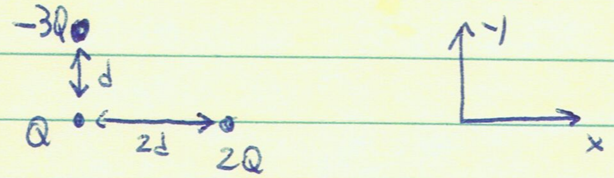
PHYSICS 24 FINAL
2013 WINTER QUARTER

1

(1)



~~Start~~ Pick origin at charge Q. $\boxed{\vec{P} = 4Qd\,\hat{x} - 3Qd\,\hat{y}}$

Can verify that choice of origin does not matter

Pick origin on charge 2Q

$$\vec{P} = -2dQ\,\hat{x} + (+6dQ\,\hat{x} - 3Qd\,\hat{y}) = \underline{4Qd\,\hat{x} - 3Qd\,\hat{y}}$$

Pick origin on charge −3Q

$$\vec{P} = -Qd\,\hat{y} + (+4Qd\,\hat{x} - 2Qd\,\hat{y}) = \underline{4Qd\,\hat{x} - 3Qd\,\hat{y}}$$

(2) (a) In rest frame (work with $c=1$)

$$\underset{\sim}{P}_{initial} = (\vec{0} \;\; M)$$

$$\underset{\sim}{P}_{final} = \underbrace{(\vec{q} \;\; \sqrt{q^2+m^2})}_{\text{electron}} + \underbrace{(-\vec{q} \;\; q)}_{\text{neutrino}} = (\vec{0} \;\; q + \sqrt{q^2+m^2})$$

Conservation of Energy: $M = q + \sqrt{q^2+m^2}$

$$M - q = \sqrt{q^2+m^2}$$

$$M^2 + q^2 - 2Mq = q^2 + m^2 \qquad \boxed{q = \frac{M^2 - m^2}{2M}}$$

(This exact problem was on the midterm also!)

(b) The boost is in the +ve x-direction
The β of the boost is the same as the β of the muon.

$$p' = \gamma M \beta \Rightarrow \boxed{\gamma\beta = p'/M} \qquad \cancel{\text{Also}}$$

$$\text{Also } \gamma = \frac{E'}{M} \Rightarrow \boxed{\gamma = \frac{\sqrt{p'^2 + M^2}}{M}}$$

The boost is in the x-direction. The momentum of the neutrino in the rest frame was $-q$ in the x-direction. There is no y and no z momentum anywhere

$$k' = \gamma(-q + \beta q)$$ (The energy of $\nu\bar{\nu} = q$ in rest frame)

$$\boxed{k' = \frac{p' - \sqrt{p'^2 + M^2}}{M}\, q}$$

③


(a) $I_1 = ?$ $\quad L \frac{dI_1}{dt} = \mathcal{E}_0 \cos\omega t$

$$\boxed{I_1 = \frac{\mathcal{E}_0}{\omega L} \sin\omega t}$$

(b) R C = Z $\qquad Z = R - \frac{i}{\omega C} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}\, e^{-i\phi}$

with $\phi = -\tan^{-1}\left(\frac{1}{\omega R C}\right)$ $\qquad$ ~~with $\phi = \tan^{-1}\left(\frac{\omega C}{R}\right)$~~

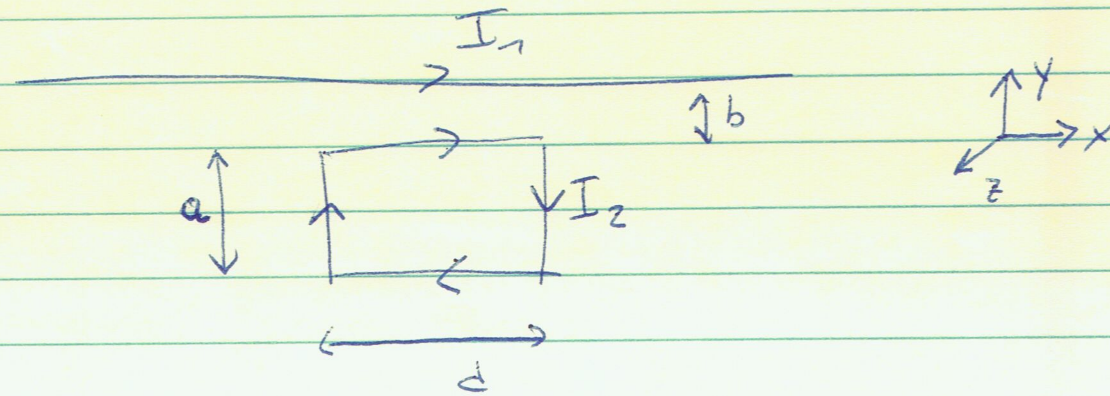
$$I_2 = \frac{\mathcal{E}_0 e^{i\omega t}}{Z} = \frac{\mathcal{E}_0 \, e^{i\omega t}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}} \, e^{-i\phi}}$$

$$I_2 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \, e^{i(\omega t + \phi)}$$

Taking real parts:

$$\boxed{I_2 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos(\omega t + \phi) \qquad \phi = \tan^{-1} \frac{1}{\omega R C}}$$

4



There is no net force on the vertical sides because the current is flowing in opposite directions and the geometry is the same

$\vec{B} = -\frac{2 I_1}{c r} \hat{z}$ in the neighborhood of the loop, where r = distance from loop

The force on each loop segment is $d\vec{F} = \frac{I}{c} d\vec{\ell} \times \vec{B}$

4

$$\vec{F} = \frac{I_2}{c}\left(d\hat{x}\right) \times \left(-\frac{2I_1}{cb}\hat{z}\right) + \frac{I_2}{c}\left(-d\hat{x}\right) \times \left(-\frac{2I_1}{c(b+a)}\hat{z}\right)$$

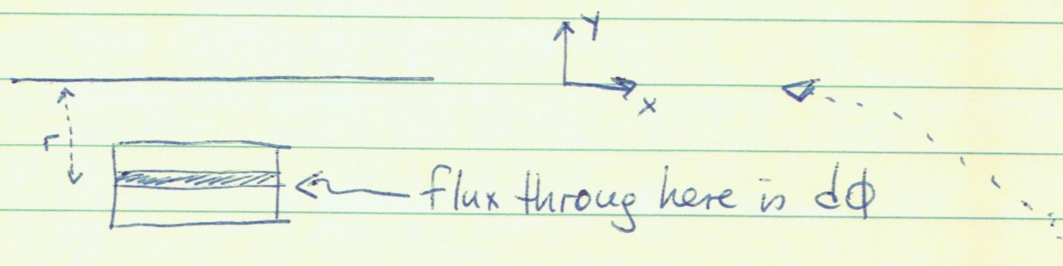
$$\vec{F} = \frac{2I_1I_2d}{c^2}\left[\frac{1}{a+b} - \frac{1}{b}\right]\left(\hat{x}\times\hat{z}\right)$$

But $\hat{x} \times \hat{z} = -\hat{y}$

$$\vec{F} = \frac{2I_1I_2d}{c^2}\left[\frac{1}{b} - \frac{1}{a+b}\right]\hat{y} = \frac{2I_1I_2d}{c^2}\,\frac{a+b-b}{(a+b)b}\,\hat{y}$$

$$\boxed{\vec{F} = \frac{2I_1I_2\,a\,d}{c^2\,b(a+b)}\,\hat{y}}$$

5



$$d\Phi = B(r)\,dA = B(r)\,d\,dy = \frac{2I_1d}{c}\,\frac{dy}{r}$$ But $|r| = |y|$

Add all the strips $$\Phi = \frac{2I_1d}{c}\int_{-b}^{-(a+b)}\frac{dy}{y}$$

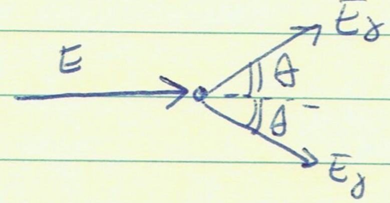
$$\boxed{\Phi = \frac{2I_1d}{c}\log\left(\frac{a+b}{b}\right)}$$

working with $c=1$

6 (a) $E_{INITIAL} = E + m$

$E_{FINAL} = 2E_\gamma$ $\Rightarrow$ CONSERVATION OF E $\boxed{E_\gamma = \frac{1}{2}(E+m)}$

(b)



Because there cannot be any vertical momentum in the final state, since the two photons have the same energy, the two angles must be the same

Conservation of $\vec{P}$: $\sqrt{E^2-m^2} = 2E_\gamma \cos\theta$ $\qquad \cos\theta = \frac{\sqrt{E^2-m^2}}{2E_\gamma}$

$\cos\theta = \frac{\sqrt{(E+m)(E-m)}}{E+m}$ $\qquad \boxed{\cos\theta = \sqrt{\frac{E-m}{E+m}}}$

7 (a) The induced current I_c must be such as to counteract ~~minimize~~ the change in flux. Since the solenoid field is to the right and is decreasing, I_c must be such that it makes a $\vec{B}$ field ~~to the~~ pointing to the right.
Using the right-hand-rule, this means I_c goes from LEFT TO RIGHT

(b) $\Phi = 5 B \pi \left(\frac{d}{2}\right)^2 = \frac{5}{4}\pi d^2 B$

5 turns of coil

$B = \frac{4\pi I_s (N/L)}{c}$ $\longrightarrow$ $\Phi(t) = \frac{5\pi^2 N d^2}{LC} I_s(t)$

induced emf

$$I_c = \frac{\mathcal{E}}{R} = \frac{1}{R}\left(-\frac{1}{c}\right)\frac{d\phi}{dt} = -\frac{1}{Rc}\frac{d\phi}{dt}$$

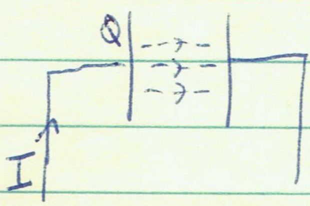
this sign is meaningless we have already figured out the direction....

$$I_c = -\frac{1}{Rc}\frac{5\pi^2 N d^2}{Lc}\frac{dI_s}{dt}.$$

But $I_s = I_0 e^{-t/\tau} \Rightarrow \frac{dI_s}{dt} = -\frac{1}{\tau} I_0 e^{-t/\tau}$

$$\boxed{I_c = \frac{5\pi^2 N d^2}{R\tau L c^2} e^{-t/\tau}}$$

8



(a) $E = \frac{V}{d} = \frac{Q/c}{d} = \frac{Q}{d}\frac{4d}{R^2} = \frac{4Q}{R^2}$

$$\frac{\partial E}{\partial t} = \frac{4}{R^2}\frac{\partial Q}{\partial t} = \frac{4I}{R^2}$$

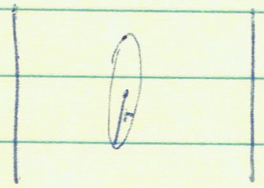
$$\Rightarrow \quad J_d = \frac{1}{4\pi}\frac{\partial E}{\partial t} = \frac{I}{\pi R^2}$$

(b) Inside the capacitor $\vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4I}{cR^2}\hat{n}$

where $\hat{n}$ is a unit vector pointing in the direction perpendicular to the plates

Stoke's theorem $\int \vec{B}\, d\vec{l} = \int (\vec{\nabla} \times \vec{B})\, d\vec{A}$

Take circular path of radius r inside capacitor



$$\int \vec{B}\, d\vec{l} = 2\pi r\, B(r)$$

These two vectors are parallel

$$\int (\vec{\nabla} \times \vec{B})\, d\vec{A} = \frac{4I}{cR^2} \int \hat{n}\, d\vec{A} = \frac{4I}{cR^2} \pi r^2$$

$$\Rightarrow 2\pi r\, B(r) = \frac{4I}{cR^2} \pi r^2 \qquad \boxed{B(r) = \frac{2Ir}{cR^2}}$$