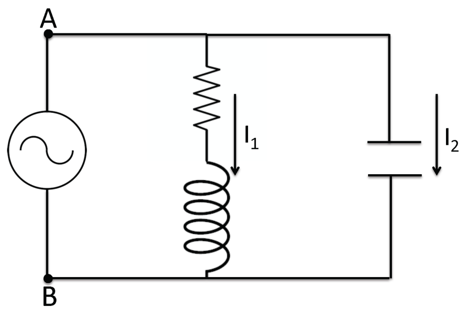
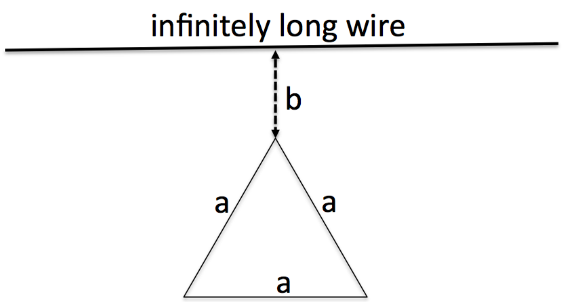
Winter 2018 – UCSB Physics 24 Final

Problem 1
Consider the circuit with a resitor R, an inductor L, and a capacitor C as shown in the figure. The AC generator provides an emf such that $V_A - V_B = E_0 \cos \omega t$. Find $I_1(t)$ and $I_2(t)$. Use the sign convention for current shown in the figure.



Problem 2
Consider a infinitely long straight wire and a triangular loop of wire as shown in the figure. Find the mutual inductance of the wire and the loop.



Problem 3
Consider a square loop of wire, side length 2L. If a current I flows in the loop, find the magnitude of the magnetic field at the center of the loop.

Problem 4
A muon of mass m_μ decays at rest into an electron of mass m_e and a photon (mass zero). (All masses here are **rest masses**).
(a) What is the magnitude of the three-momentum of the electron? And of the photon?
(b) Now imagine that in the rest frame of the muon the photon was emitted in the positive y direction. Boost the system into a frame where the muon is moving with momentum $q = 10 m_\mu c$ in the positive x direction. What is the angle that the photon momentum makes with the x axis in this frame, i.e., what is the angle between the direction of the muon and the direction of the photon.
Note that the decay $\mu \to e\gamma$ does not happen, or, if it does happen, it happens with a probability less than about $4 \cdot 10^{-13}$.

Problem 5

According to Bob, an observer on the Earth, a rocket carrying Martha from Earth to the planet Zorg travels at a speed of $0.8c$ and takes 30 years to reach Zorg. (Zorg is at rest with respect to the earth). Give answers in light-years (for distances) and years (for time intervals).

(a) What is the distance between Earth and Zorg, according to Bob.

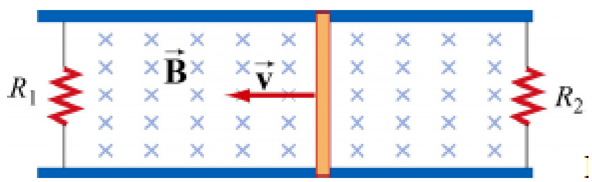
(b) How long does Martha think that the trip takes.

(c) What is the distance between Earth and Zorg, according to Martha?

(d) A third observer, Ralph, thinks that the two events of Martha passing the earth and Martha reaching Zorg occur 25 years apart. In Ralph's reference frame what is the distance between these two events? Ralph's velocity relative to the earth does not change between these two events.

Problem 6

A conducting rod of length L is free to slide on two parallel conducting bars with two resistors R_1 and R_2 connected across the ends of the bars, as shown in the figure. There is a uniform magnetic field B pointing in the page. The bar is being externally pulled to the left at a constant speed v.



(a) Make a sketch and indicate the direction of flow of the induced current.

(b) Find the current through R_1 and the current through R_2.

(c) Find the total power delivered to the resitors.

(d) Find the magnitude of the applied external force needed for the rod to maintain its constant velocity v.

Table of Basic Integrals

Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1 \tag{1}$$

$$\int \frac{1}{x} dx = \ln |x| \tag{2}$$

$$\int u dv = uv - \int v du \tag{3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| \tag{4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1} ((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2| \tag{10}$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \tag{11}$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2}x^2 - \frac{1}{2}a^2 \ln |a^2+x^2| \tag{12}$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x| \tag{15}$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a} \quad dx = \frac{2}{3}(x-a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x \pm a}} \quad dx = 2\sqrt{x \pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}} \quad dx = -2\sqrt{a-x} \tag{19}$$

$$\int x\sqrt{x-a} \quad dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, \text{ or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, \text{ or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases} \tag{20}$$

$$\int \sqrt{ax+b} \quad dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \tag{21}$$

$$\int (ax+b)^{3/2} \quad dx = \frac{2}{5a}(ax+b)^{5/2} \tag{22}$$

$$\int \frac{x}{\sqrt{x \pm a}} \quad dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a} \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}} \quad dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}} \quad dx = \sqrt{x(a+x)} - a \ln \left[\sqrt{x} + \sqrt{x+a}\right] \tag{25}$$

$$\int x\sqrt{ax+b} \quad dx = \frac{2}{15a^2}\left(-2b^2 + abx + 3a^2x^2\right)\sqrt{ax+b} \tag{26}$$

$$\int \sqrt{x(ax+b)} \quad dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2 \ln \left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right] \tag{27}$$

$$\int \sqrt{x^3(ax+b)} \quad dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln \left|a\sqrt{x} + \sqrt{a(ax+b)}\right| \tag{28}$$

$$\int \sqrt{x^2 \pm a^2} \quad dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln \left|x + \sqrt{x^2 \pm a^2}\right| \tag{29}$$

$$\int \sqrt{a^2 - x^2} \quad dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} \tag{30}$$

$$\int x\sqrt{x^2 \pm a^2} \quad dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2} \tag{31}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} \quad dx = \ln \left|x + \sqrt{x^2 \pm a^2}\right| \tag{32}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \quad dx = \sin^{-1} \frac{x}{a} \tag{33}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} \quad dx = \sqrt{x^2 \pm a^2} \tag{34}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} \quad dx = -\sqrt{a^2 - x^2} \tag{35}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} \quad dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln \left|x + \sqrt{x^2 \pm a^2}\right| \tag{36}$$

$$\int \sqrt{ax^2+bx+c} \quad dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}} \ln \left|2ax+b+2\sqrt{a\left(ax^2+bx^+c\right)}\right| \tag{37}$$

$$\int x\sqrt{ax^2+bx+c} \quad dx = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c}\left(-3b^2+2abx+8a\left(c+ax^2\right)\right) \right.$$
$$\left. +3\left(b^3-4abc\right) \ln \left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right) \tag{38}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}} \quad dx = \frac{1}{\sqrt{a}} \ln \left|2ax+b+2\sqrt{a\left(ax^2+bx+c\right)}\right| \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}} \quad dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}} \ln \left|2ax+b+2\sqrt{a\left(ax^2+bx+c\right)}\right| \tag{40}$$

$$\int \frac{dx}{\left(a^2+x^2\right)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{41}$$

Integrals with Logarithms

$$\int \ln ax \quad dx = x \ln ax - x \tag{42}$$

$$\int x \ln x \quad dx = \frac{1}{2}x^2 \ln x - \frac{x^2}{4} \tag{43}$$

$$\int x^2 \ln x \quad dx = \frac{1}{3}x^3 \ln x - \frac{x^3}{9} \tag{44}$$

$$\int x^n \ln x \quad dx = x^{n+1}\left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right), \quad n \neq -1 \tag{45}$$

$$\int \frac{\ln ax}{x} \quad dx = \frac{1}{2}(\ln ax)^2 \tag{46}$$

$$\int \frac{\ln x}{x^2} \quad dx = -\frac{1}{x} - \frac{\ln x}{x} \tag{47}$$

$$\int \ln(ax+b) \quad dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0 \tag{48}$$

$$\int \ln\left(x^2 + a^2\right) \; dx = x \ln\left(x^2 + a^2\right) + 2a \tan^{-1}\frac{x}{a} - 2x \tag{49}$$

$$\int \ln\left(x^2 - a^2\right) \; dx = x \ln\left(x^2 - a^2\right) + a \ln\frac{x+a}{x-a} - 2x \tag{50}$$

$$\int \ln\left(ax^2 + bx + c\right) \quad dx = \frac{1}{a}\sqrt{4ac - b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2 + bx + c\right) \tag{51}$$

$$\int x \ln(ax+b) \quad dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b) \tag{52}$$

$$\int x \ln\left(a^2 - b^2x^2\right) \quad dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2 - b^2x^2\right) \tag{53}$$

$$\int (\ln x)^2 \quad dx = 2x - 2x \ln x + x(\ln x)^2 \tag{54}$$

$$\int (\ln x)^3 \quad dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x \tag{55}$$

$$\int x(\ln x)^2 \quad dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x \tag{56}$$

$$\int x^2(\ln x)^2 \quad dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3 \ln x \tag{57}$$

Integrals with Exponentials

$$\int e^{ax} \quad dx = \frac{1}{a}e^{ax} \tag{58}$$

$$\int \sqrt{x}e^{ax} \quad dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\text{erf}\left(i\sqrt{ax}\right), \text{ where } \text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{59}$$

$$\int xe^x \quad dx = (x-1)\,e^x \tag{60}$$

$$\int xe^{ax} \quad dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{61}$$

$$\int x^2e^x \quad dx = \left(x^2 - 2x + 2\right)e^x \tag{62}$$

$$\int x^2e^{ax} \quad dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{63}$$

$$\int x^3e^x \quad dx = \left(x^3 - 3x^2 + 6x - 6\right)e^x \tag{64}$$

$$\int x^n e^{ax} \quad dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x \tag{65}$$

$$\int x^n e^{ax} \quad dx = \frac{(-1)^n}{a^{n+1}}\Gamma\left[1+n, -ax\right], \text{ where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{d}t \tag{66}$$

$$\int e^{ax^2} \quad dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\text{erf}\left(ix\sqrt{a}\right) \tag{67}$$

$$\int e^{-ax^2} \quad dx = \frac{\sqrt{\pi}}{2\sqrt{a}}\text{erf}\left(x\sqrt{a}\right) \tag{68}$$

$$\int xe^{-ax^2} \quad dx = -\frac{1}{2a}e^{-ax^2} \tag{69}$$

$$\int x^2e^{-ax^2} \quad dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\text{erf}\left(x\sqrt{a}\right) - \frac{x}{2a}e^{-ax^2} \tag{70}$$

Integrals with Trigonometric Functions

$$\int \sin ax \quad dx = -\frac{1}{a}\cos ax \tag{71}$$

$$\int \sin^2 ax \quad dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{72}$$

$$\int \sin^3 ax \quad dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{73}$$

$$\int \sin^n ax \quad dx = -\frac{1}{a}\cos ax \;\; {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] \tag{74}$$

$$\int \cos ax \quad dx = \frac{1}{a}\sin ax \tag{75}$$

$$\int \cos^2 ax \quad dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{76}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{77}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] \tag{78}$$

$$\int \cos x \sin x \quad dx = \frac{1}{2}\sin^2 x + c_1 = -\frac{1}{2}\cos^2 x + c_2 = -\frac{1}{4}\cos 2x + c_3 \tag{79}$$

$$\int \cos ax \sin bx \quad dx = \frac{\cos\left[(a-b)x\right]}{2(a-b)} - \frac{\cos\left[(a+b)x\right]}{2(a+b)}, a \neq b \tag{80}$$

$$\int \sin^2 ax \cos bx \quad dx = -\frac{\sin\left[(2a-b)x\right]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin\left[(2a+b)x\right]}{4(2a+b)} \tag{81}$$

$$\int \sin^2 x \cos x \quad dx = \frac{1}{3}\sin^3 x \tag{82}$$

$$\int \cos^2 ax \sin bx \quad dx = \frac{\cos\left[(2a-b)x\right]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos\left[(2a+b)x\right]}{4(2a+b)} \tag{83}$$

$$\int \cos^2 ax \sin ax \quad dx = -\frac{1}{3a}\cos^3 ax \tag{84}$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin\left[2(a-b)x\right]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin\left[2(a+b)x\right]}{16(a+b)} \tag{85}$$

$$\int \sin^2 ax \cos^2 ax \quad dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{86}$$

$$\int \tan ax \quad dx = -\frac{1}{a}\ln\cos ax \tag{87}$$

$$\int \tan^2 ax \quad dx = -x + \frac{1}{a}\tan ax \tag{88}$$

$$\int \tan^n ax \quad dx = \frac{\tan^{n+1} ax}{a(1+n)} \times_2 F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \tag{89}$$

$$\int \tan^3 ax dx = \frac{1}{a}\ln\cos ax + \frac{1}{2a}\sec^2 ax \tag{90}$$

$$\int \sec x \quad dx = \ln\left|\sec x + \tan x\right| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \tag{91}$$

$$\int \sec^2 ax \quad dx = \frac{1}{a}\tan ax \tag{92}$$

$$\int \sec^3 x \quad dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln\left|\sec x + \tan x\right| \tag{93}$$

$$\int \sec x \tan x \quad dx = \sec x \tag{94}$$

$$\int \sec^2 x \tan x \quad dx = \frac{1}{2}\sec^2 x \tag{95}$$

$$\int \sec^n x \tan x \quad dx = \frac{1}{n}\sec^n x, n \neq 0 \tag{96}$$

$$\int \csc x \quad dx = \ln\left|\tan\frac{x}{2}\right| = \ln\left|\csc x - \cot x\right| + C \tag{97}$$

$$\int \csc^2 ax \quad dx = -\frac{1}{a}\cot ax \tag{98}$$

$$\int \csc^3 x \quad dx = -\frac{1}{2}\cot x\csc x + \frac{1}{2}\ln\left|\csc x - \cot x\right| \tag{99}$$

$$\int \csc^n x\cot x \quad dx = -\frac{1}{n}\csc^n x, n \neq 0 \tag{100}$$

$$\int \sec x\csc x \quad dx = \ln\left|\tan x\right| \tag{101}$$

Products of Trigonometric Functions and Monomials

$$\int x\cos x \quad dx = \cos x + x\sin x \tag{102}$$

$$\int x\cos ax \quad dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \tag{103}$$

$$\int x^2\cos x \quad dx = 2x\cos x + \left(x^2 - 2\right)\sin x \tag{104}$$

$$\int x^2\cos ax \quad dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3}\sin ax \tag{105}$$

$$\int x^n\cos x dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1, -ix) + (-1)^n\Gamma(n+1, ix)\right] \tag{106}$$

$$\int x^n\cos ax \quad dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right] \tag{107}$$

$$\int x\sin x \quad dx = -x\cos x + \sin x \tag{108}$$

$$\int x\sin ax \quad dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} \tag{109}$$

$$\int x^2\sin x \quad dx = \left(2 - x^2\right)\cos x + 2x\sin x \tag{110}$$

$$\int x^2\sin ax \quad dx = \frac{2 - a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} \tag{111}$$

$$\int x^n\sin x \quad dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1, -ix) - (-1)^n\Gamma(n+1, -ix)\right] \tag{112}$$

$$\int x\cos^2 x \quad dx = \frac{x^2}{4} + \frac{1}{8}\cos 2x + \frac{1}{4}x\sin 2x \tag{113}$$

$$\int x\sin^2 x \quad dx = \frac{x^2}{4} - \frac{1}{8}\cos 2x - \frac{1}{4}x\sin 2x \tag{114}$$

$$\int x\tan^2 x \quad dx = -\frac{x^2}{2} + \ln\cos x + x\tan x \tag{115}$$

$$\int x\sec^2 x \quad dx = \ln\cos x + x\tan x \tag{116}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x\sin x \quad dx = \frac{1}{2}e^x(\sin x - \cos x) \tag{117}$$

$$\int e^{bx}\sin ax \quad dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax) \tag{118}$$

$$\int e^x\cos x \quad dx = \frac{1}{2}e^x(\sin x + \cos x) \tag{119}$$

$$\int e^{bx}\cos ax \quad dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax) \tag{120}$$

$$\int xe^x\sin x \quad dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) \tag{121}$$

$$\int xe^x\cos x \quad dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) \tag{122}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax \quad dx = \frac{1}{a}\sinh ax \tag{123}$$

$$\int e^{ax}\cosh bx \quad dx = \begin{cases} \frac{e^{ax}}{a^2-b^2}[a\cosh bx - b\sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \tag{124}$$

$$\int \sinh ax \quad dx = \frac{1}{a}\cosh ax \tag{125}$$

$$\int e^{ax}\sinh bx \quad dx = \begin{cases} \frac{e^{ax}}{a^2-b^2}[-b\cosh bx + a\sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \tag{126}$$

$$\int \tanh ax\, dx = \frac{1}{a}\ln\cosh ax \tag{127}$$

$$\int e^{ax}\tanh bx \quad dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)}(\quad {}_2F_1)\left[1+\frac{a}{2b}, 1, 2+\frac{a}{2b}, -e^{2bx}\right] & \\ \qquad -\frac{e^{ax}}{a}(\quad {}_2F_1)\left[1, \frac{a}{2b}, 1+\frac{a}{2b}, -e^{2bx}\right] & a \neq b \\ \frac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \tag{128}$$

$$\int \cos ax\cosh bx \quad dx = \frac{1}{a^2+b^2}\left[a\sin ax\cosh bx + b\cos ax\sinh bx\right] \tag{129}$$

$$\int \cos ax\sinh bx \quad dx = \frac{1}{a^2+b^2}\left[b\cos ax\cosh bx + a\sin ax\sinh bx\right] \tag{130}$$

$$\int \sin ax\cosh bx \quad dx = \frac{1}{a^2+b^2}\left[-a\cos ax\cosh bx + b\sin ax\sinh bx\right] \tag{131}$$

$$\int \sin ax \sinh bx \quad dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] \tag{132}$$

$$\int \sinh ax \cosh axdx = \frac{1}{4a} [-2ax + \sinh 2ax] \tag{133}$$

$$\int \sinh ax \cosh bx \quad dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] \tag{134}$$