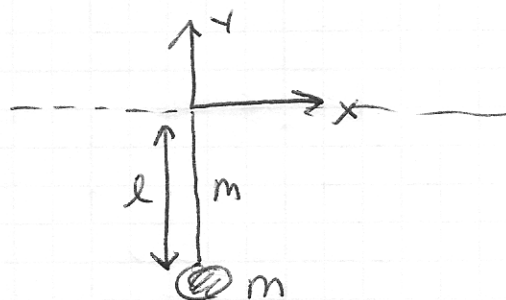


PHYSICS 105 B MIDTERM

(1) (e)



$$x_{cm} = 0 \quad y_{cm} = -\frac{1}{2m}(ml + \frac{m\ell}{2}) = -\frac{3}{4}\ell \quad \checkmark$$

(b) $I = I_{BALL} + \cancel{I_{BALL}} + I_{ROD}$

$$I_{BALL} = m\ell^2$$

$$I_{ROD} = \int_0^\ell \left(\frac{m}{\ell}\right) y^2 dy = \frac{m}{\ell} \left[\frac{y^3}{3}\right]_0^\ell = \frac{1}{3}m\ell^2$$

$$\Rightarrow I = m\ell^2 + \frac{1}{3}m\ell^2 = \frac{4}{3}m\ell^2 \quad \checkmark$$

(c) $I = I_{cm} + M y_{cm}^2$ where $M = 2m = \text{total mass}$

$$I_{cm} = I - 2m y_{cm}^2 = \frac{4}{3}m\ell^2 - 2m \frac{9}{16}\ell^2 = \left(\frac{4}{3} - \frac{9}{8}\right)m\ell^2 = \frac{5}{24}m\ell^2 \quad \checkmark$$

(d) Linear for CM: $2m\vec{v} = \vec{J}$

Rotation about CM: $I_{cm} \omega = J \left(\ell h - \frac{3}{4}\ell\right)$

← CM coordinate

(e) Instantaneous initial velocity of hook after blow is given by $V_{hook} = V_L + V_R$

Where $V_L = \frac{J}{2m}$ = linear motion and V_R is from the rotational motion - Let's find the absolute value of V_R first - We'll worry about the sign later

$$\cancel{V_R = \omega \cdot r} \quad V_R = \omega |r_{cm}| = \frac{3}{4} \ell \omega$$

But from part (d), $\omega = \frac{J}{I_{cm}} |h - \frac{3}{4} \ell|$

Take absolute value

$$V_R = \frac{3}{4} \ell \cdot \frac{J}{I_{cm}} |h - \frac{3}{4} \ell| = \frac{3}{4} \ell \frac{J}{\frac{5}{24} m \ell^2} |h - \frac{3}{4} \ell|$$

$$V_R = \frac{72}{20} \frac{J}{m \ell} |h - \frac{3}{4} \ell| = \frac{18}{5} \frac{J}{m \ell} |h - \frac{3}{4} \ell|$$

Now we need V_L and V_R to be equal in magnitude and opposite in sign - V_L is to the left - V_R ^{must be} to the right, which I can achieve if the hammer blow is struck below the CM, i.e. $h > \frac{3}{4} \ell$

Then

$$\frac{J}{2m} = \frac{18}{5} \frac{J}{m \ell} (h - \frac{3}{4} \ell)$$

$$\frac{5}{36} \ell = h - \frac{3}{4} \ell$$

$$h = \frac{5+27}{36} \ell$$

$$h = \frac{32}{36} \ell$$

$$\boxed{h = \frac{8}{9} \ell} \checkmark$$

② (a) Can solve this in two ways

(1) using forces - First spring must support the two masses, second spring must support the second mass

$$\Rightarrow \begin{cases} 2K(L_1 - L_0) = (2m + m)g \\ K(L_2 - L_0) = mg \end{cases} \Rightarrow \begin{cases} L_1 = L_0 + \frac{3mg}{2K} \\ L_2 = L_0 + \frac{mg}{K} \end{cases}$$

(2) using potential energy

$$U_{\text{gravity}} = -2mgL_1 - mg(L_1 + L_2) = -3mgL_1 - mgL_2$$

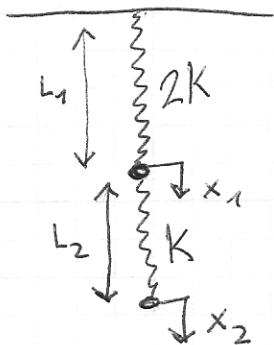
$$U_{\text{spring}} = \frac{1}{2}(2K)(L_1 - L_0)^2 + \frac{1}{2}K(L_2 - L_0)^2$$

$$\text{Equilibrium if } \frac{\partial U}{\partial L_1} = \frac{\partial U}{\partial L_2} = 0$$

$$\frac{\partial U}{\partial L_1} = -3mg + 2K(L_1 - L_0) = 0 \Rightarrow L_1 = L_0 + \frac{3mg}{2K}$$

$$\frac{\partial U}{\partial L_2} = -mg + K(L_2 - L_0) = 0 \Rightarrow L_2 = L_0 + \frac{mg}{K}$$

(b) Two dof



$$(c) T = \frac{1}{2}(2m)\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 = m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2$$

$$(d) \quad U_{\text{gravity}} = -2mg(L_1 + x_1) - mg(L_1 + L_2 + x_2)$$

$$U_{\text{gravity}} = -mg(2x_1 + x_2) + \text{constant}$$

$$U_{\text{spring}} = \frac{1}{2}(2k)(L_1 + L_0 + x_1)^2 + \frac{1}{2}k(L_2 - L_0 + (x_2 - x_1))^2$$

$$\text{But } L_1 - L_0 = \frac{3mg}{2k} \quad L_2 - L_0 = \frac{mg}{k}$$

$$U_{\text{spring}} = k\left(\frac{3mg}{2k} + x_1\right)^2 + \frac{1}{2}k\left(\frac{mg}{k} + (x_2 - x_1)\right)^2$$

$$U_{\text{spring}} = Kx_1^2 + 3mgx_1 + \frac{9m^2g^2}{4k} + \frac{1}{2}k(x_2 - x_1)^2 + mg(x_2 - x_1) + \frac{m^2g^2}{2k}$$

$$U_{\text{spring}} = Kx_1^2 + \frac{1}{2}k(x_2 - x_1)^2 + mg(x_2 + 2x_1) + \text{constant}$$

$$U_{\text{TOTAL}} = U_{\text{gravity}} + U_{\text{spring}} = Kx_1^2 + \frac{1}{2}k(x_2 - x_1)^2$$

$$U_{\text{TOTAL}} = \frac{3}{2}Kx_1^2 + \frac{1}{2}Kx_2^2 - Kx_1x_2$$

$$(e) \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_1} = \frac{\partial L}{\partial x_1} \Rightarrow \boxed{2m\ddot{x}_1 = -3Kx_1 + Kx_2}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_2} = \frac{\partial L}{\partial x_2} \Rightarrow \boxed{m\ddot{x}_2 = -Kx_2 + Kx_1}$$

$$(f) \quad M = \begin{pmatrix} 2m & 0 \\ 0 & m \end{pmatrix} \quad K = \begin{pmatrix} 3K & -K \\ -K & K \end{pmatrix}$$

$$\det(M\omega^2 - K) = 0$$

gives $\det \begin{pmatrix} 3k - 2m\omega^2 & -k \\ -k & k - m\omega^2 \end{pmatrix} = 0$

$$(3k - 2m\omega^2)(k - m\omega^2) - k^2 = 0$$

$$3k^2 - 3km\omega^2 - 2km\omega^2 + 2m^2\omega^4 - k^2 = 0$$

$$2m^2\omega^4 - 5km\omega^2 + 2k^2 = 0$$

$$\omega^2 = \frac{5km \pm km\sqrt{25-16}}{4m} \quad \begin{cases} \omega_1^2 = 2k/m \\ \omega_2^2 = k/2m \end{cases}$$

(g) For $\omega_1^2 = \frac{2k}{m}$

$$(k - \omega_1^2 m) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0 \text{ becomes } \begin{pmatrix} -k & -k \\ -k & -k \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0$$

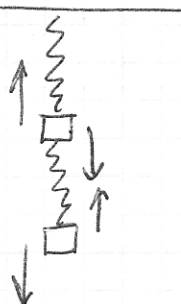
$$a_1 = -a_2$$

$$x_1 = A \cos(\omega_1 t - \delta)$$

$$x_2 = -x_1$$

Oscillations are out of phase

with same amplitude



For $\omega_2^2 = \frac{k}{2m}$

$$(k - \omega_2^2 m) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0 \text{ becomes } \begin{pmatrix} 2k & -k \\ -k & \frac{1}{2}k \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0$$

$a_1 = \frac{1}{2}a_2$ Oscillations are in phase with amplitudes that differ by a factor of two —

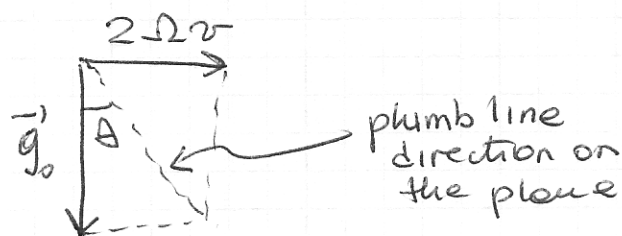
- ③ At the pole, no centrifugal force. The plumb line on the surface points towards center of earth -

Coriolis force $\vec{F}_c = -2m\vec{\Omega} \times \vec{v}$

Here $\vec{\Omega} \perp \vec{v}$ - Also $\vec{\Omega}$ along $\vec{R}'$, where $\vec{R}'$ is the vector that joins the center of the earth and the north pole

Then $|\vec{F}_c| = 2m\Omega v$

$\vec{g}_0$ along $\vec{R}' \Rightarrow \vec{F}_c \perp \vec{g}_0$



$$\tan A \approx A \approx \frac{2\Omega v}{g_0}$$

④ (a) $L = m v b$

(b) $U=0$ at $\infty \Rightarrow E = \frac{1}{2} m v^2$

(c) hyperbola

(d) $\frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 - \frac{GmM}{r} = \frac{1}{2} m v^2$ (conservation of energy)

But $L = m r^2 \dot{\theta}$

$m v b = m r^2 \dot{\theta} \Rightarrow \dot{\theta} = \frac{b}{r^2} v$

$\frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \frac{b^2}{r^4} v^2 - \frac{GmM}{r} = \frac{1}{2} m v^2$

$\frac{1}{2} m \dot{r}^2 + \frac{m b^2 v^2}{2 r^2} - \frac{GmM}{r} = \frac{1}{2} m v^2$

At point of closest approach $\dot{r} = 0$

$$\frac{mb^2v^2}{2r^2} - \frac{GMm}{r} = \frac{1}{2}mv^2$$

$$\frac{2GM}{r} = \left(\frac{b^2}{r^2} - 1 \right) v^2$$

$$\frac{2GM}{v^2} = \frac{b^2}{r} - r$$

$$\frac{2GM}{v^2} r = b^2 - r^2$$

$$r^2 + \frac{2GM}{v^2} r - b^2 = 0$$

$$r = \frac{-\frac{2GM}{v^2} \pm \sqrt{4\frac{G^2M^2}{v^4} + 4b^2}}{2}$$

$$r = \sqrt{\frac{G^2M^2}{v^4} + b^2} - \frac{GM}{v^2}$$