

Physics 105B – Spring 2007 – Final

Problem 1 (15 points)

By studying the scattering of α particles from a thin gold foil, Rutherford and collaborators were able to show that the Thomson's model of the atom was not correct. Briefly, and without doing any calculations, explain which features of their result were in contradiction with Thomson's model and why

Problem 2 (15 points)

A bead of mass m is constrained to move without friction on a hoop of radius R . The hoop rotates with constant angular velocity ω about a vertical axis which coincides with a diameter of the hoop, see Figure 1.

(a) By applying Newton's laws, including fictitious forces in the rest frame of the hoop, show that

$$\ddot{\theta} + \left(\frac{g}{R} - \omega^2 \cos \theta \right) \sin \theta = 0$$

If you are unable to derive this result, you can still use it and proceed with the remainder of the problem. *Hint: it may help to consider forces perpendicular and tangent to the hoop.* **(7 points)**

(b) Find the critical angular velocity Ω below which the bottom of the hoop provides a position of stable equilibrium for the bead. **(4 points)**

(c) Find the equilibrium position for $\omega > \Omega$ **(4 points)**

(d) **Extra Credit:** Show that the equilibrium position that you found in part (c) is a position of stable equilibrium. What is the frequency of small oscillations. **(5 bonus points)**

Problem 3 (18 points)

Three identical point bodies of mass M are placed in the (xy) plane as follows:

- (i) coordinates $(0; a)$
- (ii) coordinates $(a; a)$
- (iii) coordinates $(a; 0)$

(a) Find the inertia tensor with respect to the (xyz) -axes **(8 points)**

(b) Find the three principal moments of inertia **(7 points)**

(c) Without doing any algebra, use your intuition about principal axes to guess and sketch the direction of the three principal axes. After you have guessed the three principal axes, calculate the moments of inertia about these three axes and verify that they correspond to the three principal moments of inertia that you calculated in part (b) **(3 points)**

Problem 4 (16 points)

A beam of particles is directed towards a $1/r^2$ scattering center which is at rest in the laboratory frame. A detector with surface area $A_1 = 1 \text{ cm}^2$ is placed at an angle of $\theta_1 = 90^\circ$ with respect to the incoming beam and at a distance $d_1 = 1 \text{ m}$. In a given time interval, this detector is hit by $N_1 = 10,000$ scattered particles. A second detector of surface area

$A_2 = 2 \text{ cm}^2$ is also placed at an angle of $\theta_2 = \theta_1 = 90^\circ$ but at a distance $d_2 = 2 \text{ m}$. A third detector of surface area $A_3 = 1 \text{ cm}^2$ is placed at an angle of $\theta_3 = 60^\circ$ at a distance $d_3 = 3 \text{ m}$.
 (a) By how many particles you expect the 2nd detector to be hit in the same time interval?

(8 points)

(b) Same question, but for the 3rd detector **(8 points)**

(Note: by “a $1/r^2$ scattering center” we mean that the force between any one of the particles and the scattering center varies as $1/r^2$, where r is the distance between the particle and the scattering center.

Problem 5 (16 points)

An inclined plane of mass M_2 is sliding on a smooth horizontal frictionless surface, while a body of mass M_1 is sliding down the inclined plane, see Figure 2

(a) Write down the lagrangian using generalized coordinates X_1 and X_2 (see Figure 2 for definitions of X_1 and X_2) **(5 points)**

(b) Find expressions for P_1 and P_2 , the generalized momenta corresponding to X_1 and X_2 . **(4 points)**

(c) Verify that X_2 does not appear in the lagrangian, and therefore P_2 is conserved. What does P_2 correspond to? **(3 points)**

(d) Write down the Hamiltonian (to simplify the algebra take $M_1 = M_2 = M$ for this part of the problem only). Do this for the case where the system is released from rest, i.e., use the appropriate value for the conserved quantity P_2 which corresponds to this initial condition. **(4 points)**

Problem 6 (20 points)

A superhero standing at the back of a train of proper length $L_0 = 150 \text{ m}$ shoots a rocket to the front of the train. The superhero measures the speed of the rocket to be $c/5$. A bystander on the platform sees the train going by at a speed of $3c/5$. What values does the bystander measure for

(a) The length of the train **(5 points)**

(b) The speed of the rocket **(5 points)**

(c) The time taken by the rocket to hit the front of the train **(5 points)**

(d) The distance traveled by the rocket **(5 points)**

Take $c = 3 \cdot 10^8 \text{ m/sec}$.

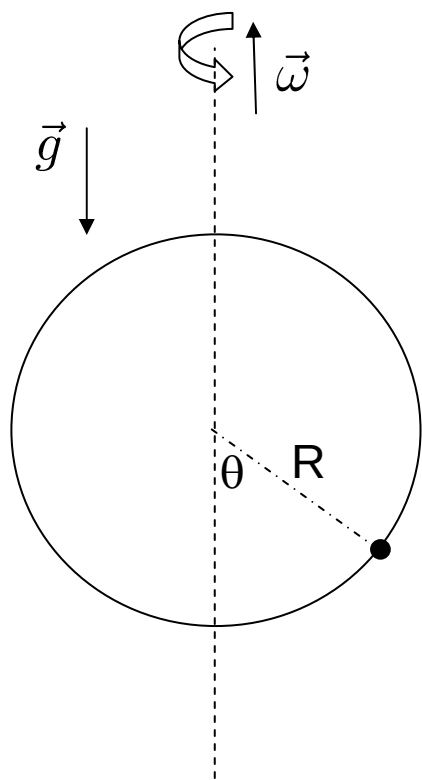


Figure 1

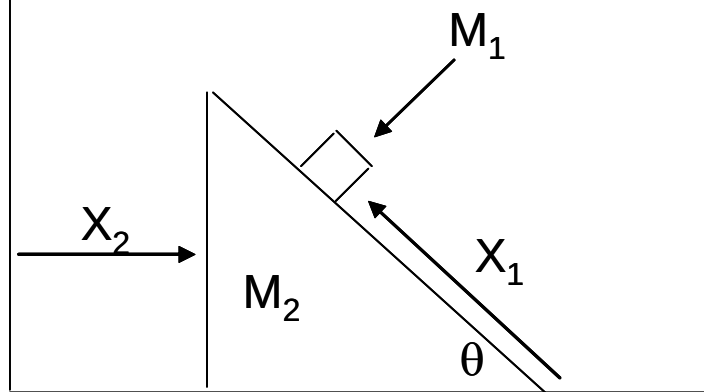


Figure 2