PHYSICS 24 SET 1

(1) $\tanh^2\alpha + \operatorname{sech}^2\alpha = 1$

Let $\tanh\alpha = \frac{v}{c}$

$$\frac{v^2}{c^2} + \frac{1}{\cosh^2\alpha} = 1 \Rightarrow \boxed{\cosh\alpha = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma}$$

Then $\cosh^2\alpha - \sinh^2\alpha = 1$

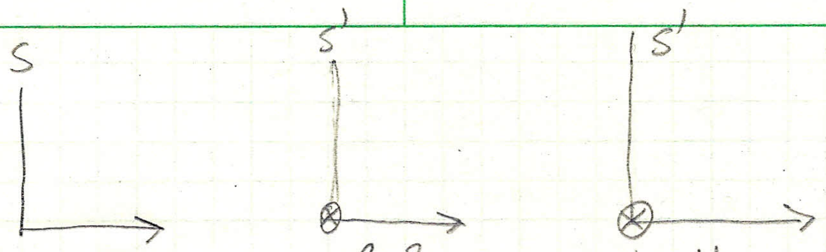
$$\sinh^2\alpha = \cos^2\alpha - 1 = \frac{1}{1 - v^2/c^2} - 1 = \frac{v^2/c^2}{1 - v^2/c^2}$$

$$\Rightarrow \boxed{\sinh\alpha = \frac{v/c}{\sqrt{1 - v^2/c^2}} = \frac{\gamma v}{c}}$$

$$\begin{cases} x' = x\cosh\alpha - ct\sinh\alpha = \gamma(x - vt) \\ t' = t\cosh\alpha - \frac{x}{c}\sinh\alpha = \gamma(t - \frac{v}{c^2}x) \end{cases}$$

✓

(2) After time t measured in S, the clock will be at a distance $\boxed{x = tv}$ from the origin of S. This distance is also measured in S.
The observer actually will have seen the clock as it was located at an earlier time because it takes time for light to travel from the clock to the observer.
Sketch on the next page



$x = \ell = ?$ $\qquad x = vt$

$t_1 \leftarrow$ time in $S = ?$

$t_1' \leftarrow$ time in $S' \leftarrow$ this is what we want

Here are some equations

$$\begin{cases} t_1' = \gamma\left(t_1 - \dfrac{v\ell}{c^2}\right) \\ x = vt \\ \ell = vt_1 \\ c(t - t_1) = \ell \end{cases}$$

We have four equations and four unknowns (x, ℓ, t_1, t_1')
We can solve for t_1' !!

- eliminate ℓ using 3rd eqtn

$$\begin{cases} t_1' = \gamma\left(t_1 - \dfrac{v^2 t_1}{c^2}\right) = \gamma\left(1 - \frac{v^2}{c^2}\right) t_1 \\ x = vt \\ c(t - t_1) = v t_1 \end{cases}$$

- use the 3rd equation $\quad t_1 = \dfrac{1}{1 + v/c}\, t$

- Plug it into the 1st equation

$$t_1' = \gamma\left(1 - \frac{v^2}{c^2}\right) t_1 = \frac{\gamma\left(1 - v^2/c^2\right)}{1 + v/c}\, t \qquad \boxed{t_1' = \gamma\left(1 - v/c\right) t}$$

3) Time dilation $\tau_{LAB} = \gamma \tau_0 = \frac{1}{\sqrt{1-0.999^2}} 2\mu se$

$\tau_{LAB} = 22.37 \times 2\ \mu sec = 44.7\ \mu sec$

4) K&K, 12.5

This is worked out in section 12.4 of K&K for the case $v = 0.9c$

The answer then is $v = \frac{0.99c + 0.99c}{1+(0.99)^2} = \underline{0.99995c}$

5) K&K 12.6

Lorentz contraction $\boxed{L = L_0 \sqrt{1 - v'^2/c^2}}$ (1)

where v' is the velocity of the rod in S'

$v' = \frac{u - v}{1 - uv/c^2}$ eqtn 12.2a in K&K

$v'^2 = \frac{u^2 + v^2 - 2uv}{(1 - uv/c^2)^2}$

$1 - \frac{v'^2}{c^2} = 1 - \frac{u^2 + v^2 - 2uv}{c^2(1 - uv/c^2)^2} = \frac{c^2 \cancel{- 2uv} + u^2v^2/c^2 - u^2 - v^2 \cancel{+ 2uv}}{c^2(1 - uv/c^2)^2}$

$1 - \frac{v'^2}{c^2} = \frac{c^2 + u^2v^2/c^2 - u^2 - v^2}{1/c^2 (c^2 - uv)^2} = \frac{c^4 + u^2v^2 - u^2c^2 - v^2c^2}{(c^2 - uv)^2}$

$1 - \frac{v'^2}{c^2} = \frac{(c^2 - u^2)(c^2 - v^2)}{(c^2 - uv)^2}$ And substituting into (1)

$$\boxed{L = L_0 \frac{\sqrt{(c^2 - u^2)(c^2 - v^2)}}{c^2 - uv}}$$

6 K&K 12.10

$$v = \frac{\sqrt{3}}{2} c \Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 2$$

$\boxed{\text{Farmers Frame}}$

Let's look at 3 events and their time sequence –

1. Front of pole reaches door at $x_1 = 0$ $t_1 = 0$
2. Front of pole reaches back of barn

   $x_2 = \frac{3}{4} l_0$

   $$t_2 = \frac{x_2}{v} = \frac{2}{\sqrt{3}} \frac{3}{4} \frac{l_0}{c} = \frac{\sqrt{3}}{2} \frac{l_0}{c} \approx 0.87 \frac{l_0}{c}$$

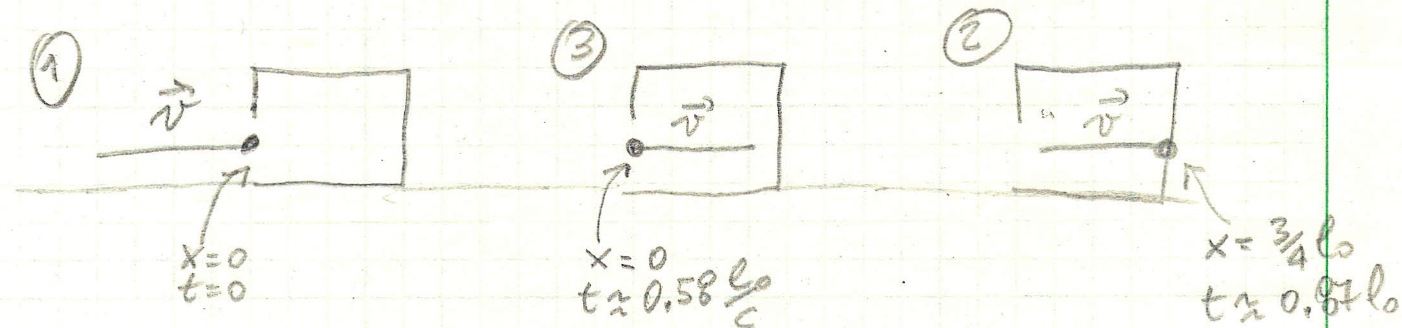
3. End of pole reaches door

   $x_3 = 0$

   $$t_3 = \frac{l_0/\gamma}{v} = \frac{1}{2} \frac{2}{\sqrt{3}} \frac{l_0}{c} = \frac{1}{\sqrt{3}} \frac{l_0}{c} \approx 0.58 \frac{l_0}{c}$$

So the sequence of events is 1 → 3 → 2

Sketch



<u>The Farmer is right !!</u>

Same events but from pole vaulter frame

In this frame the pole is at rest and the barn is moving

1. Door reaches front of pole $x_1'=0$ $t_1'=0$

2. Back of barn reaches front of pole

$$x_2'=0$$

$$t_2' = \frac{1/\gamma\left(\frac{3}{4}l_0\right)}{v} = \frac{1}{2}\,\frac{3}{4}\,\frac{2}{\sqrt{3}}\,\frac{l_0}{c} = \frac{\sqrt{3}}{4}\,\frac{l_0}{c} \approx 0.43\,\frac{l_0}{c}$$

Sanity Check

$$x_2' = \gamma(x_2 - v t_2) = 2\left(\frac{3}{4}l_0 - \frac{\sqrt{3}}{2}c\,\frac{\sqrt{3}}{2}\,\frac{l_0}{c}\right) = 2\left(\frac{3}{4}l_0 - \frac{3}{4}l_0\right) = 0$$

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = 2\left(\frac{\sqrt{3}}{2}\frac{l_0}{c} - \frac{1}{c^2}\frac{\sqrt{3}}{2}c\,\frac{3}{4}l_0\right) = 2\left(\frac{\sqrt{3}}{2}\frac{l_0}{c} - \frac{3\sqrt{3}}{8}\frac{l_0}{c}\right)$$

$$t_2' = 2\left(\frac{\sqrt{3}}{8}\frac{l_0}{c}\right) = \frac{\sqrt{3}}{4}\frac{l_0}{c}$$

Same as above!!

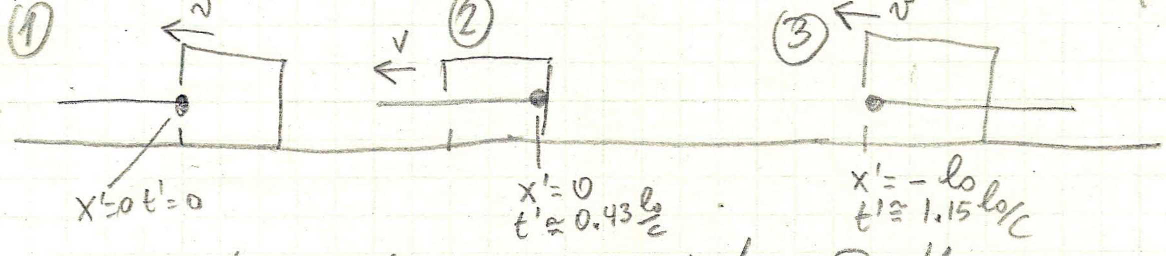
3. Front door reaches back of pole $x_3' = -l_0$

$$t_3' = \frac{l_0}{v} = \frac{2}{\sqrt{3}}\frac{l_0}{c} \approx 1.15\,\frac{l_0}{c}$$

Sanity check $x_3' = \gamma(x_3 - v t_3) = 2\left(0 - \frac{\sqrt{3}}{2}c\,\frac{1}{\sqrt{3}}\,\frac{l_0}{c^2}\right) = -l_0$

$t_3' = \gamma\left(t_3 - \frac{v}{c^2}x_3\right) = \gamma t_3 = 2\,\frac{1}{\sqrt{3}}\,\frac{l_0}{c} = \frac{2}{\sqrt{3}}\frac{l_0}{c}$ Same as above!!

Sequence of events is ① → ② → ③ _ <u>Note</u> different sequence than farmer's frame



The pole vaulter is right! Both are right!