ISOSPIN SYMMETRY

Nuclear physics, 1930's

m(proton) $\simeq$ m(neutron)

As far as strong interactions, they are ~ same

Of course, very different for E&M _ p is charged _

But strong interactions (eg in nuclei) is much "stronger" than E&M (or weak)

There is then a good symmetry that if we interchange $p \leftrightarrow n$ nothing changes (as far as strong interaction is concerned)

Heisenberg proposed that n & p could be considered single entity of "NUCLEON"

$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\qquad$ (does it look like spin?)

Physics invariant under $p \leftrightarrow n$ _ But more than that _ Invariant under rotation of states

$$\begin{pmatrix} p \\ n \end{pmatrix} \longrightarrow U \begin{pmatrix} p \\ n \end{pmatrix}$$

U = 2 × 2 complex matrix

⇒ 4 complex numbers ⇒ 8 real numbers

But it must be unitary $U^\dagger U = 1$

That's 4 constraints ⇒ 8-4 real parameters

One of the parameter is trivial

$$U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{i\alpha}$$

This U is not really a rotation. Just adding a meaningless phase. Therefore 3 real parameters

It can be shown that U can be written as

$$e^{i\vec{\alpha}\vec{I}} \quad \text{or} \quad e^{-i\vec{\beta}\vec{I}} \quad (\vec{\alpha} = -\vec{\beta})$$

where $\vec{\alpha} = (\alpha_1, \alpha_2 \alpha_3)$ real vector

$$\vec{I} = \frac{1}{2}\vec{\sigma}$$

$\vec{\sigma}$ Pauli matrices $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\vec{I}$ are the generators of rotations in isospin space.

Compare with rotation operator $R = e^{-\frac{i\vec{\theta}\cdot\vec{L}}{\hbar}}$

For $L = \frac{1}{2}$, ie spin $(S = \frac{1}{2})$

$$R = e^{-\frac{i\vec{\theta}\cdot\vec{S}}{\hbar}} \qquad \vec{S} = \frac{\hbar}{2}\vec{\sigma} \quad (!!)$$

except for factors of $\hbar$ $\vec{S}$ and $\vec{I}$ have the same matrix representation. Careful, they are not the same operator!

Therefore p and n behave just like spin up and spin down in their algebra.

Can combine p and n just like we combine spins to make states of $I = 1, 0$ (two nucleons), $I = \frac{3}{2}$ or $\frac{1}{2}$ (three nucleons) etc

Same Clebsch Gordan coefficients

BUT THERE IS MORE!

If we have rotational symmetry:

- ang momentum is conserved (J)
- we conventionally choose basis states that are eigenstates of J^2 with eigenvalues $j(j+1)\hbar^2$ and j integer or half integer, <u>and</u> eigenstates of J_3 with eigenvalues $j\hbar, (j-1)\hbar, \ldots -j(\hbar)$

If we have isospin symmetry

- isospin is conserved (I)
- eigenstates of I^2, I_3 with same properties as above (except for factors $\hbar$)

The conservation of I is realized in scattering processes, eg $AB \to CD$ or $AB \to CDE \ldots$ and in decay processes $A \to BC(D, E \ldots)$

I_3 is also conserved (of course)

And the strength of the process (cross

section in case of scattering, inverse of lifetime in the case of decays) is <u>independent</u> of I_3

Why? because rotating in isospin space changes I_3, and this rotation is a symmetry

Now we understand isospin a bit better
The fundamental isospin doublet is the up/down quark doublet $(I = \frac{1}{2})$

$\begin{pmatrix} u \\ d \end{pmatrix}$ They have different charges $Q(u) = \frac{2}{3}e$ $Q(d) = -\frac{1}{3}e$, almost same mass, and same strong interaction

If I combine 3 quarks

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = (1 + 0) \times \frac{1}{2} = \frac{3}{2} + \frac{1}{2} + \frac{1}{2}$$

I can get $I = \frac{3}{2}$ or $\frac{1}{2}$ -

The proton and neutron are the $I=\frac{1}{2}$ combination of uud and udd

You would think that then there should also be 3-quark combinations with $I=\frac{3}{2}$ - You would be right

Δ^{++} uuu $I=\frac{3}{2}$ $I_3=\frac{3}{2}$

Δ^{+} uud $I=\frac{3}{2}$ $I_3=\frac{1}{2}$

Δ^{0} udd $I=\frac{3}{2}$ $I_3=-\frac{1}{2}$

same content as proton & neutron but different states!!

Δ^{-} ddd $I=\frac{3}{2}$ $I_3=-\frac{3}{2}$

Other isospin multiplets exist by including the antiquark doublet $\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$

(The minus sign is "technical") ☺

eg π^+ $u\bar{d}$ $I=1$ $I_3=1$

π^0 $(u\bar{u}-d\bar{d})/\sqrt{2}$ $I=1$ $I_3=0$

π^- $\bar{u}d$ $I=1$ $I_3=-1$

Remember: Isospin symmetry is not an *exact* symmetry even neglecting E&M because the masses of the u and d quark are not exactly the same –

Isospin symmetry can be extended to a larger symmetry of rotations between 3 states, once we consider the strange quark

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \longrightarrow U \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

An even more approximate symmetry since $m(s) \gg m(u), m(d)$

$m(u) \cong 2\ MeV/c^2$

$m(d) \simeq 5\ MeV/c^2$

$m(s) \simeq 96\ MeV/c^2$

Gell-Mann and Zweig were able to make sense of the plethora of particles that were being discovered in experiments, by postulating the existence of "quarks" as fundamental building blocks
(Although Gell Mann did not think that they were real, at least initially - He thought that they were "mathematical objects")

The prediction and subsequent discovery of the Ω^- particle (a $|sss\rangle$ state) was the triumph of the theory