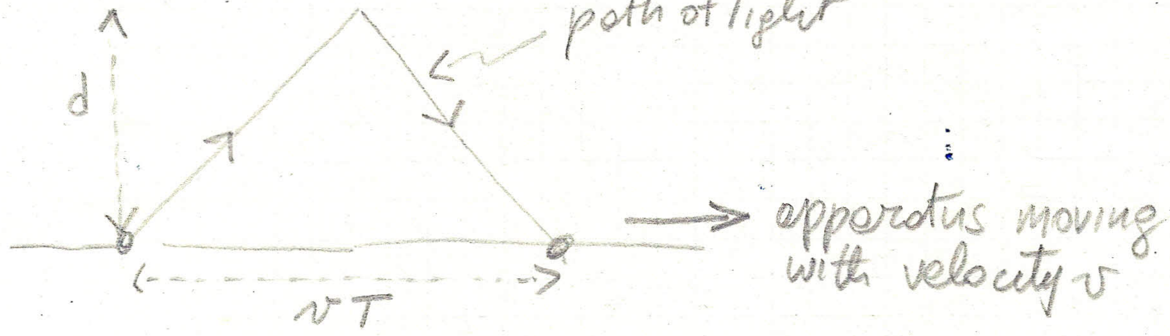
$$u - V = V - \frac{uV^2}{c^2} \qquad \frac{uV^2}{c^2} - 2V + u = 0$$

$$uV^2 - 2c^2 V + uc^2 = 0$$

$$V = \frac{2c^2 \pm \sqrt{4c^4 - 4u^2c^2}}{2u} = \frac{c^2 \pm \sqrt{c^4 - u^2c^2}}{u}$$

Take the soltn with the minus sign to insure $\underline{V < C}$

6 (a) From the point of view of the observer at rest



T = period _ Path length of light $= cT$

Let's look at the triangle

$$\frac{1}{4}c^2T^2 = \frac{1}{4}v^2T^2 + d^2$$

$$(c^2 - v^2)T^2 = 4d^2 \qquad \boxed{T = \frac{2d}{c\sqrt{1 - v^2/c^2}} = \gamma \tau_0}$$

where $\tau_0 = \frac{2d}{c}$

(b) Clock rest frame is unprimed - X-axis is up -

Consider three events

(1) Light is emitted $x_1 = x_1' = 0$
$t_1 = t_1' = 0$

(2) Light bounces off mirror $x_2 = d \quad t_2 = d/c$

$$x_2' = \gamma(x_2 - V t_2) = \gamma\left(d - \frac{v}{c}d\right) = \gamma\left(1 - \frac{v}{c}\right)d$$

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = \gamma\left(\frac{d}{c} - \frac{v}{c^2}d\right) = \gamma\left(1 - \frac{v}{c}\right)\frac{d}{c}$$

(3) Light gets back to clock $x_3 = 0 \quad t_3 = 2\frac{d}{c}$

$$T = t_3' = \gamma\left(t_3 - \frac{v}{c^2}x_3\right) = \gamma\frac{2d}{c} = \underline{\underline{\gamma\tau_0}}$$