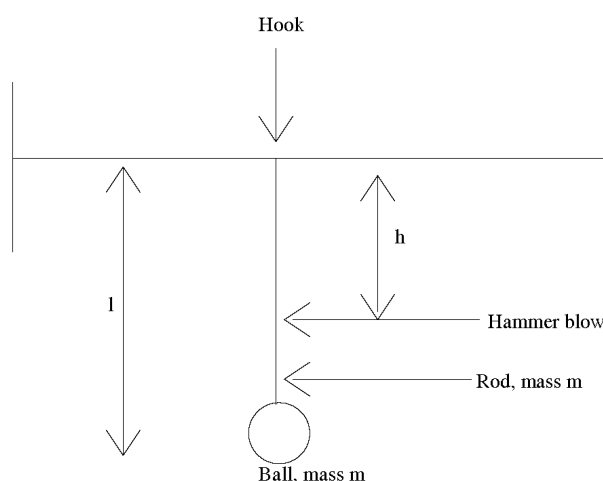


Physics 105B – Spring 2007 – Midterm

Problem 1 (25 points)

A rod of mass m and length l hangs vertically from a horizontal frictionless wire, as shown in the Figure below. Attached to the end of the rod is a small ball, also of mass m . The rod is free to slide along the wire. The radius of the ball is negligibly small.



- (a) Find the location of the center of mass of rod plus ball, taking the hook as the origin of the coordinate system. Make sure to indicate clearly the orientation of the axes. **(5 points)**
- (b) Find the moment of inertia of rod + ball around the hook. **(5 points)**
- (c) Use the parallel axis theorem and the result of part (b) to find the moment of inertia around the center of mass. If you do not know how to apply the parallel axis theorem, try to answer this question in a different way for partial credit. **(5 points)**
- (d) The rod-ball system is struck by a quick horizontal impulsive hammer blow a distance h from the hook. Find the linear velocity of the CM and the angular velocity for rotation about the center of mass at the instant of the blow in terms of the magnitude of the impulse J (ignore gravity). **(5 points)**
- (e) Find h such that the hook does not move along the wire at the instant of the blow. **(5 points)**

Reminder: if a force F is applied for a time Δt , the impulse J is defined as $J = F\Delta t$. J is also a vector, which points in the same direction as the force.

Problem 2 (30 points)

A mass $2m$ is suspended from a fixed support by a spring of relaxed length L_0 and spring constant $2k$. A second mass m is suspended from the first mass by a spring of constant k and relaxed length L_0 . The masses of the springs are negligible. Consider only motion along the line joining the two masses.

- (a) What are the lengths of the two springs in equilibrium? **(5 points)**

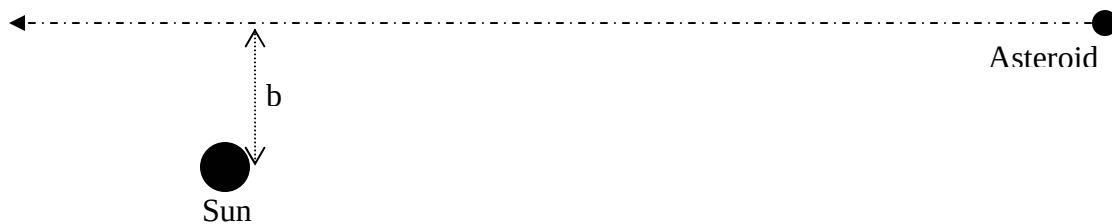
- (b) How many degrees of freedom are in the problem? Pick a convenient set of generalized coordinates for the system, and indicate them clearly in a sketch. **(3 points)**
- (c) Write the kinetic energy of the system in terms of the generalized coordinates and the generalized velocities. **(4 points)**
- (d) Write the potential energy of the system. Do not worry about additive constants. **(5 points)**
- (e) Find the equations of motion for small oscillations. **(4 points)**
- (f) Find the frequencies of the normal modes. **(5 points)**
- (g) Describe the motion in the normal modes. If you cannot solve the equations, guess the pattern of motion for partial credit. **(4 points)**

Problem 3 (20 points)

An airplane flies across the North Pole with velocity V , following a meridian of longitude (which rotates with the earth). Find the angle between the direction of a plumb line hanging freely in the airplane as it passes over the pole and one hanging freely at the surface of the earth over the pole. Express your answer in terms of V , the magnitude of the earth's gravitational field vector at the surface of the earth (g_0), and the angular velocity of rotation of the earth Ω .

Problem 4 (25 points)

An asteroid of mass m approaches the sun (mass M) from infinitely far away with velocity V ($m \ll M$). In the absence of any gravitational interaction, the asteroid would pass within a distance b of the sun, see figure below (the dashed line indicates the trajectory that the asteroid would have followed in the absence of gravitational interactions).



- (a) What is the magnitude of the angular momentum of the asteroid about the sun? **(6 points)**
- (b) What is its energy (take the constant in the definition of gravitational potential energy such that the potential energy between two objects is zero when they are infinitely far apart). **(6 points)**
- (c) Without doing any calculation, can you describe qualitatively the trajectory of the asteroid. **(5 points)**
- (d) What will be the distance of closest approach between the asteroid and the sun? **(8 points)**

Express all your answers in terms of m , M , v , b , and G .