Feldman Cousins

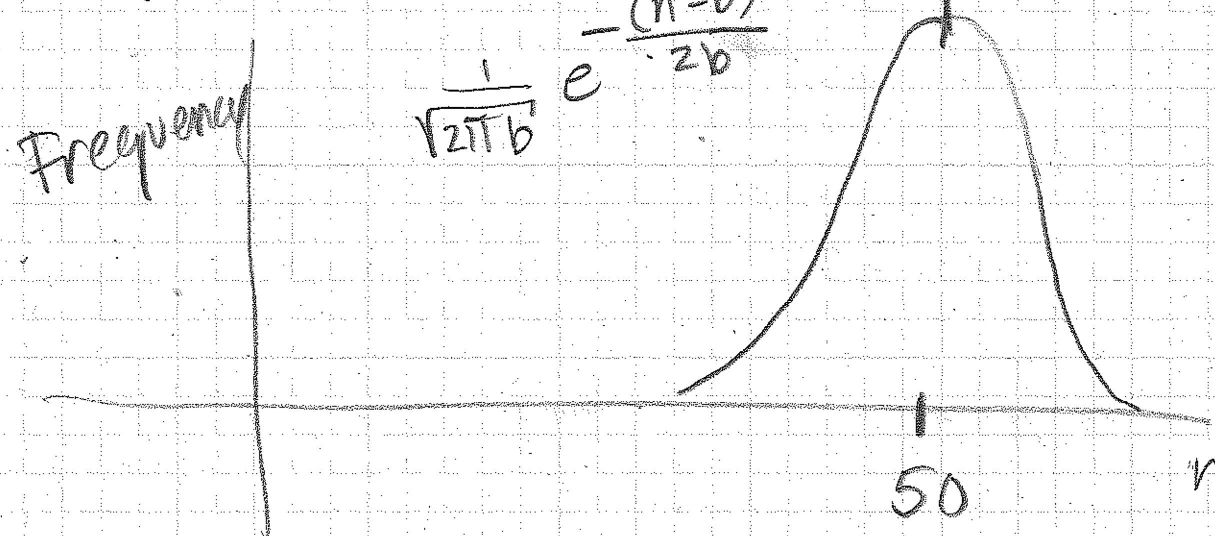
"Unified" Approach, Classical

Look at Gaussian Limit First

"Know" background b .... say $b = 50$

Expect: (signal = 0)

$$\frac{1}{\sqrt{2\pi b}}\, e^{-\frac{(n-b)^2}{2b}}$$



Bayesian: measure

$$\mu = s + b$$

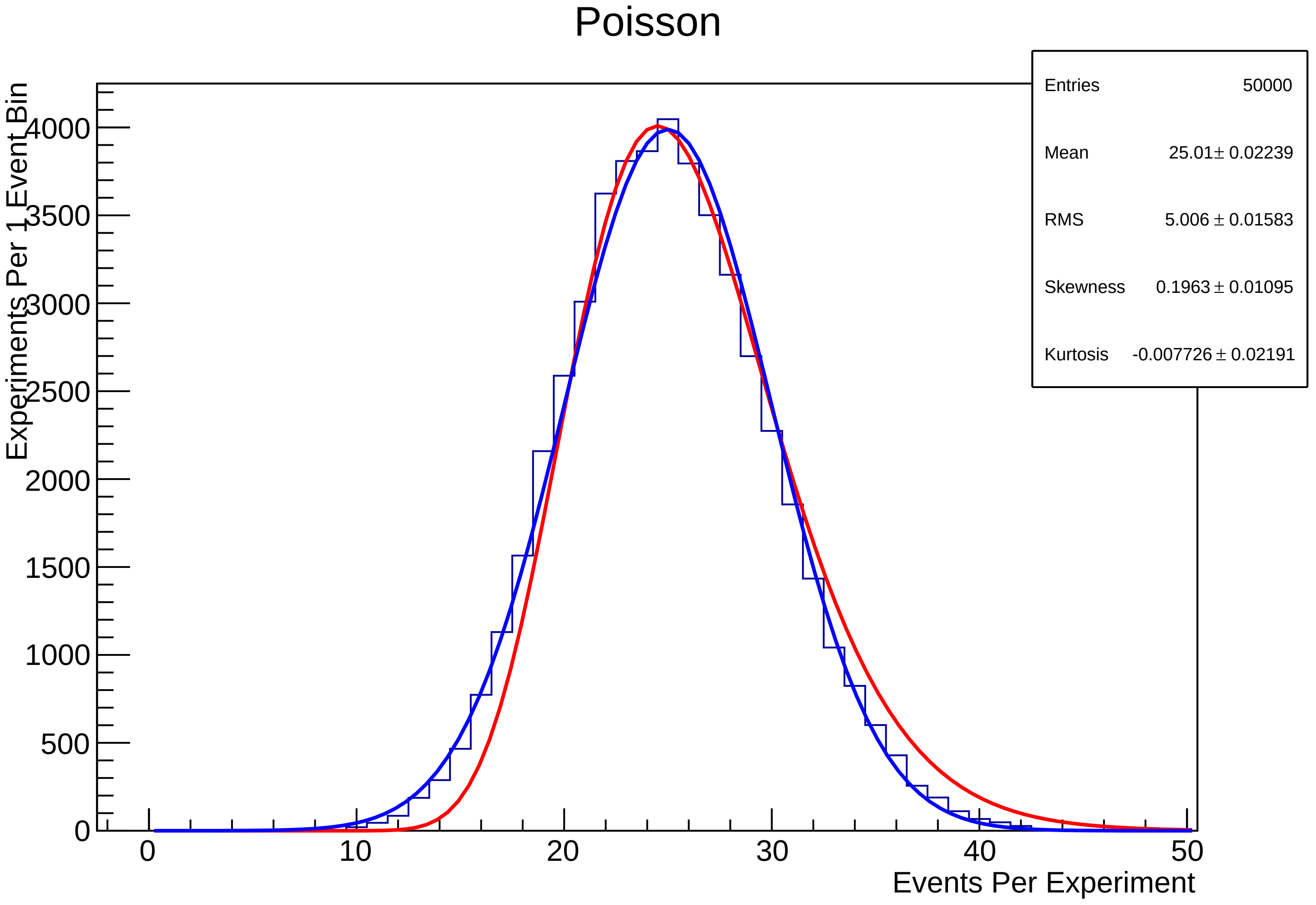
$$s = \mu - b$$

See n events,

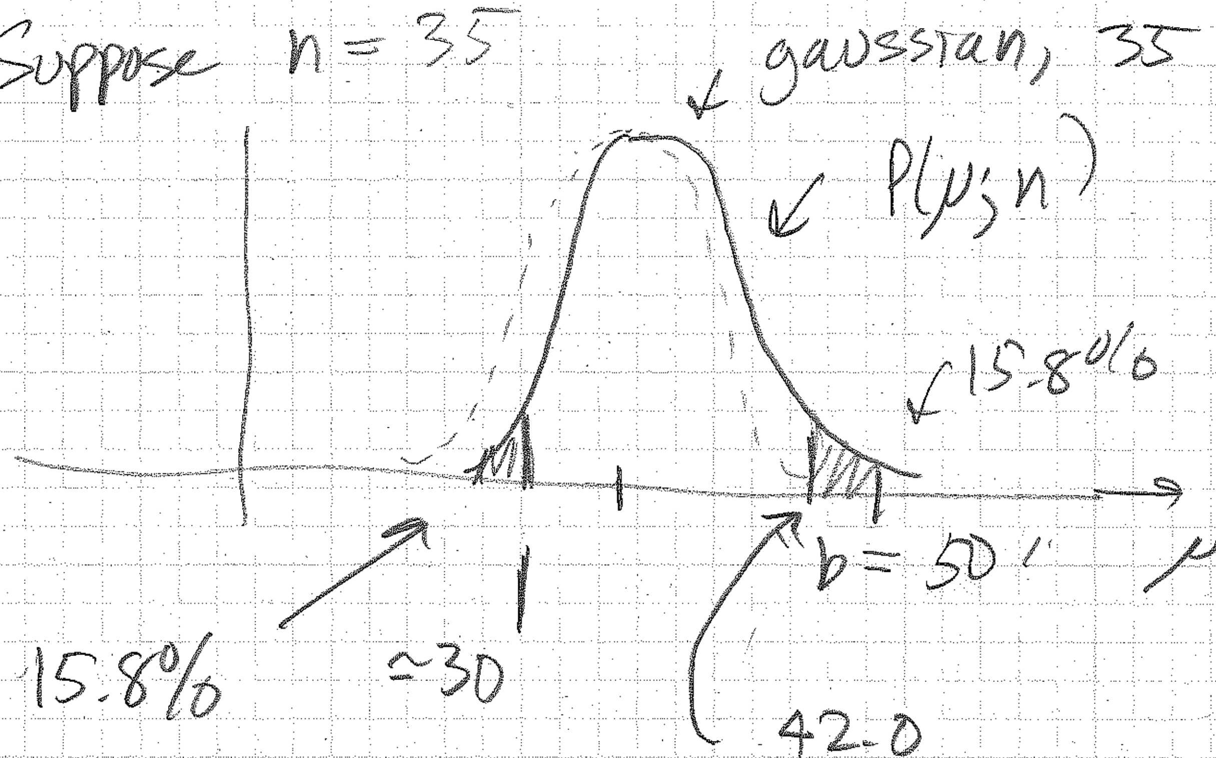
$$P(\mu; n) = \frac{1}{\sqrt{2\pi\mu}}\, e^{-\frac{(n-\mu)^2}{2\mu}} \times (\text{prior density}, \mu)$$

NOT ~~REALLY~~ GAUSSIAN!

assume flat...


Poisson
Entries 50000
Mean 25.01 ± 0.02239
RMS 5.006 ± 0.01583
Skewness 0.1963 ± 0.01095
Kurtosis -0.007726 ± 0.02191
Experiments Per 1 Event Bin
0
500
1000
1500
2000
2500
3000
3500
4000
0
10
20
30
40
50
Events Per Experiment

Suppose $n = 35$



Bayesian 35^{+7}_{-5}

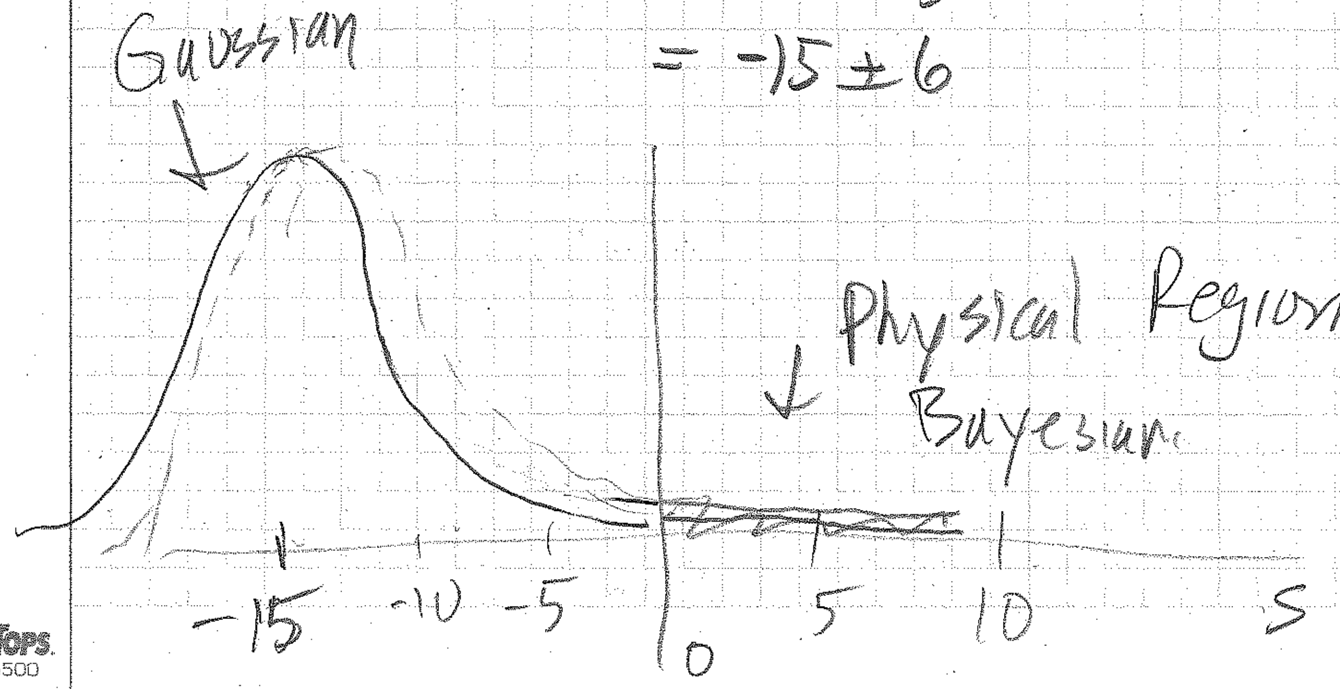
"Simple" 35 ± 6

!

Really care about $s = \mu - b$

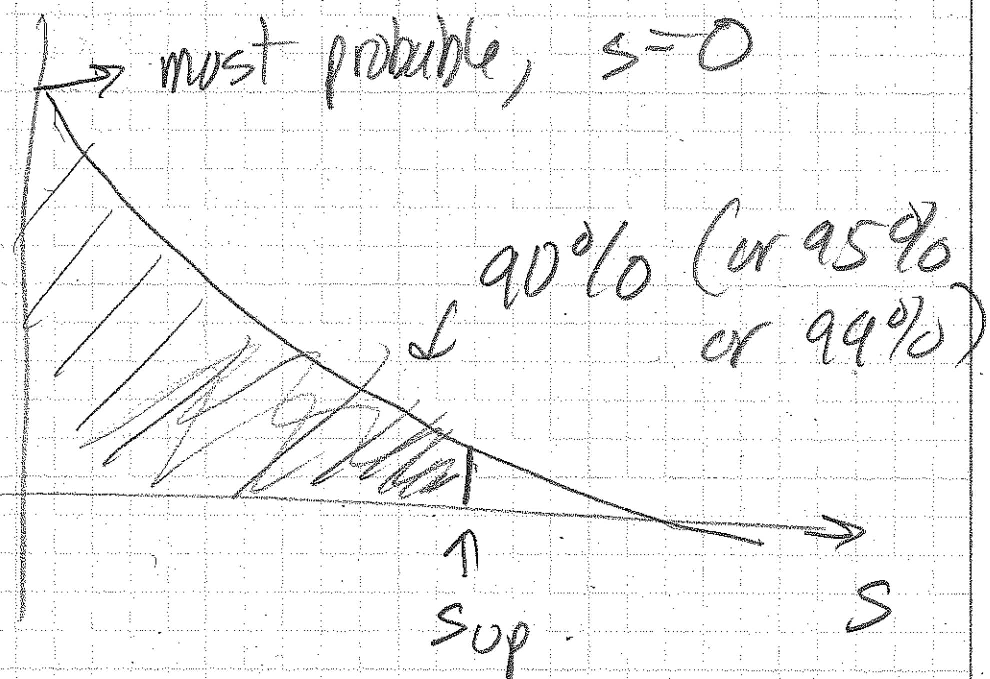
$s = -15^{+7}_{-5}$

$= -15 \pm 6$

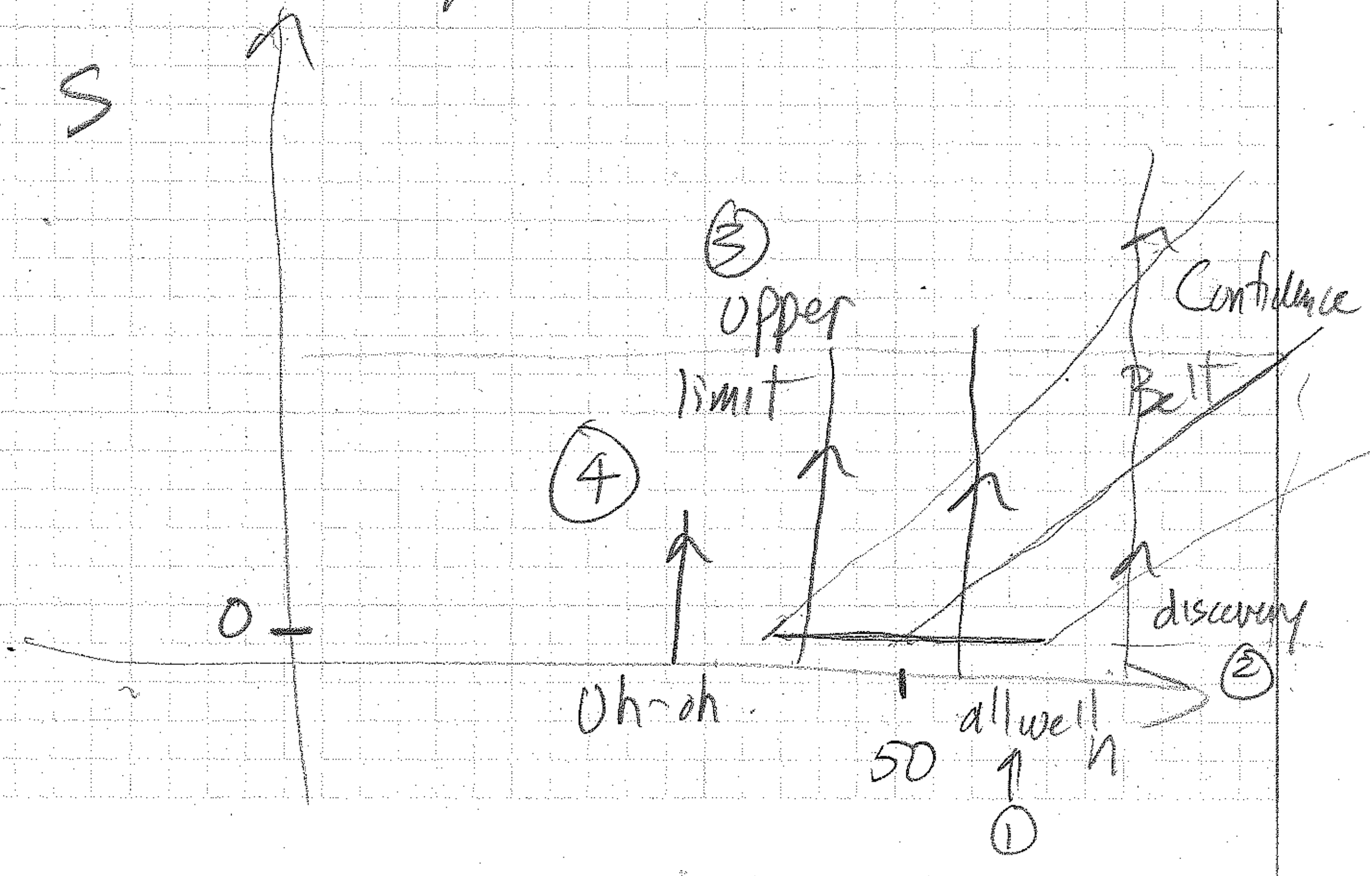


Bayesian : $P(s<0) = 0$

$P(s>0) = \text{constant}$

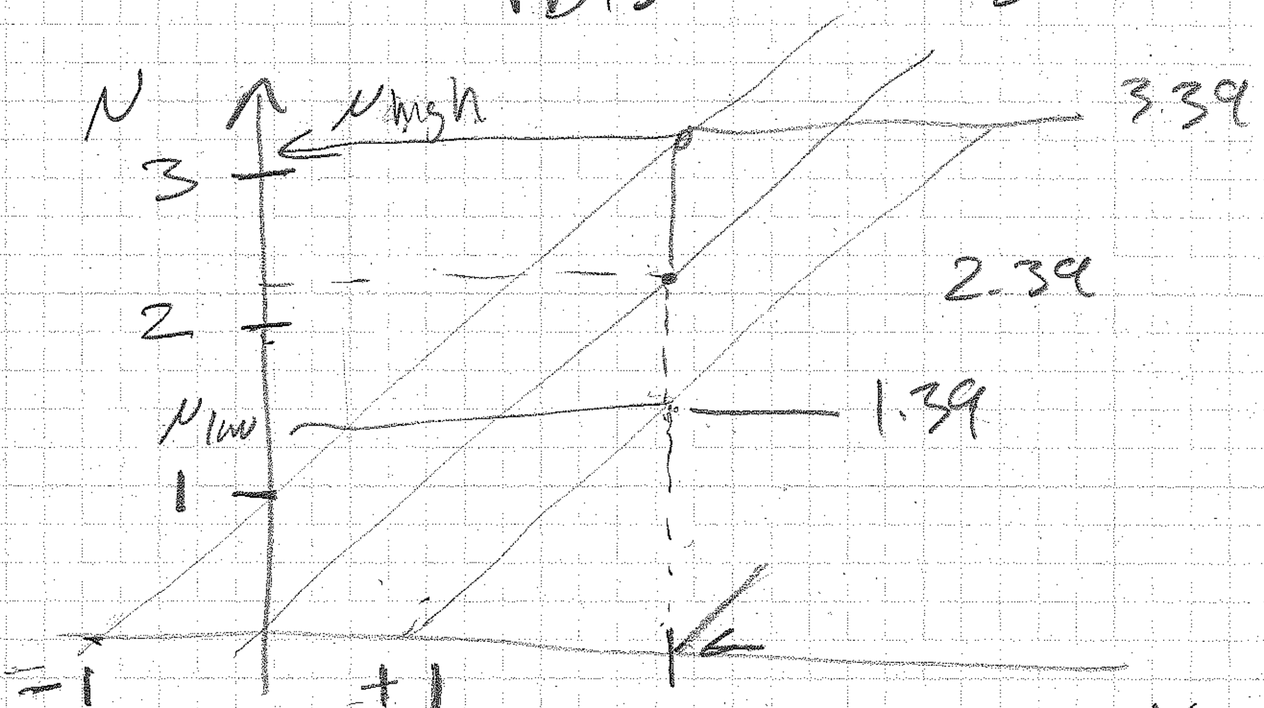


Classical (Very Gaussian)



Normalize Coordinates

$$x = \frac{n-b}{\sqrt{b+s}} \qquad \mu = \frac{s}{\sqrt{b+s}}$$



$x_0 = 2.39$

say, get $n_0 = 70$

$s = 70 - 50 = 20$

$$x_0 = \frac{70-50}{\sqrt{50+20}} = 2.39$$

$$\mu_{low} \quad 1.39 = \frac{s}{\sqrt{50+s}} \;,\; s = 10.843$$

$$\mu_{up} = 3.39 = \frac{s}{\sqrt{50+s}} \;,\; s = 30.397$$

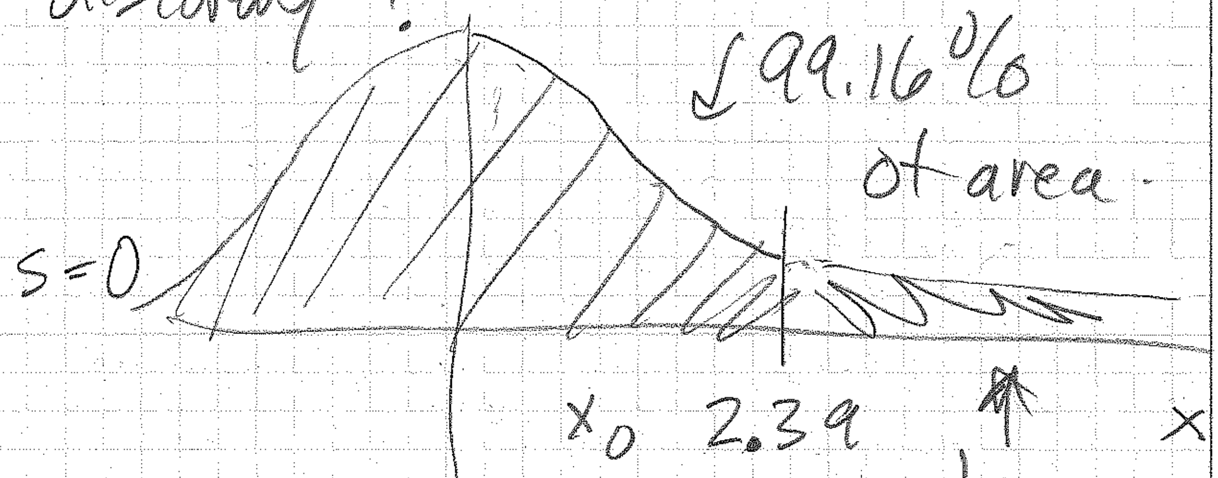
$1\sigma \qquad (20 \pm \overset{\sqrt{70}}{8.37}) \qquad$ 68% interval

$b = 50$
$n_0 = 70$

	Classical	Sloppy
μ_{low}	10.843	11.633
μ_{high}	30.397	28.367

68% intervals

What confidence level is this a discovery?



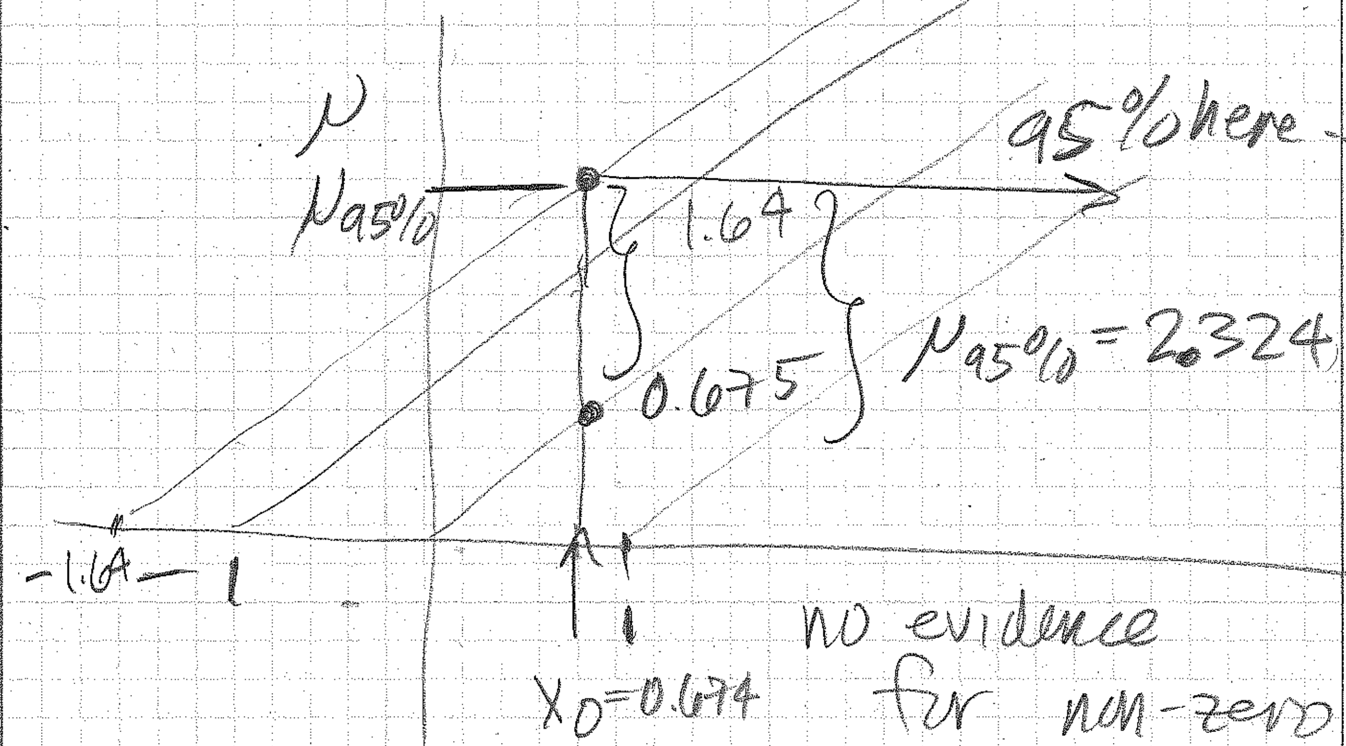
less than 1% chance of $s=0$

What if you see $b=50$ $n_0 = 55$ events?

no good evidence for signal

$$s = 55 - 50 = 5$$

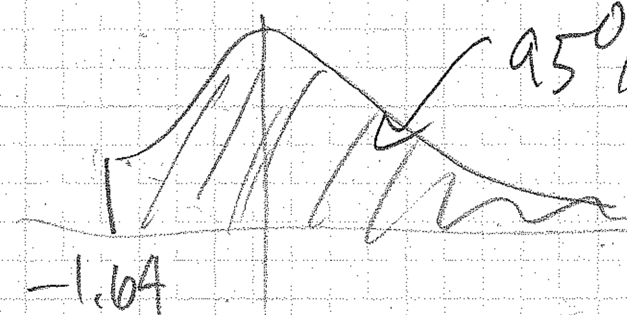
$$x_0 = \frac{5}{\sqrt{55}} = 0.674$$



no evidence for non-zero

might quote, say, 95% CL on μ_{high} ...

$$\frac{s}{\sqrt{50+s}} = 2.324$$



$$\boxed{s \simeq 19.35}$$

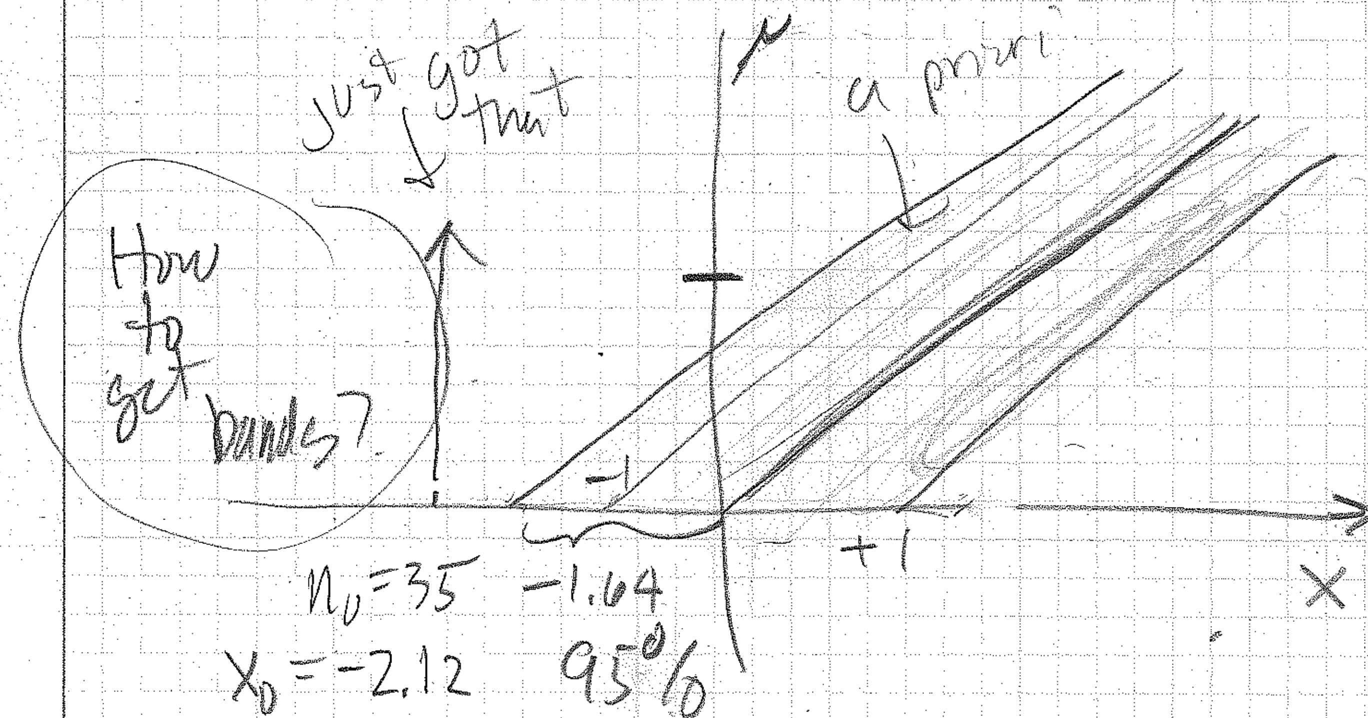
SLOPPY: $55 + 1.64 \cdot \sqrt{55} - 50 = 17.16$

What happens to 95% CL if

$$x_0 < -1.64$$

$$\frac{n_0 - 50}{\sqrt{50}} < -1.64$$

$$n_0 < 50 - 1.64 \cdot \sqrt{50} = 38?$$



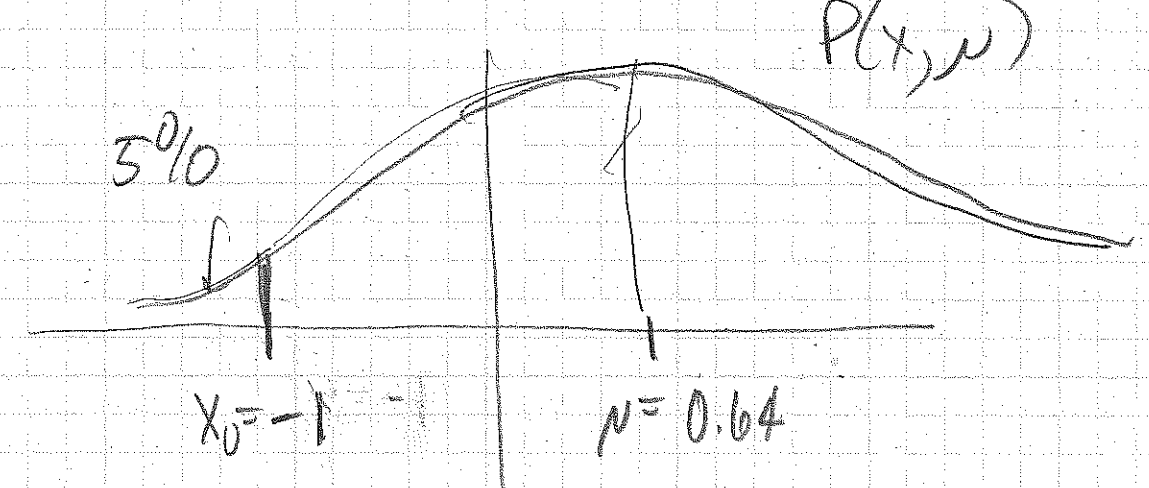
Feldman Cousins Gave useful prescription

Observation #1 This is "base case" very unlikely

Observation #2 Limits involve variation from "base case"

$$P(x,\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}$$

Normally would



Look at likelihood ratio ....

$$\frac{P(x,\mu)}{P(x,\mu=0)} = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}}{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}} \quad (x<0)$$

$$= e^{\mu(x-\frac{1}{2}\mu)} \quad x < 0$$

$\mu = 0, 1!$

TOPS. 35500

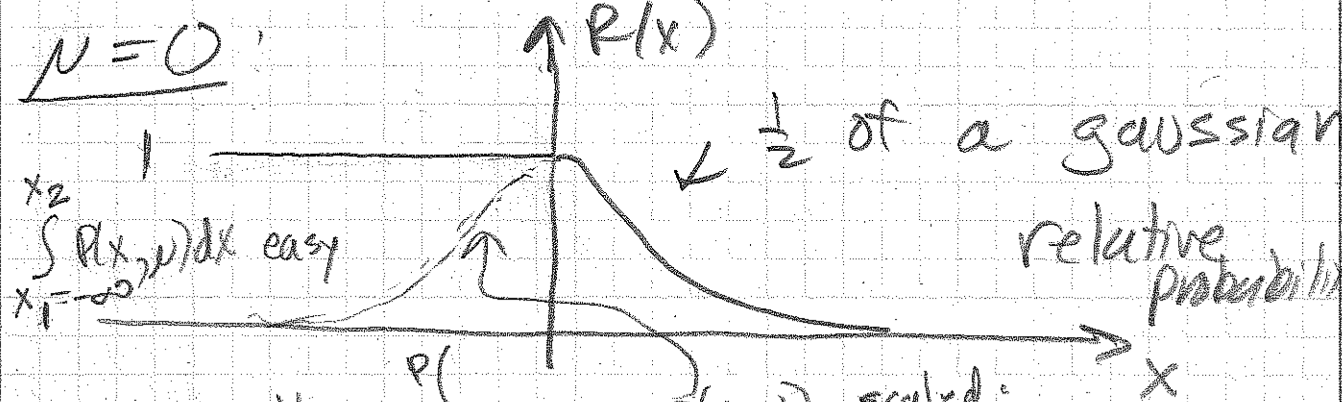
If $x > 0$, best guess for μ is $\mu = x$,

$$P(x,x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-x)^2} = \frac{1}{\sqrt{2\pi}}$$

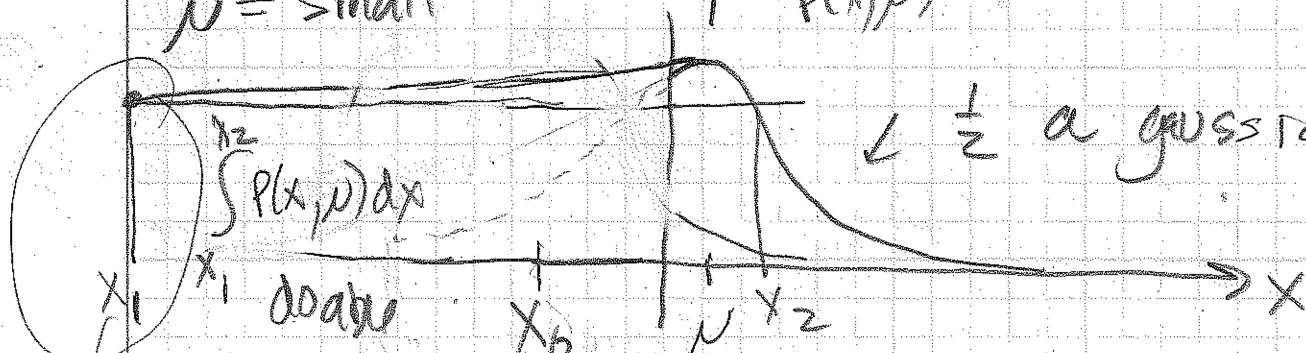
$$R(x) = \frac{P(x,\mu)}{P(x,\mu=x)} = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}}{\frac{1}{\sqrt{2\pi}}} = e^{-\frac{1}{2}(x-\mu)^2}$$

$x \geq 0$

$\underline{\mu = 0}$

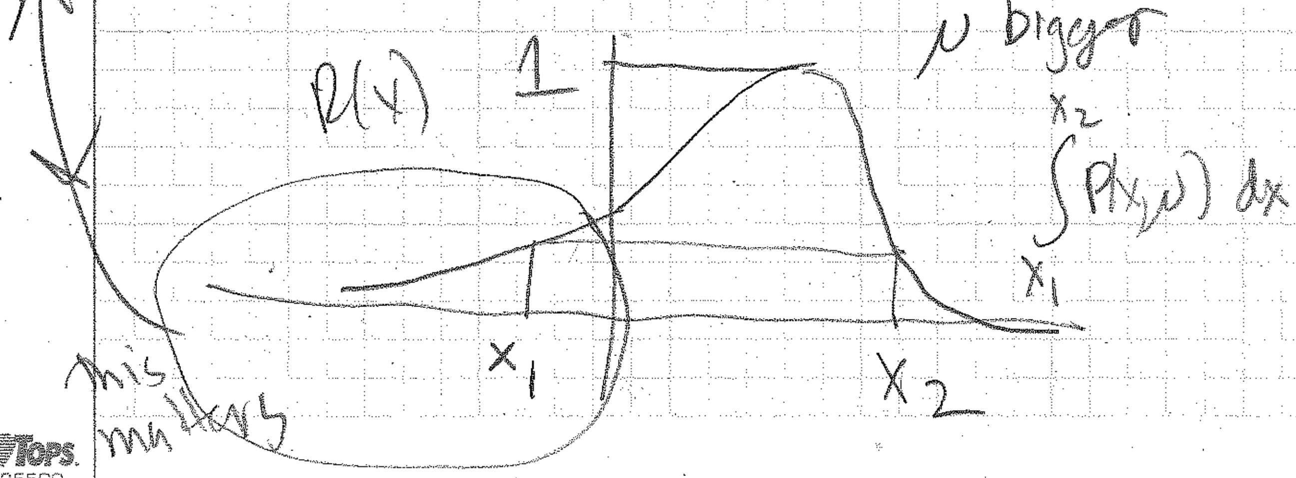


μ = small



x_1 & x_2 are equally probable!

like $-\infty$ close to



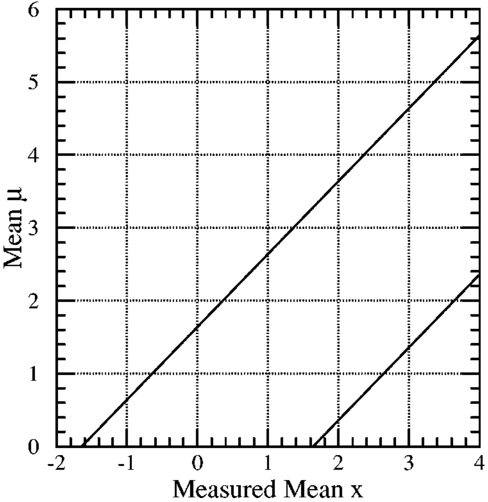


FIG. 3. Standard confidence belt for 90% C.L. central confidence intervals for the mean of a Gaussian, in units of the rms deviation.

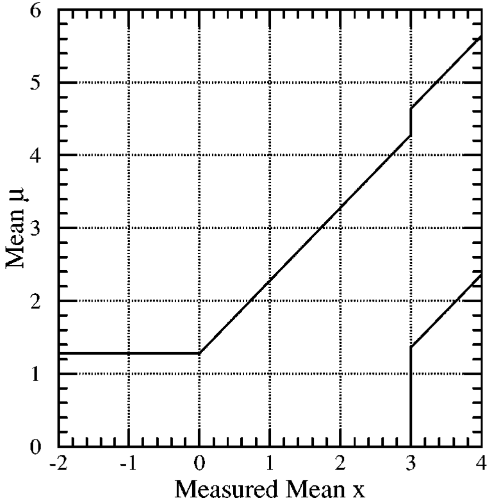


FIG. 4. Plot of confidence belts implicitly used for 90% C.L. confidence intervals (vertical intervals between the belts) quoted by flip-flopping physicist X, described in the text. They are not valid confidence belts, since they can cover the true value at a frequency less than the stated confidence level. For $1.36 < \mu < 4.28$, the coverage (probability contained in the horizontal acceptance interval) is 85%.

TABLE X. Our confidence intervals for the mean μ of a Gaussian, constrained to be non-negative, as a function of the measured mean x_0, for commonly used confidence levels. Italicized intervals correspond to cases where the goodness-of-fit probability (Sec. IV C) is less than 1%. All numbers are in units of σ.

x_0	68.27% C.L.	90% C.L.	95% C.L.	99% C.L.	x_0	68.27% C.L.	90% C.L.	95% C.L.	99% C.L.
−3.0	*0.00, 0.04*	*0.00, 0.26*	*0.00, 0.42*	*0.00, 0.80*	0.1	0.00, 1.10	0.00, 1.74	0.00, 2.06	0.00, 2.68
−2.9	*0.00, 0.04*	*0.00, 0.27*	*0.00, 0.44*	*0.00, 0.82*	0.2	0.00, 1.20	0.00, 1.84	0.00, 2.16	0.00, 2.78
−2.8	*0.00, 0.04*	*0.00, 0.28*	*0.00, 0.45*	*0.00, 0.84*	0.3	0.00, 1.30	0.00, 1.94	0.00, 2.26	0.00, 2.88
−2.7	*0.00, 0.04*	*0.00, 0.29*	*0.00, 0.47*	*0.00, 0.87*	0.4	0.00, 1.40	0.00, 2.04	0.00, 2.36	0.00, 2.98
−2.6	*0.00, 0.05*	*0.00, 0.30*	*0.00, 0.48*	*0.00, 0.89*	0.5	0.02, 1.50	0.00, 2.14	0.00, 2.46	0.00, 3.08
−2.5	*0.00, 0.05*	*0.00, 0.32*	*0.00, 0.50*	*0.00, 0.92*	0.6	0.07, 1.60	0.00, 2.24	0.00, 2.56	0.00, 3.18
−2.4	*0.00, 0.05*	*0.00, 0.33*	*0.00, 0.52*	*0.00, 0.95*	0.7	0.11, 1.70	0.00, 2.34	0.00, 2.66	0.00, 3.28
−2.3	0.00, 0.05	0.00, 0.34	0.00, 0.54	0.00, 0.99	0.8	0.15, 1.80	0.00, 2.44	0.00, 2.76	0.00, 3.38
−2.2	0.00, 0.06	0.00, 0.36	0.00, 0.56	0.00, 1.02	0.9	0.19, 1.90	0.00, 2.54	0.00, 2.86	0.00, 3.48
−2.1	0.00, 0.06	0.00, 0.38	0.00, 0.59	0.00, 1.06	1.0	0.24, 2.00	0.00, 2.64	0.00, 2.96	0.00, 3.58
−2.0	0.00, 0.07	0.00, 0.40	0.00, 0.62	0.00, 1.10	1.1	0.30, 2.10	0.00, 2.74	0.00, 3.06	0.00, 3.68
−1.9	0.00, 0.08	0.00, 0.43	0.00, 0.65	0.00, 1.14	1.2	0.35, 2.20	0.00, 2.84	0.00, 3.16	0.00, 3.78
−1.8	0.00, 0.09	0.00, 0.45	0.00, 0.68	0.00, 1.19	1.3	0.42, 2.30	0.02, 2.94	0.00, 3.26	0.00, 3.88
−1.7	0.00, 0.10	0.00, 0.48	0.00, 0.72	0.00, 1.24	1.4	0.49, 2.40	0.12, 3.04	0.00, 3.36	0.00, 3.98
−1.6	0.00, 0.11	0.00, 0.52	0.00, 0.76	0.00, 1.29	1.5	0.56, 2.50	0.22, 3.14	0.00, 3.46	0.00, 4.08
−1.5	0.00, 0.13	0.00, 0.56	0.00, 0.81	0.00, 1.35	1.6	0.64, 2.60	0.31, 3.24	0.00, 3.56	0.00, 4.18
−1.4	0.00, 0.15	0.00, 0.60	0.00, 0.86	0.00, 1.41	1.7	0.72, 2.70	0.38, 3.34	0.06, 3.66	0.00, 4.28
−1.3	0.00, 0.17	0.00, 0.64	0.00, 0.91	0.00, 1.47	1.8	0.81, 2.80	0.45, 3.44	0.16, 3.76	0.00, 4.38
−1.2	0.00, 0.20	0.00, 0.70	0.00, 0.97	0.00, 1.54	1.9	0.90, 2.90	0.51, 3.54	0.26, 3.86	0.00, 4.48
−1.1	0.00, 0.23	0.00, 0.75	0.00, 1.04	0.00, 1.61	2.0	1.00, 3.00	0.58, 3.64	0.35, 3.96	0.00, 4.58
−1.0	0.00, 0.27	0.00, 0.81	0.00, 1.10	0.00, 1.68	2.1	1.10, 3.10	0.65, 3.74	0.45, 4.06	0.00, 4.68
−0.9	0.00, 0.32	0.00, 0.88	0.00, 1.17	0.00, 1.76	2.2	1.20, 3.20	0.72, 3.84	0.53, 4.16	0.00, 4.78
−0.8	0.00, 0.37	0.00, 0.95	0.00, 1.25	0.00, 1.84	2.3	1.30, 3.30	0.79, 3.94	0.61, 4.26	0.00, 4.88
−0.7	0.00, 0.43	0.00, 1.02	0.00, 1.33	0.00, 1.93	2.4	1.40, 3.40	0.87, 4.04	0.69, 4.36	0.07, 4.98
−0.6	0.00, 0.49	0.00, 1.10	0.00, 1.41	0.00, 2.01	2.5	1.50, 3.50	0.95, 4.14	0.76, 4.46	0.17, 5.08
−0.5	0.00, 0.56	0.00, 1.18	0.00, 1.49	0.00, 2.10	2.6	1.60, 3.60	1.02, 4.24	0.84, 4.56	0.27, 5.18
−0.4	0.00, 0.64	0.00, 1.27	0.00, 1.58	0.00, 2.19	2.7	1.70, 3.70	1.11, 4.34	0.91, 4.66	0.37, 5.28
−0.3	0.00, 0.72	0.00, 1.36	0.00, 1.67	0.00, 2.28	2.8	1.80, 3.80	1.19, 4.44	0.99, 4.76	0.47, 5.38
−0.2	0.00, 0.81	0.00, 1.45	0.00, 1.77	0.00, 2.38	2.9	1.90, 3.90	1.28, 4.54	1.06, 4.86	0.57, 5.48
−0.1	0.00, 0.90	0.00, 1.55	0.00, 1.86	0.00, 2.48	3.0	2.00, 4.00	1.37, 4.64	1.14, 4.96	0.67, 5.58
0.0	0.00, 1.00	0.00, 1.64	0.00, 1.96	0.00, 2.58	3.1	2.10, 4.10	1.46, 4.74	1.22, 5.06	0.77, 5.68

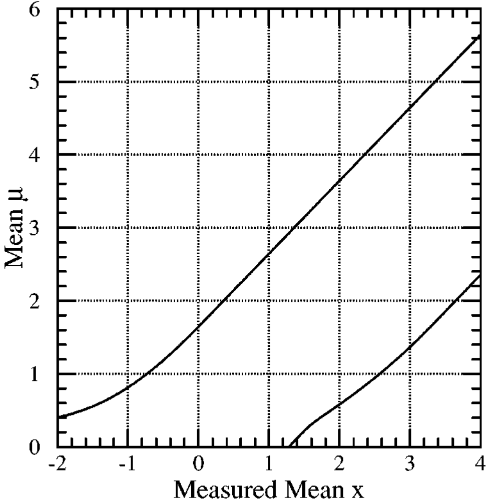
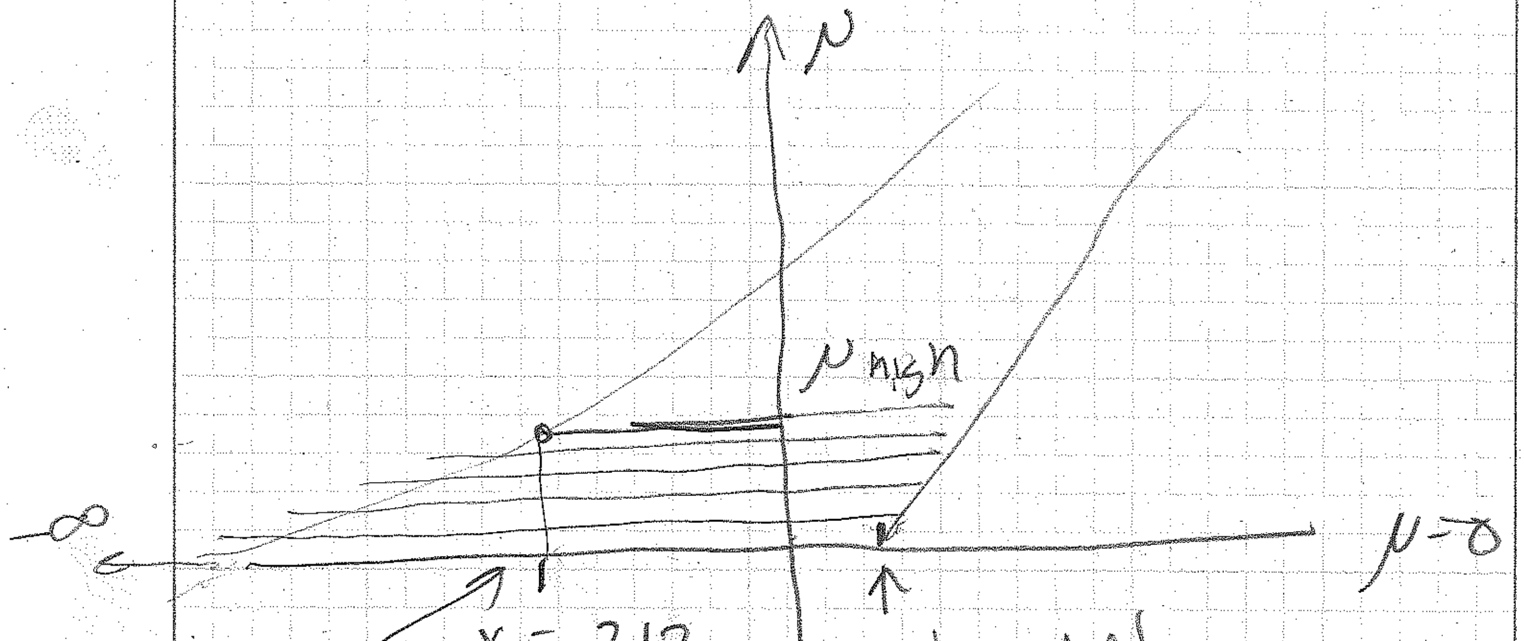


FIG. 10. Plot of our 90% confidence intervals for the mean of a Gaussian, constrained to be non-negative, described in the text.



single sided!

$\mu_{Low} = 0$

68% $z_0 \simeq 0.48\sigma$

95% is 1.64

68.3% $\Rightarrow \mu_{High} = 0.06 = \frac{s}{\sqrt{50+s}}$ | $s \simeq 0.426$

95% $\mu_{High} = 0.59 = \frac{s}{\sqrt{50+\cancel{s}}}$

$s \simeq 4.35$

Also a "presciption"!