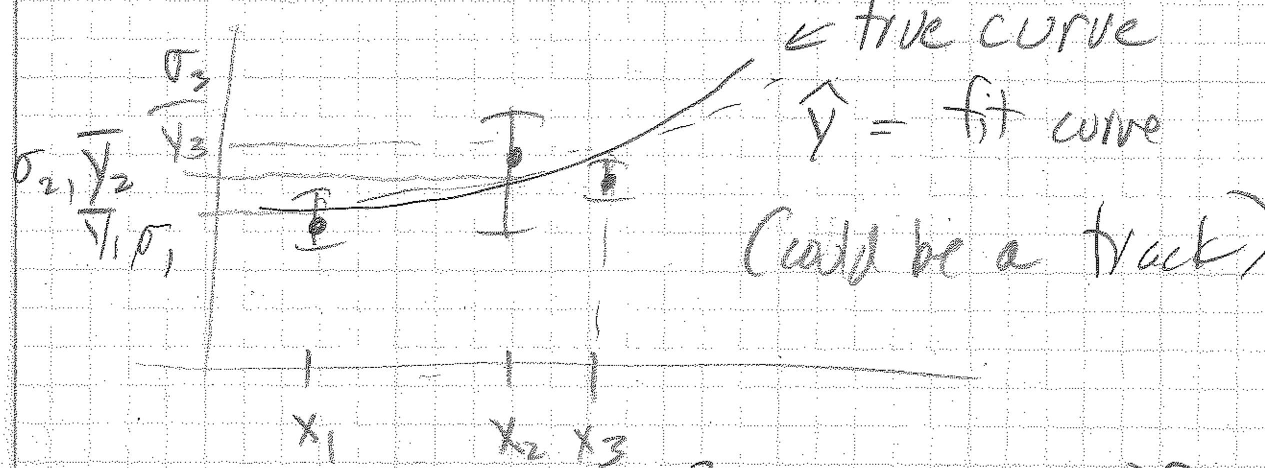
Unbinned Likelihood

"Curve Fit"

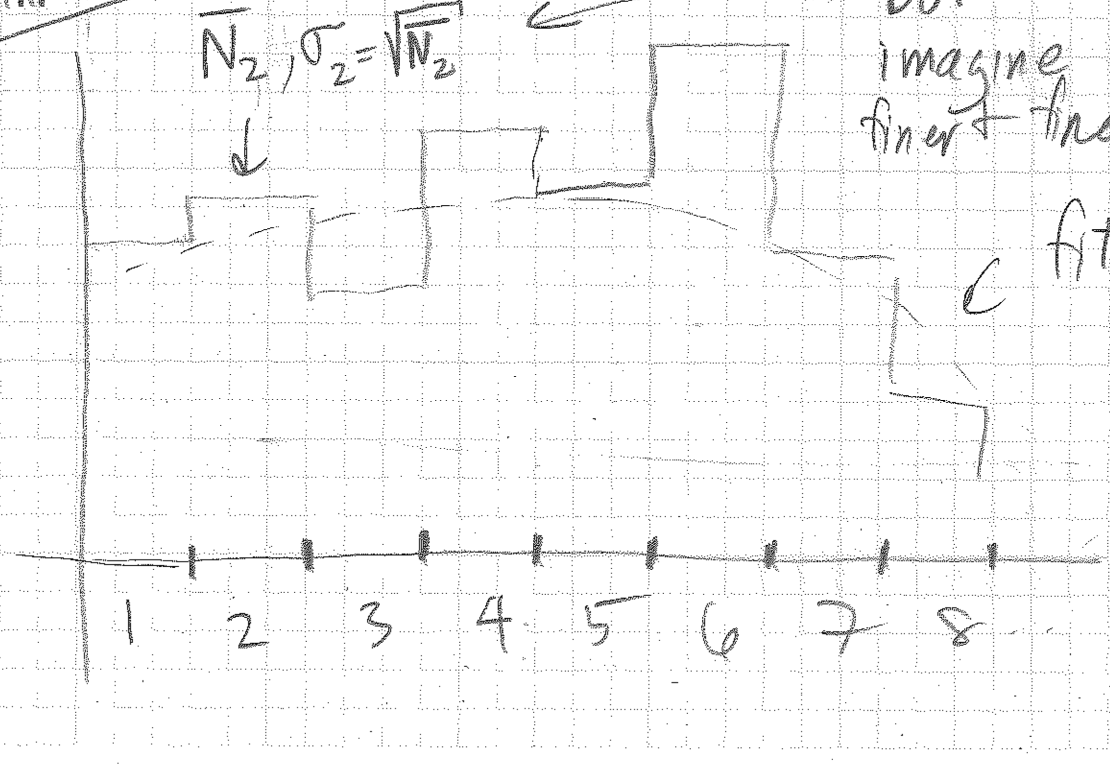


$$\mathcal{L} = \frac{1}{\sqrt{2\pi\sigma_1}} e^{-\frac{(y_1-\bar{y}_1)^2}{2\sigma_1^2}} \times \frac{1}{\sqrt{2\pi\sigma_2}} e^{-\frac{(y_2-\bar{y}_2)^2}{2\sigma_2^2}} \cdots$$

$$-2\ln\mathcal{L} = \text{constant} + \sum \frac{(y_i-\bar{y}_i)^2}{\sigma_i^2}$$

Binned

but imagine finer + finer binning



$$\mathcal{L}_2(N_2;\overline{N}_2) = \frac{\overline{N}_2^{\,N_2}}{N_2!}\, e^{-\overline{N}_2}$$

assume $= \mathcal{L}_i(\overline{N}_2; N_2)$

$$\ln \mathcal{L}_2(\overline{N}_2) = N_2 \ln \overline{N}_2 - \overline{N}_2 + \text{constant}$$

$$\sum \ln \mathcal{L}_i(\overline{N}_i) = \sum N_i \ln \overline{N}_i - \sum \overline{N}_i$$

$\overline{N}_i = N_{tot}\, f(\vec{\theta}, x_i)$

f normalized

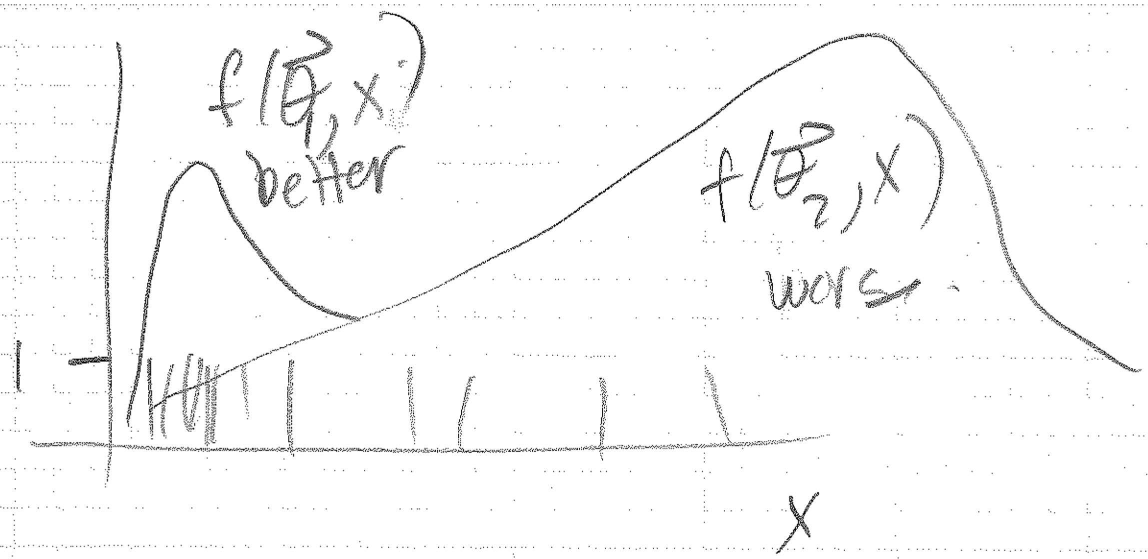
$\sum \overline{N}_i$ should give N_{tot}

make bins small enough that $N_i = 0$ or 1

$$\sum \ln \mathcal{L}_i(\vec{\theta}) = \text{constant} + \sum_i \ln f(\vec{\theta}, x_i)$$

$$\mathcal{L}_i(\vec{\theta}) = \prod_i f(\vec{\theta}, x_i)$$

data points



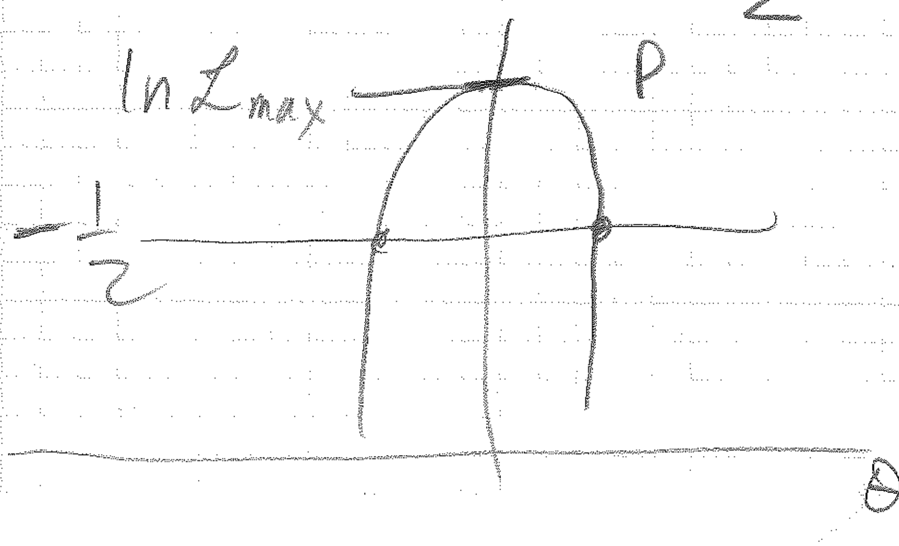
But more:

maximize $\sum \ln f(\vec{\theta}, x_i)$

then vary around

$\hat{\vec{\theta}}$ to get

$\Delta \ln L = \frac{1}{2}$



And even more: as statistics $\to \infty$, achieves smallest statistical error on θ_1!

$$I_{\theta_1} = \int \left(\frac{\partial \ln f}{\partial \theta_1}\right)^2 f(\vec{\theta}_{T}, x)\, dx$$

$$\sigma_{\theta_1, \min} = \frac{1}{\sqrt{I_{\theta_1}}} \quad \text{(Cramer Rao)}$$

$\to \sqrt{N}$ implied

example of

"efficient" estimator – one that gives

$$\sigma_{\text{estimator}} \to \frac{1}{\sqrt{N I_{\theta_1}}}$$

<u>Examples</u>: mean for $e^{-x/\lambda}$, $e^{\frac{-(x-\mu)^2}{\sigma^2}}$