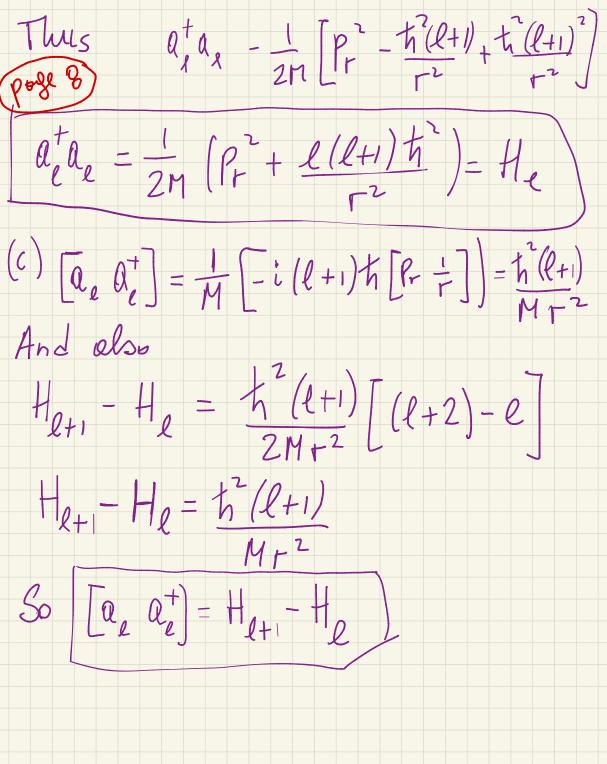


 $\begin{bmatrix} P_r + \\ F \end{bmatrix} = -i\hbar \begin{bmatrix} \partial_r + \\ \partial_r + \end{bmatrix} = \frac{i\hbar}{\Gamma^2}$ Then $\begin{bmatrix} a_e & a_e^+ \end{bmatrix} = \frac{1}{2mhw} \left(-i(l+1)h\left[l_r + \right] + \frac{1}{2mhw} \left(-i(l+1)h\left[l_r + \right] + \frac{1}{2mhw} \right] \right)$ + imw [Pr r] + +i(l+1)th[+Pr] + -imw[r Pr]) $\begin{bmatrix} a_1 & a_1 \\ a_2 & b_1 \end{bmatrix} = \frac{1}{2m\hbar\omega} \begin{bmatrix} 2\hbar^2(l+1) + 2\hbar m\omega \end{bmatrix}$ $\left[\begin{bmatrix} a_{\ell} & a_{\ell}^{\dagger} \end{bmatrix} = \left(\frac{\ell + i}{m W r^{2}} + 1 \right)^{t} \\ \frac{1}{m W r^{2}} + 1 \right]$

(b) $\hbar w a_{\ell}^{\dagger} a_{\ell} = \frac{1}{2m} \left(-i R - (l+1) \frac{h}{r} + m w r \right)$. $\left(\frac{+ir}{r}-\frac{l+1}{r}+mwr\right)$ $= \frac{1}{2m} \left(P_r + i(l+1)h P_r(l+1) - imw P_r r \right)$ $-i(l+1)h + P_r + (l+1) + +$ $-2(lti)\hbar mwtimwrptm^2wr^2)$ $= \frac{1}{2m} \left(\frac{P_r}{r} + i \frac{h}{r} \left(\frac{l+1}{r} \right) \left[\frac{P_r}{r} + \frac{l}{r} \right] - i \frac{mw}{r} \left[\frac{P_r}{r} \right] \right)$ $+\left(l+1\right)\frac{h}{r^{2}}-2\left(l+1\right)hmw+mw^{2}r^{2}\right)$ $= \frac{1}{2m} \left(\frac{1}{r^2} - \frac{h^2(l+1)}{r^2} - \frac{hmw}{r^2} + \frac{h^2(l+1)^2}{r^2} + \frac{h^2(l+1)^$ $-2(l+i)\hbar m\omega + m\omega r)$

 $hw a_{\ell}^{\dagger}a_{\ell} = \frac{f_{r}^{2}}{2m} + \frac{h^{2}l(\ell+1)}{2mr^{2}} + \frac{1}{2}mw^{2}r^{2} - (\ell+\frac{3}{2})hw$ (poge 6) $hwa_{\ell}^{+}a_{\ell} = H_{\ell} - (\ell + \frac{3}{2})hw$ $=) H_{\ell} = \hbar \omega \left[a_{\ell}^{\dagger} a_{\ell} + \left(l + \frac{3}{2} \right) \right]$ (C) Take Ret1 such that He+1 Re+, = ERe+1 Then $a_{\ell}^{\dagger} H_{\ell+1} R_{\ell+1} = E a_{\ell}^{\dagger} R_{\ell+1}$ From lecture QeHe = He+ Re + hwRe Teking hermitien conjugate of both sides $H_{e} q_{e}^{\dagger} = q_{e}^{\dagger} H_{e+} + h w a_{e}^{\dagger}$ Take both left and right hand side and

meke then oct on Reti (poge 7) This gives $\mathcal{H}_{\ell}\left(\mathfrak{a}_{\ell}^{+}\mathfrak{k}_{\ell+1}\right) = \left(\mathcal{E} + \mathcal{L}_{W}\right)\left(\mathfrak{a}_{\ell}^{+}\mathfrak{k}_{\ell+1}\right)$ Thus de Re+1 is a state of energy Et trus and en eigenfunction of He (3) (e) This is taking what we have come in class and setting V(r)=10 (b) $\theta_{e}^{\dagger}\theta_{e} = \frac{1}{2m} \left(P_{F}^{2} + h(l+i)f_{+}^{2} \cdot (l+i)[P_{F} +] \right)$ The commutator [Pr +] = it From problem 2(0)

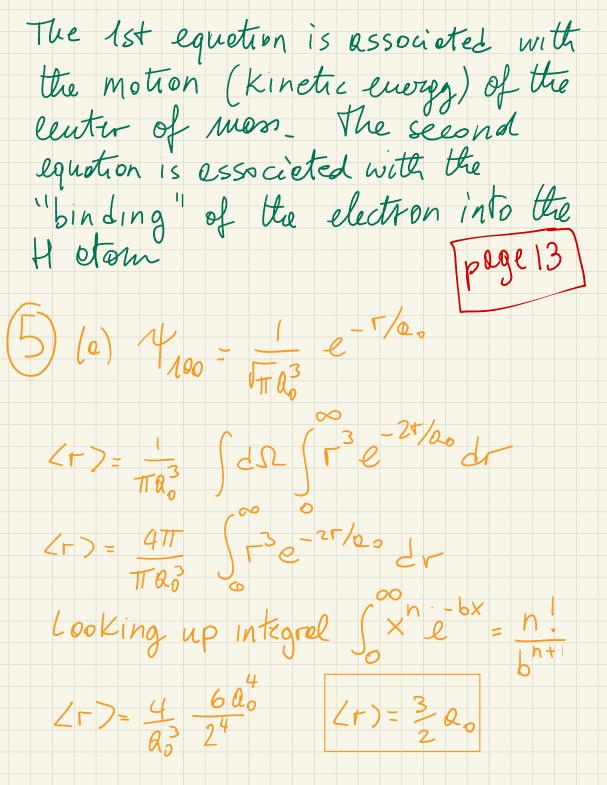


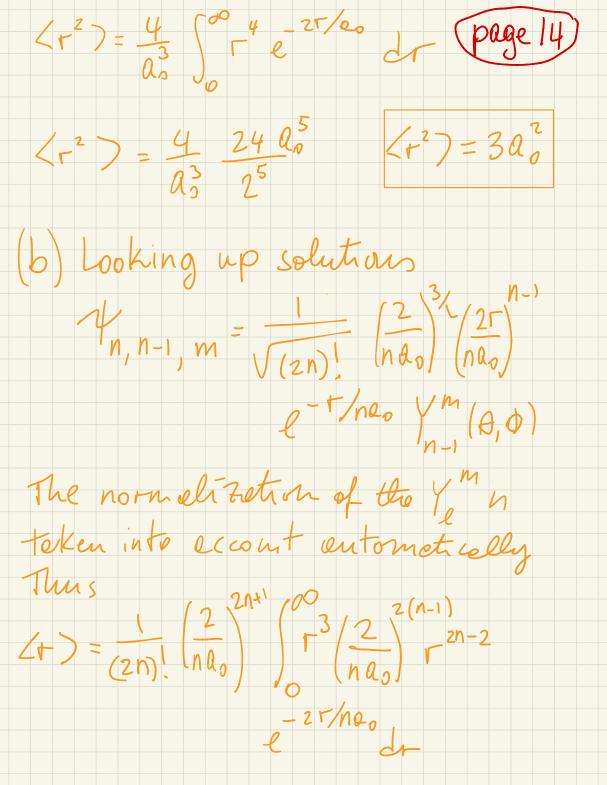
(d) $[a_e H_e] = [a_e a_e^+ a_e] = Puge q$ $= a_e a_e^{\dagger} a_e - a_e^{\dagger} a_e a_e = [a_e a_e^{\dagger}] a_e$ $\begin{bmatrix} a_e & H_e \end{bmatrix} = \begin{pmatrix} H_{e+1} - H_e \end{pmatrix} a_e$ QeHe-Hele= Herile-Hele $a_{e}H_{e} = H_{e+1}a_{e}$ Then if we have a solution Here = Ere acting with a on both sides gives Het, (aeke) = E (aeke) So alke is a solth with the serve E but with l-1 l+1

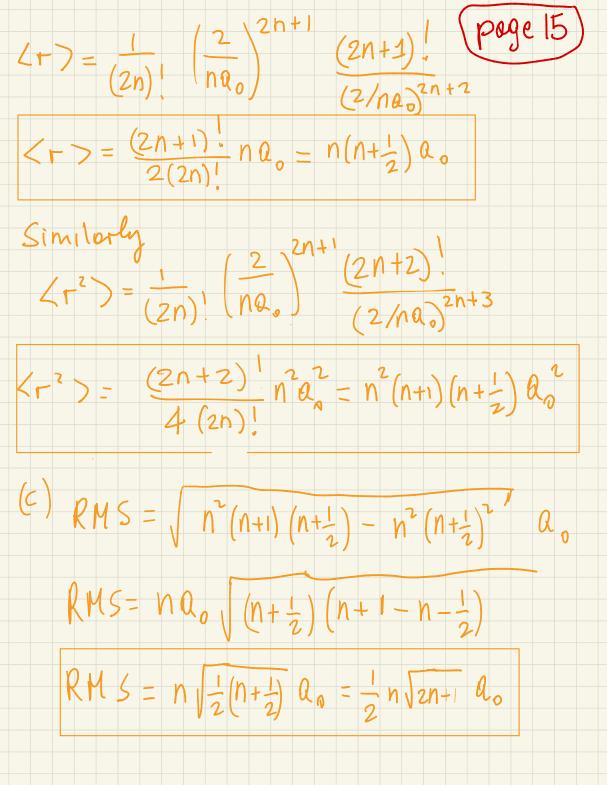
(e) Sturting with a solution (El) We can build more and more (page 10) solutions with some E but higher end higher l_ This seems like en impossibility since eg closically L= mVr so you'd think that increasing L means increasing $E = \frac{1}{2}mv^2$ end you connot do that at intinitum_ But there is a "r fector in L_ So os L cou become as longe as you want without heving v become inconsistent with fixed E= 1 muz, just by increasing

(4) From the chain rule page 11 (a) $\frac{\partial}{\partial x_e} = \frac{\partial \overline{x}}{\partial \overline{x}_e} \frac{\partial}{\partial \overline{x}} + \frac{\partial \overline{r}}{\partial \overline{z}_e} \frac{\partial}{\partial \overline{r}} = \frac{m_e}{m_e + m_p} \frac{\partial}{\partial \overline{x}} + \frac{\partial}{\partial \overline{r}}$ Then taking the dot product of each side with itseff $\nabla_{e}^{2} = \left(\frac{m_{e}}{m_{e}+m_{p}}\right)^{2} \nabla_{x}^{2} + \nabla_{r}^{2} + \frac{2m_{e}}{m_{e}+m_{p}} \frac{\partial^{2}}{\partial x^{2} \partial r^{2}}$ Doing the some monipulations with Jxp gives $\nabla_{p}^{2} = \left(\frac{m_{p}}{m_{e}+m_{p}}\right)^{2} \overline{\nabla_{x}^{2}} + \nabla_{r}^{2} - \frac{2m_{p}}{m_{e}+m_{p}} \frac{\partial^{2}}{\partial \vec{x}} \overline{J_{r}^{2}}$ (b) $\frac{1}{m_e} \nabla_e^2 + \frac{1}{m_p} \nabla_p^2 = \frac{m_e}{M^2} \nabla_X^2 + \frac{m_p}{M^2} \nabla_X^2 + (\frac{1}{m_e} + \frac{1}{m_p}) \nabla_1^2$ where $M = m_p + m_e$ $\Rightarrow = \frac{1}{M} \nabla_x^2 + \frac{1}{\Gamma} \nabla_r^2$ where $\mu = \frac{M_e M_p}{M}$

Substituting into the original H we get the desited expression puge 12 $\frac{t^2}{2M} \frac{\nabla^2 \psi}{\chi} - \frac{t^2}{2M} \frac{\nabla^2 \psi}{\chi} - \frac{t^2}{4\pi \epsilon_0} \frac{\psi}{\chi} = E \psi$ (c) $\Psi = F(\vec{x})\phi(\vec{r})$ Plug into the SE, divide by FØ, get $-\frac{1}{F}\frac{h^2}{2H}\frac{\nabla_r^2F}{\nabla_r}-\frac{1}{\Phi}\frac{h^2}{2V}\frac{\nabla_r^2\phi}{\Phi}-\frac{e^2}{2}=E$ Split it into $-\frac{t^2}{2M} \nabla_x^2 F(\overline{x}') = E_x F(\overline{x})$ $E_{\rm H}\phi(\vec{r})$ $-\frac{t^2}{2\mu}\nabla_{\Gamma}^2\phi(\vec{r}) - \frac{1}{4\pi\epsilon_{\sigma}\Gamma}\phi(t) =$ with $E = E_{k} + E_{r}$







(d) For lorge n, <r> nºa page 16) Compore it with (r)=328, for the ground state -So, for n= 100, rodius Tr 104 lorger, volume in 1012 lorger (6) compared with the H stom, everything looks the some in the SE expect that e -> Ze_ Since the energy eigenvelues come from Solving SE, we simply have to veplace e with Ze in the Rydberg Constant_ Now, the Rydborg constant goes like et, so $R(z) = Z^2 R(z=1)$ $E(z) = Z^{\prime}E(z=1)$

