

Physics 105B Midterm Practice Problems

Problem 1

Four masses are placed in the (x,y) plane as follows:

- (i) mass=2m, coordinates (a, b)
- (ii) mass=m, coordinates (-a, b)
- (iii) mass=2m, coordinates (-a, -b)
- (iv) mass=m, coordinates (a, -b)

- (a) Find the inertia tensor
- (b) Now set a=b (for simplicity) and find the three principal moments of inertia
- (c) Without doing any algebra, can you use your intuition about principal axes of inertia to indicate (for example, in a sketch) the directions of the three principal axes?

Problem 2

A uniform spherical planet of radius R rotates with constant angular velocity ω . The effective gravitational acceleration g_{eff} is some constant g at the poles and 0.8g at the equator. Find the mass of the planet in terms of ω , R, and the gravitational constant G.

Problem 3

A yo-yo of mass M is composed of two identical uniform disks of radius R held together through their centers by a massless shaft of radius r. A massless string is wound in the middle of the shaft, and the loose end is held in somebody's hand. Upon release, the yo-yo descends until the string is unwound. Find the tension T of the string as the yo-yo is descending.

You can use the fact that the moment of inertia of a uniform disk of mass μ and radius a for rotation around an axis that passes through the center of the disk and perpendicular to the plane of the disk is $I = \frac{1}{2} \mu a^2$. Don't forget that in this problem M is the mass of the yo-yo, i.e., the combined mass of the two disks.

Problem 4

A bead of mass m is constrained to move without friction on a hoop of radius R. The hoop rotates with constant angular velocity ω about a vertical axis which coincides with a diameter of the hoop, see figure below

- (a) By applying Newton's laws, including fictitious forces, in the rest frame of the hoop, show that

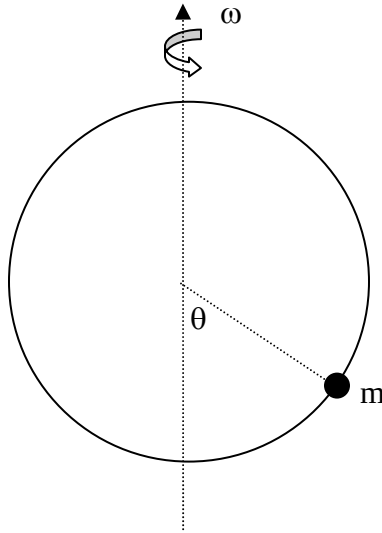
$$d^2\theta/dt^2 + (g/R - \omega^2 \cos\theta)\sin\theta = 0$$

If you are unable to derive this result, you can still use it, and proceed with the remainder of the problem

Hint: consider forces both perpendicular and tangent to the hoop

- (b) Find the critical angular velocity Ω below which the bottom of the hoop provides a position of stable equilibrium for the hoop
- (c) Find the equilibrium position for $\omega > \Omega$.

- (d) Extra Credit: Show that the equilibrium position in part (c) is a position of stable equilibrium. What is the frequency of small oscillations?



Problem 5

A bug of mass m crawls out without slipping along the radius of a turntable rotating with angular velocity ω . If the bug is at a distance d from the center and it is traveling with velocity v , what is the magnitude of the total force on the bug.

Problem 6

A body moves in an elliptical orbit under the influence of an attractive inverse square ($F = -k/r^2$) force. The major axis of the ellipse is $2a$, the minor axis is $2b$, and the eccentricity is ϵ . (Note: $b^2 = (1 - \epsilon^2)a^2$).

- sketch the ellipse and indicate where the velocity of the body will be minimum and maximum (justify!)
- show that $V_{\max}/V_{\min} = (1 + \epsilon)/(1 - \epsilon)$, where $V_{\max}$ and $V_{\min}$ are the maximum and minimum values of the velocity.