PHYSICS 24
FINAL

1 Before: $M \xrightarrow{\quad p \quad}$ Use $c=1$

After: $\xleftarrow{\quad k_2 \quad} \; \xrightarrow{\quad k_1 \quad}$

Conservation of $\vec{p}$: $k_1 - k_2 = p$ (1)

Conservation of E : $k_1 + k_2 = \sqrt{p^2+m^2}$ (2)

From equation (1): $k_1 = p + k_2$ (3)

Stick it into equation 2

$$p + 2k_2 = \sqrt{p^2+m^2}$$

$$\boxed{k_2 = \frac{1}{2}\left[\sqrt{p^2+m^2} - p\right]}$$

Then, from equation (1)

$$\boxed{k_1 = \frac{1}{2}\left[\sqrt{p^2+m^2} + p\right]}$$

2) S = rest frame of spaceship 2

S' = rest frame of probe A

Let direction of motion of probe A in S be x

Probe B has $V_{xB} = 0.5c \times \cos\theta = 0.3c$

$V_{yB} = 0.5c \times \sin\theta = 0.4c$

$\sin\theta = \sqrt{1-\cos^2\theta} = \frac{4}{5}$

u = velocity of S' wrt S = $0.8c$ in the x-direction

We are going to need this quantity

$$\sqrt{1-u^2/c^2} = \sqrt{1-0.8^2} = 0.6$$

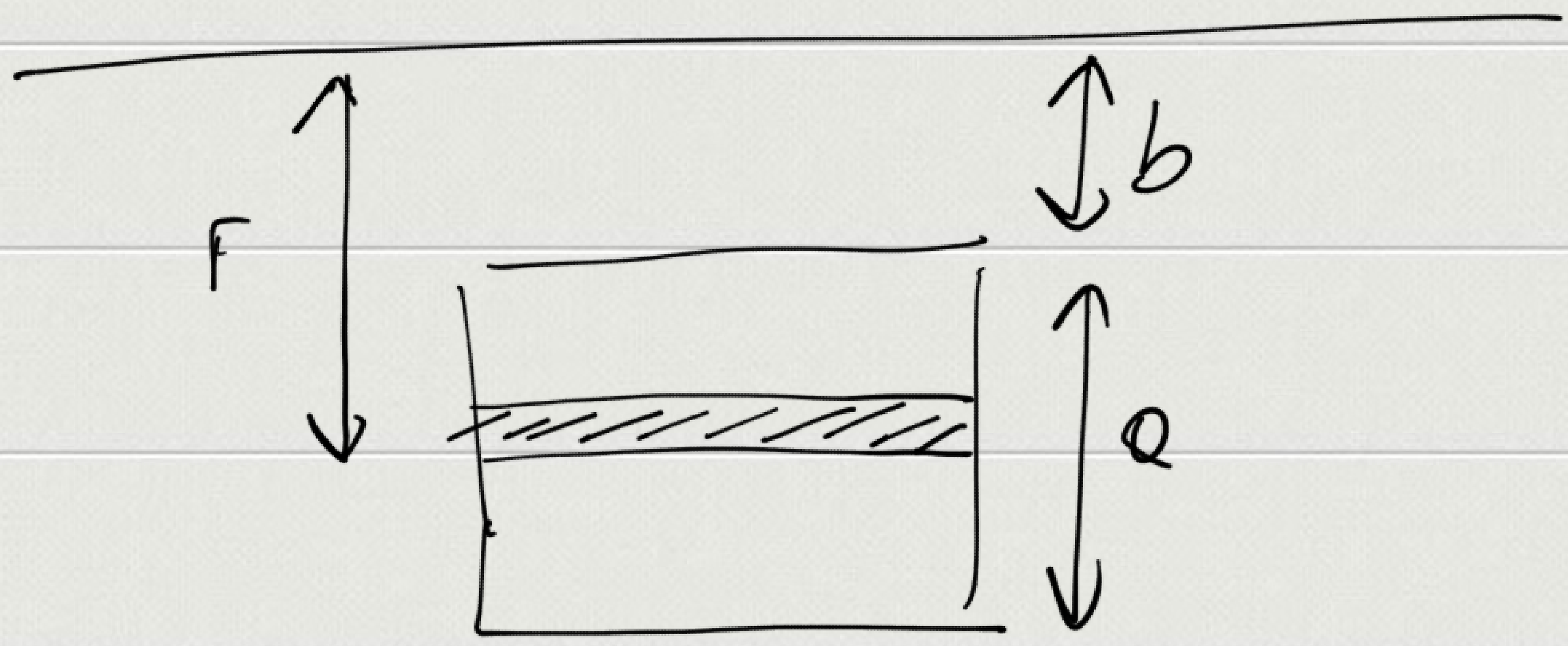
$$V'_{xB} = \frac{V_{xB} - u}{1 - V_{xB}u/c^2} = \frac{0.3-0.8}{1 - 0.3 \cdot 0.8}c = -0.658c$$

$$V'_{yB} = \frac{V_{yB}\sqrt{1-u^2/c^2}}{1 - V_{xB}u/c^2} = \frac{0.4 \cdot 0.6}{1-0.3\,0.8} = 0.316c$$

$$V'_B = \sqrt{0.658^2 + 0.316^2} = \underline{\underline{0.730\,c}} \checkmark$$

3 We calculate the flux of the B-field due to the wire through the square loop for a current I going through the wire. Then

$$M = \Phi / I$$



$d\Phi$ = flux through shaded area = $B(r)\, dA$

$$d\Phi = B(r)\, a\, dr \qquad B(r) = \frac{\mu_0 I}{2\pi r}$$

$$d\Phi = \frac{\mu_0 I a}{2\pi} \frac{dr}{r}$$

$$\Phi = \frac{\mu_0 I a}{2\pi} \int_b^{a+b} \frac{dr}{r} = \frac{\mu_0 I a}{2\pi} \log \frac{a+b}{a}$$

$$\Rightarrow M = \frac{\Phi}{I} \qquad \boxed{M = \frac{\mu_0 a}{2\pi} \log \frac{a+b}{a}}$$

4 (a) $J = \frac{I}{\pi R^2} = \frac{\alpha t}{\pi R^2}$ 4

$$E = \frac{J}{\sigma} = \frac{\alpha t}{\pi R^2 \sigma}$$

Displacement current $J_D = \varepsilon_0 \frac{\partial E}{\partial t}$ $(r<R)$

$$\boxed{J_D = \frac{\varepsilon_0 \alpha}{\pi R^2 \sigma}}$$

Independent of r for $r<R$. Independent of t

(b) Maxwell eqtn $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \vec{J}_D \right)$$

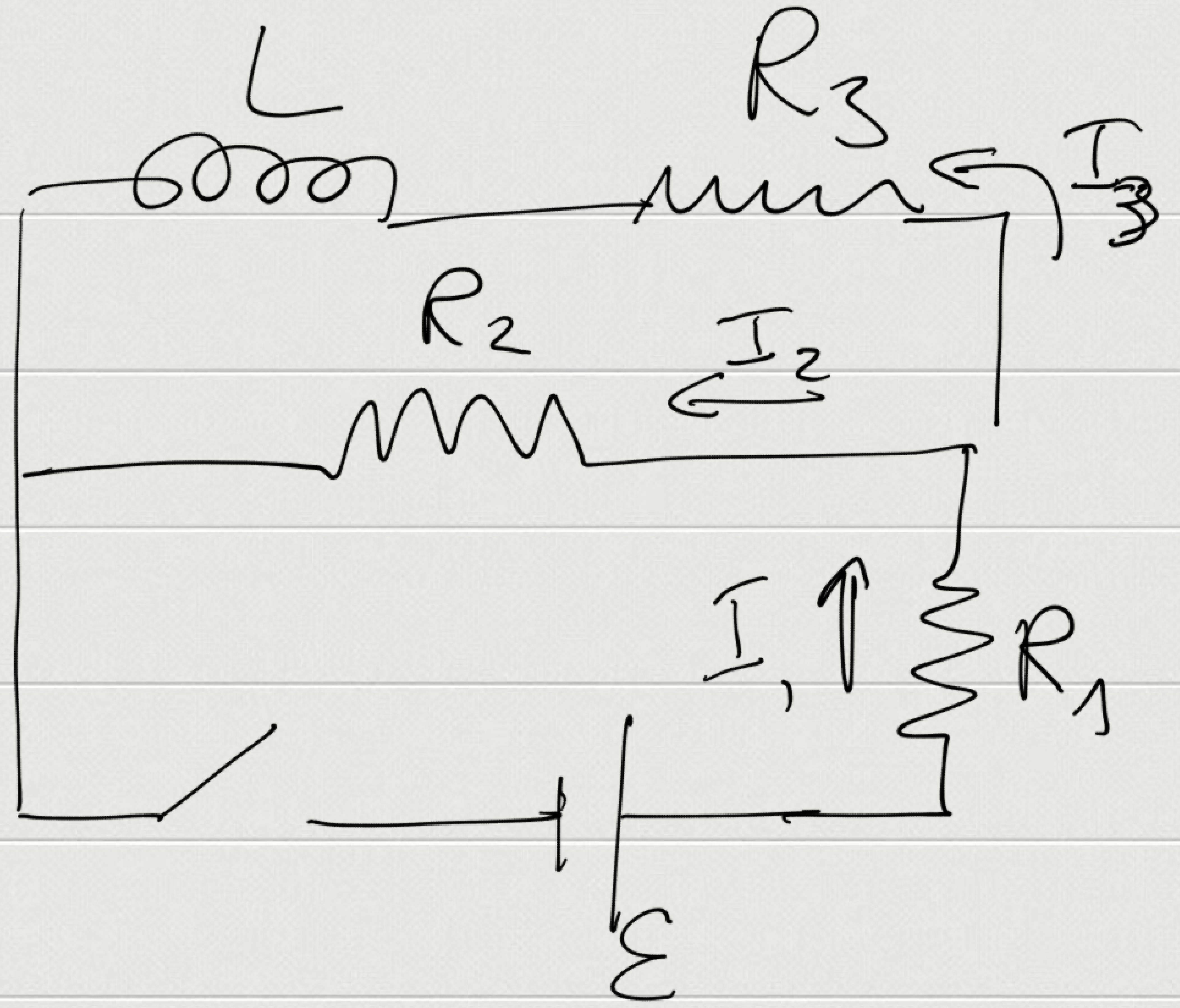
When turning this into an integral equation this becomes $\int \vec{B} \, d\vec{l} = \mu_0 \left(I_{enclosed} + I_{D\_enclosed} \right)$

Take path as circle of radius $r > R$

Also $I_{D\_enclosed} = J_D \cdot \pi R^2 = \frac{\varepsilon_0 \alpha}{\sigma}$

$$B \, 2\pi r = \mu_0 \alpha t + \frac{\mu_0 \varepsilon_0 \alpha}{\sigma}$$

$$\boxed{B = \frac{\mu_0 \alpha}{2\pi r} \left[t + \frac{\varepsilon_0}{\sigma} \right]}$$



(a) Immediately after the switch is closed $I_3 = 0$

$$I_1 = I_2 = \frac{\varepsilon}{R_1 + R_2}$$

(b) A long time after the switch is closed L doesnt do anything anymore

$$I_1 = I_2 + I_3 \quad (1)$$

$$I_1 R_1 + I_2 R_2 = \varepsilon \quad (2)$$

$$I_1 R_1 + I_3 R_3 = \varepsilon \quad (3)$$

From (2) & (3) $\Rightarrow I_2 R_2 = I_3 R_3 \quad (4)$

Use eqtn (1) to substitute into (2)

$$I_2R_1 + I_3R_1 + I_2R_2 = \mathcal{E}$$

But from (4) $I_3 = I_2 \frac{R_2}{R_3}$

$$I_2R_1 + I_2\frac{R_2R_1}{R_3} + I_2R_2 = \mathcal{E}$$

$$\boxed{I_2 = \frac{R_3\,\mathcal{E}}{R_1R_3 + R_1R_2 + R_2R_3}}$$

$$\boxed{I_3 = \frac{R_2\,\mathcal{E}}{R_1R_3 + R_1R_2 + R_2R_3}}$$

Then, using equation (1):

$$\boxed{I_1 = \frac{(R_2+R_3)\,\mathcal{E}}{R_1R_3 + R_1R_2 + R_2R_3}}$$

6 (a) $I_R = \frac{\mathcal{E}_0}{R} \cos\omega t$

(b) C and L in series have impedance

$$Z = i\omega L + \frac{1}{i\omega C} = i\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = e^{i\pi/2}\left(\omega L - \frac{1}{\omega C}\right)$$

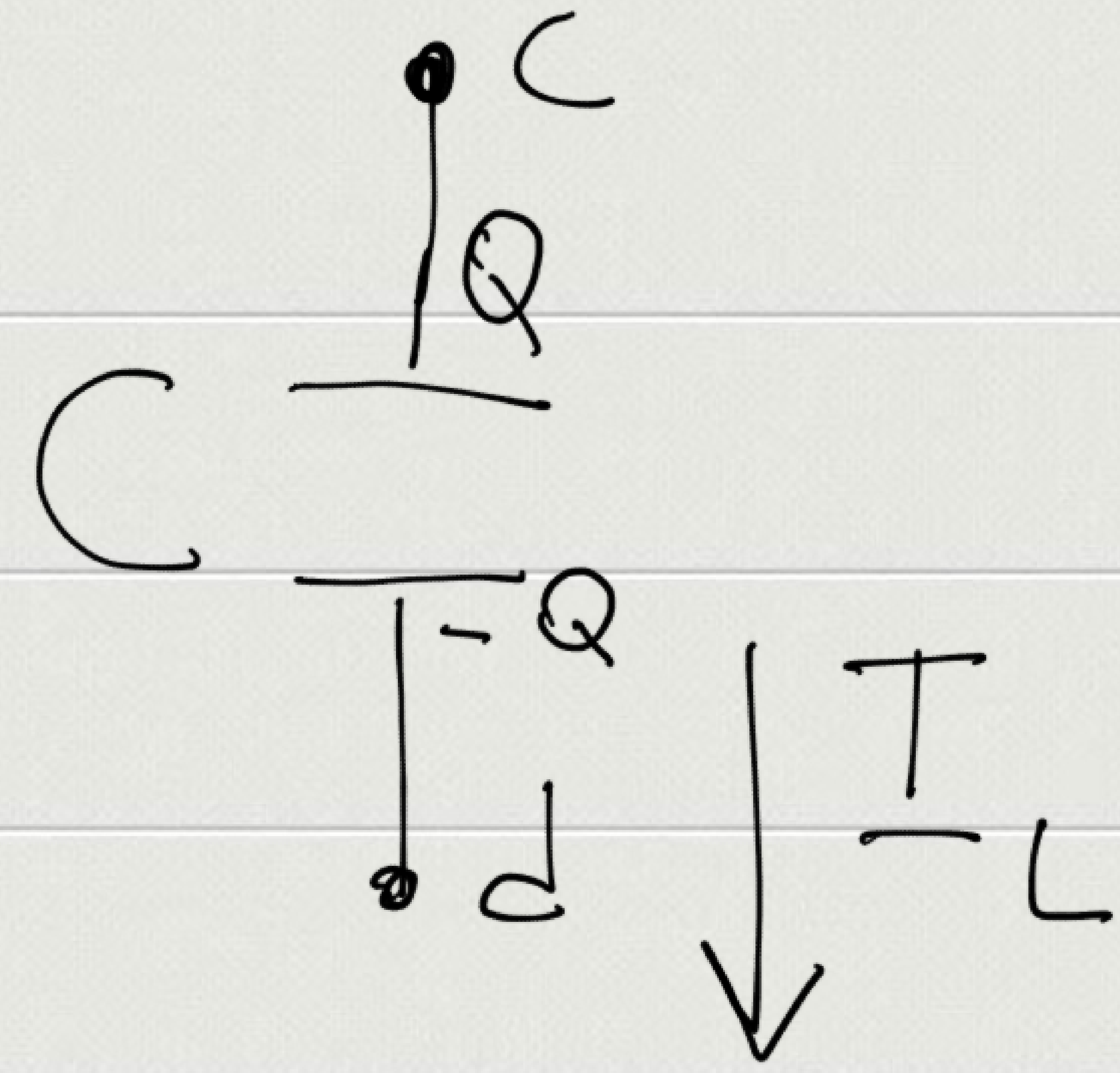
$$I_L = \mathrm{Real}\left(\frac{\mathcal{E}_0 e^{i\omega t}}{Z}\right)$$

$$I_L = \mathrm{Real}\, \frac{\mathcal{E}_0}{\omega L - \frac{1}{\omega C}}\, e^{i(\omega t - \pi/2)}$$

$$I_L = \frac{\mathcal{E}_0}{\omega L - \frac{1}{\omega C}} \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{I_L = \frac{\mathcal{E}_0}{\omega L - 1/\omega C} \sin\omega t}$$

(c)



$$V_c - V_d = \frac{Q}{C}$$

$$I_L = \frac{dQ}{dt} = \frac{\mathcal{E}_0}{\omega L - \frac{1}{\omega C}} \sin\omega t$$

$$\Rightarrow Q = \frac{\mathcal{E}_0}{\frac{1}{\omega C} - \omega L} \frac{1}{\omega} \cos\omega t = \frac{\mathcal{E}_0 C}{1-\omega^2 LC} \cos\omega t$$

$$\Rightarrow \boxed{V_c - V_d = \frac{\mathcal{E}_0}{1-\omega^2 LC} \cos\omega t} \quad (1)$$

Alternatively: $(V_c - V_d) + \frac{L\,dI}{dt} = \mathcal{E}_0 \cos\omega t$

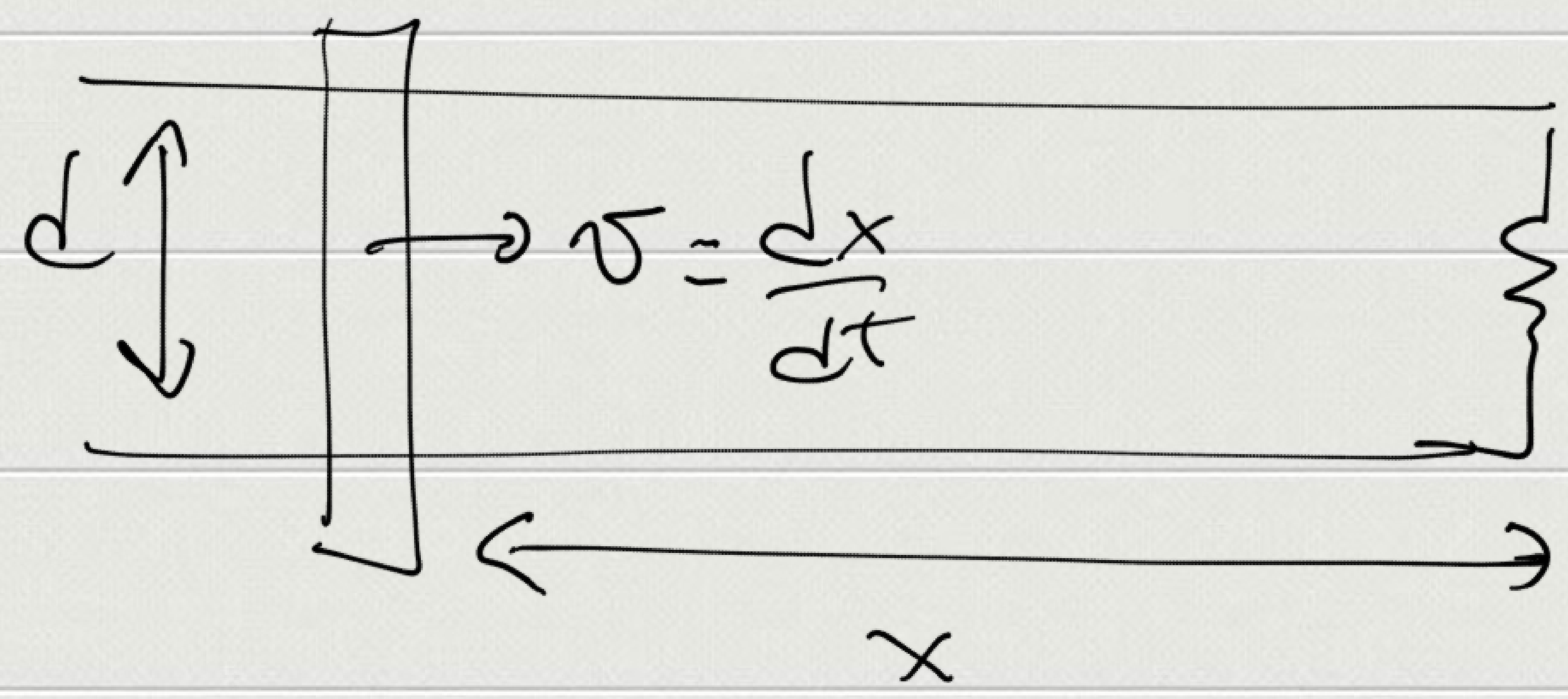
$$V_c - V_d = \mathcal{E}_0 \cos\omega t - L\omega\mathcal{E}_0 \cos\omega t / (\omega L - 1/\omega C)$$

$$V_c - V_d = \mathcal{E}_0 \cos\omega t \left[1 - \frac{1}{1 - \frac{1}{\omega^2 LC}}\right]$$

$$\boxed{V_c - V_d = \frac{\mathcal{E}_0}{1-\omega^2 LC} \cos\omega t}$$

Sanity check: for $L=0$, $V_c - V_d$ should just equal the emf. Indeed, it does!!

7 (a) $\mathcal{E} = -\frac{d\phi}{dt}$ $I = \frac{\mathcal{E}}{R}$ 9



$\phi = x d B$ $\frac{d\phi}{dt} = dB\frac{dx}{dt} = dBv$

$\Rightarrow \mathcal{E} = -dBv$ $\boxed{I = \frac{Bdv}{R}}$

where I dropped the minus sign

(b) $F = IBd = \frac{B^2 d^2 v}{R}$

(c) $\text{Power} = Fv = \frac{B^2 d^2 v^2}{R}$

Note also $\text{Power} = I^2 R = \frac{B^2 d^2 v^2}{R^2} R$

$\text{Power} = \frac{B^2 d^2 v^2}{R}$ ✓