1) Counting with no background
   - a) "inverse" problem, $1/\sqrt{n}$
   - b) Bayesian View
   - c) Frequentist or Classical View

2) Simple inclusion of background
   - a) Relative error degrades
   - b) how do you estimate background ?

3) Limits + discovery
   - a) Look for signal on top of expected background
   - b) When seems like no excess over background, what do you do ?
   - c) How do you discover, when it seems like "too many" events for background ?

1) <u>No background.</u> somehow, "Gedanken"
Run your experiment, cuts, etc.
See, say, 25 events.
Feldman-Cousins calls this $n_0 = 25$
notation

What you know (with scant error) —
you saw 25 events. Not much of an error on that. But.. who cares?

What everyone wants to know: μ_t, (FC notation), the "true" value of the mean # of events you expected. Very crudely, you should know..

Symmetric ≈68% chance $25-\sqrt{25} < \mu_t < 25+\sqrt{25}$

$20 < \mu_t < 30$

or $\mu_t = 25 \pm 5$ "1σ"

≈95% chance $25-2\sqrt{25} < \mu_t < 25+2\sqrt{25}$

$15 < \mu_t < 35$

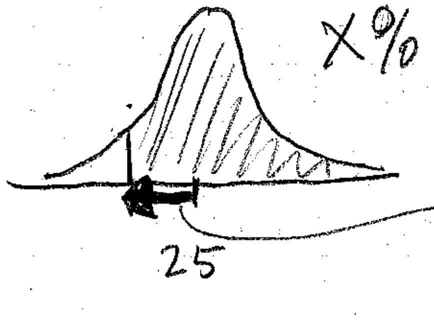
"Double Sided"

Single Sided!:

90% only 1.28σ!

$25 - 1.28\cdot\sqrt{5} = 22.1$

95% $1.64\sigma = 16.8$



Bayesian, Classical

$$\underset{\text{deduce}}{P(n_0};\underset{\text{known}}{\mu_+)} \cong \frac{1}{\sqrt{2\pi\mu_+}} e^{-\frac{(n_0-\mu_+)^2}{2\mu_+}}$$

limiting case of $\approx \frac{\mu_+^{n_0}}{n_0!} e^{-\mu_+}$

but everyone wants to know $P(\underset{\text{deduce}}{\mu_+}; \underset{\text{known}}{n_0})$

__Bayesian__: OK, get to work, but: flaws!

__Classical/Frequentist__: who cares what everyone wants, do what is right

Think different.

Bayes Theorem

$$P(A+B) = \underbrace{P(A;B)}_{\text{probability of A, given B}} \underbrace{P(B)}_{\text{prob of B}} = \underbrace{P(B;A)}_{\text{prob B, given A}} \underbrace{P(A)}_{\text{prob of A}}$$

simplest case: $P(A;B) = \underset{\text{don't care abut B}}{P(A)}$ $\quad P(B;A) = \underset{\text{don't care abut A}}{P(B)}$

statistical independence

$$P(A+B) = P(A)P(B) = P(B)P(A)$$

__ok__: $\underbrace{P(\mu_+;n_0)}_{\text{want}} \underset{\text{"just a constant" no is known!}}{P(n_0)} = \underbrace{P(n_0;\mu_+)}_{\text{Poisson}} \underset{\text{BELIEF like: } \mu_+>0}{P(\mu_+)}$ "prior density"

Lots of arguments over $P(\mu_+)$. It you just forget about it --

$$P(\mu_+; n_0) \cong \frac{1}{\sqrt{2\pi\mu_+}} e^{-\frac{(n_0-\mu_+)^2}{2\mu_+}} \left(\frac{P(\mu_+)}{P(n_0)}\right)$$

↑ NOTE SWAP

↑ some people can't give it up

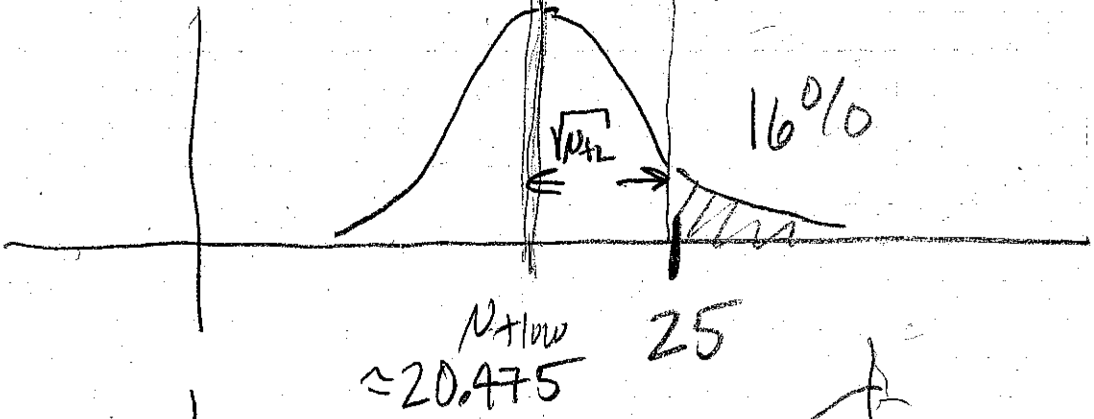
Alternative **Think differently** (argue about which $\sqrt{\mu}$ ↑ $\sqrt{n_0}$?

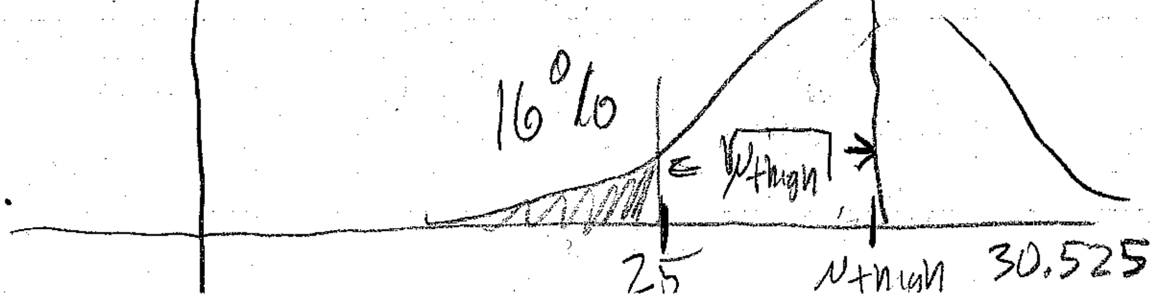
"Neyman Construction"

Want, say, symmetric 68% limits.

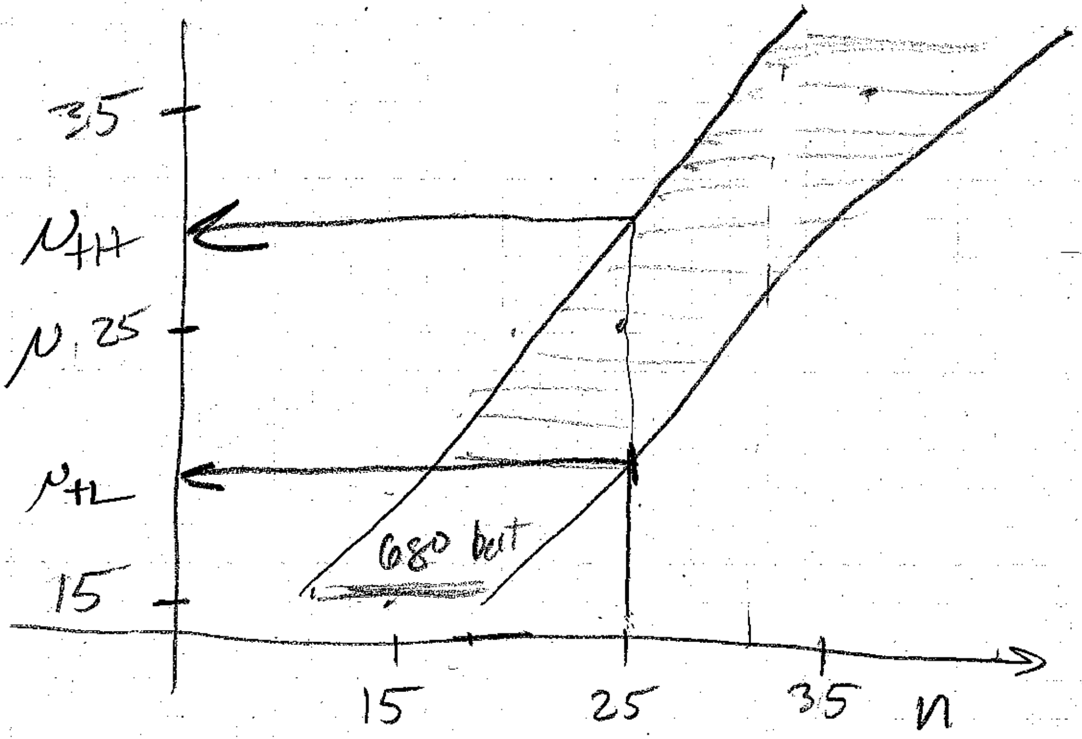
↑ 32% → 16%, 16%

→ imagine that μ_{+low} that, upon repeated executions of your experiment, would give 16% of n's above n_0





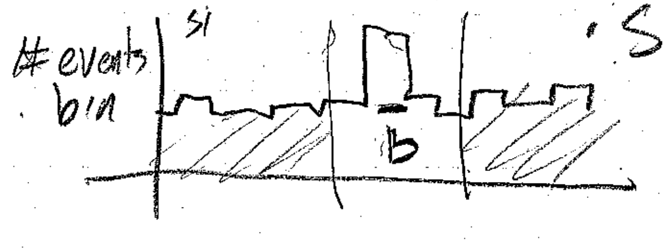
Visualization ("Neyman" Construction)



Advantage: very "experimental"

Me: Bayes when it feels good
Classical "

Background: confounding
"Somehow" know it (must estimate errors)
$\mu_+ = 25$, $b = 50.0000$ ← AVERAGE
How? • Beam On/Off
• Sidebands



Crudely: Now measure $75 \pm \sqrt{75}$

75 ± 8.7

but 50.0000 of those are background (on average)

would estimate $\mu_+ = 25 \pm 8.7$

↑ Bigger than 5 because background fluctuates

Relative error now...

$\frac{8.7}{25} \approx 35\%$ → not $\sqrt{75+25} = 10$

was: $\frac{5}{25} \approx 20\%$

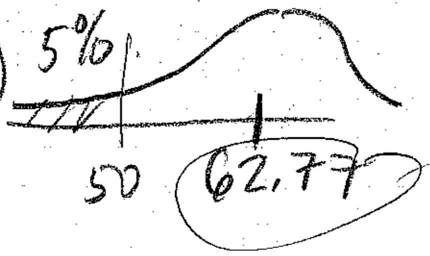
Bad, but not as bad maybe as you expected (2B/1sig 50/25)

95% discovery ①

→ $50 + 1.64 \cdot \sqrt{50}$

≈62

②



which? (don't argue, do a better experiment!)

→ Limitsmanship!

THE RUB

→ In previous experiment, expect 50 bkgd, want to measure an unknown signal, maybe now not 25, but small.

→ Suppose you see 35 events.

→ what do you conclude?

Ⓐ Is $\mu_+ = 35 - 50 = -15$?

→ "central value is !"

→ $\mu_+ < 0$ "non physical"

Ⓑ Is b really equal to 50?

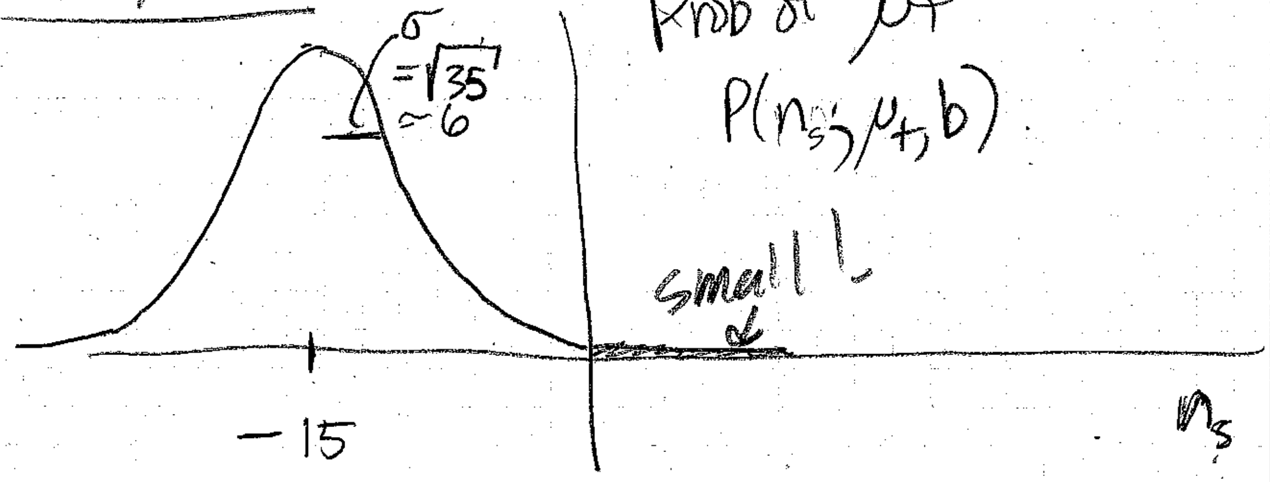
worry

$$\frac{35-50}{\sqrt{50}} \simeq \frac{-15}{7} \sim -2\sigma$$

"could happen, ~ 1 in 40"

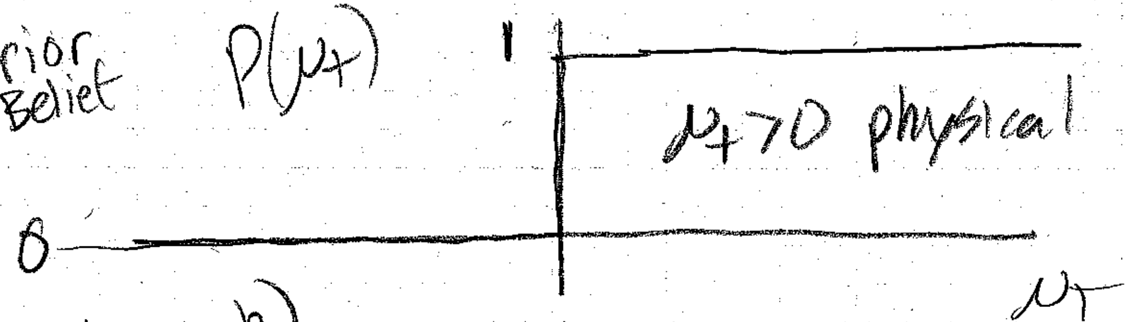
Dealing with this situation is what causes interminable discussions between Bayesians and Classicals.

Bayesian

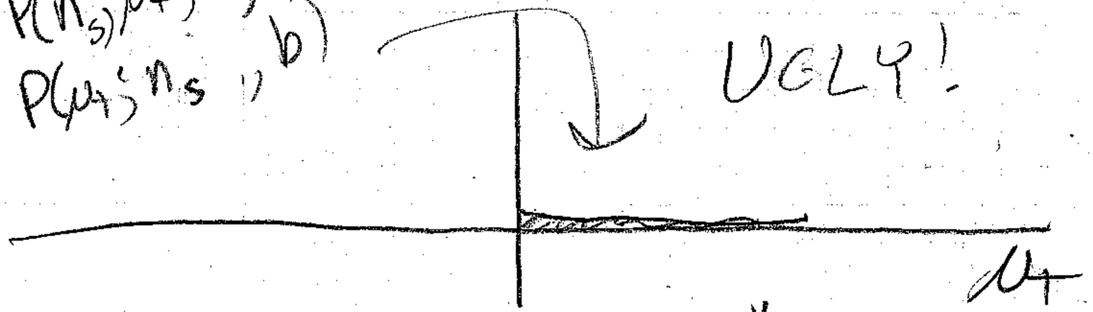


Prob of μ_+
$P(n_s; \mu_+, b)$

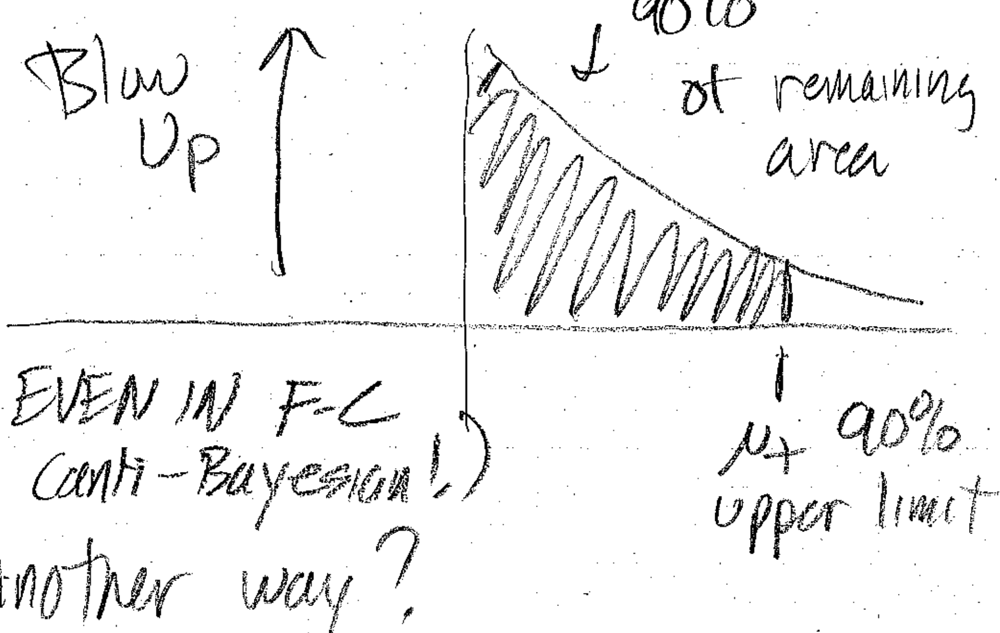
Prior Belief $P(\mu_+)$



$P(\mu_+)\,P(n_s; \mu_+, b)$
$= P(\mu_+; n_s, b)$



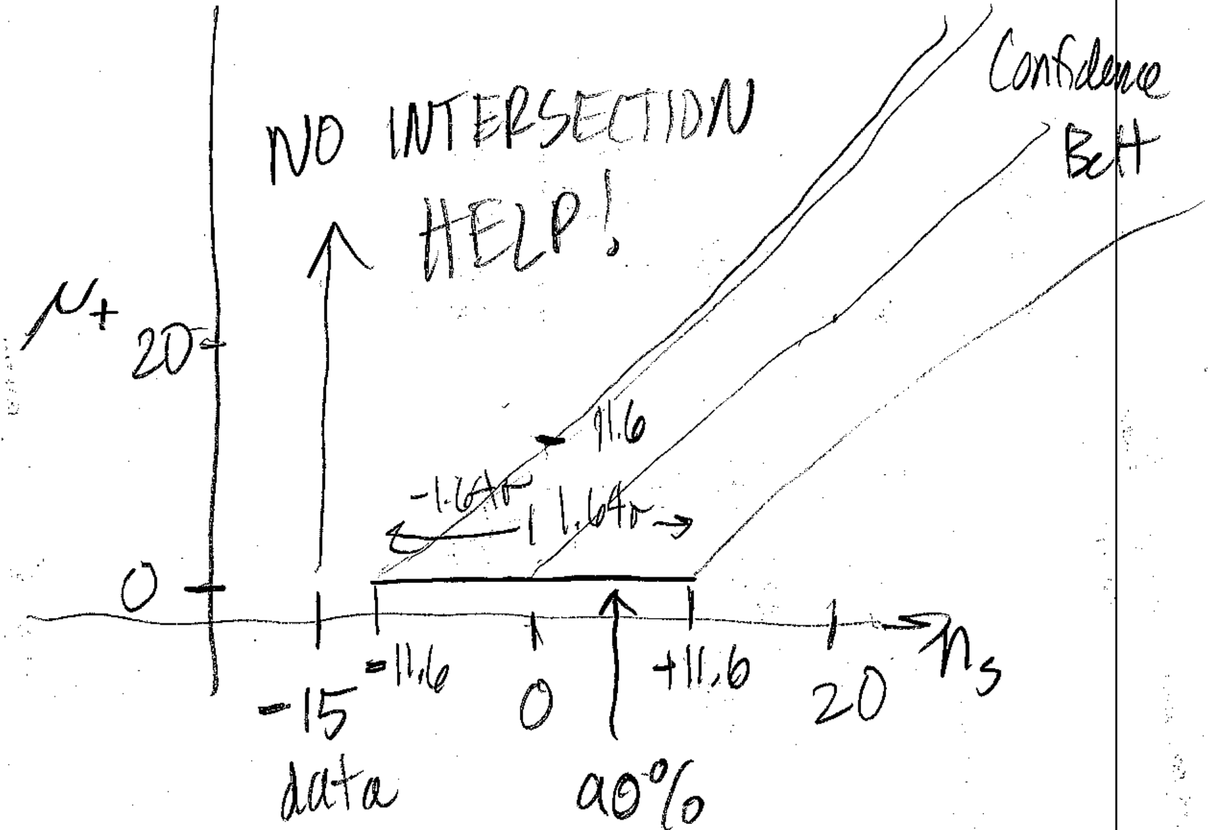
Blow Up



NOT EVEN IN F-C
(anti-Bayesian!)

Another way?

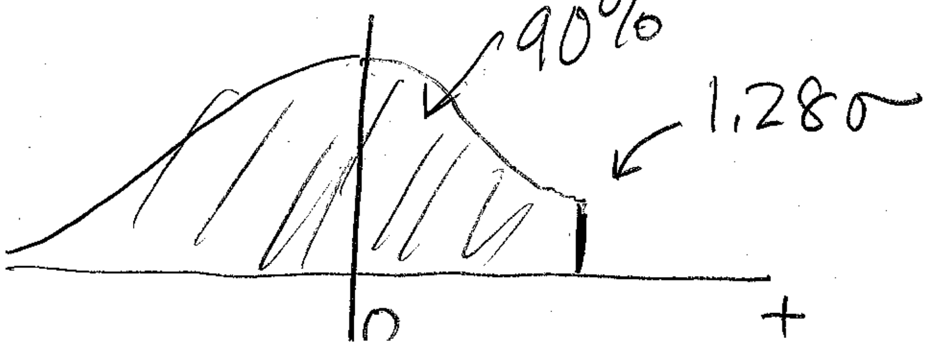
Another way? Yes, $\boxed{\text{Frequentist}}$

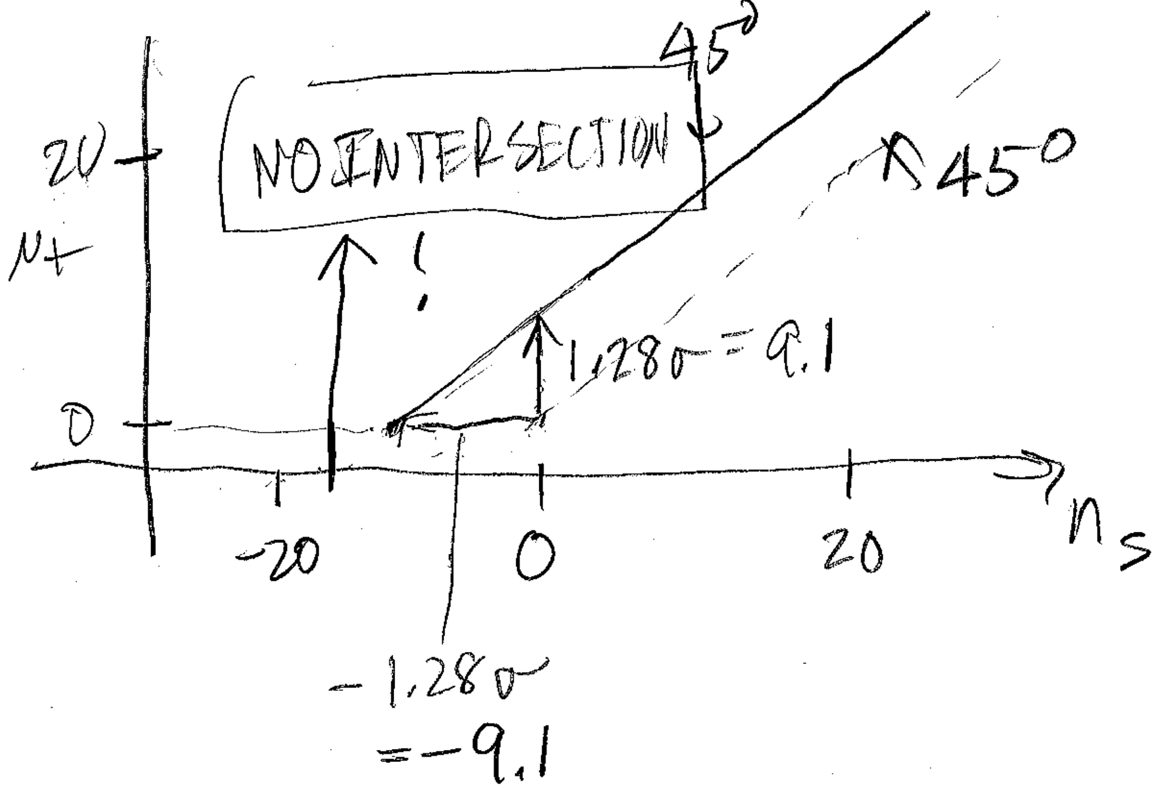


determined by...
Background!
$1.64 \times \sqrt{50} = 11.6$

Doesn't work either...

WAIT.. UPPER LIMIT CONFIDENCE BELT ACTUALLY DESIRED..





How to fix this?

→ Feldman-Cousins.

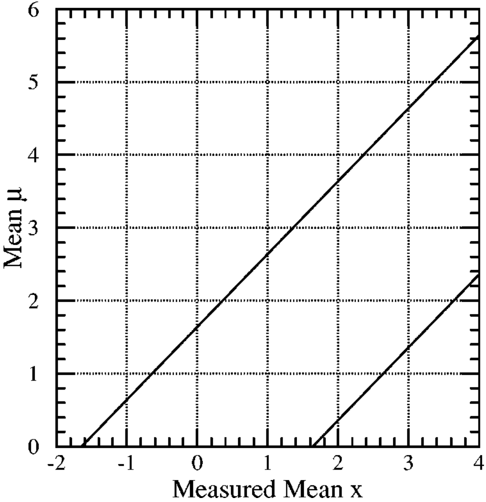


FIG. 3. Standard confidence belt for 90% C.L. central confidence intervals for the mean of a Gaussian, in units of the rms deviation.

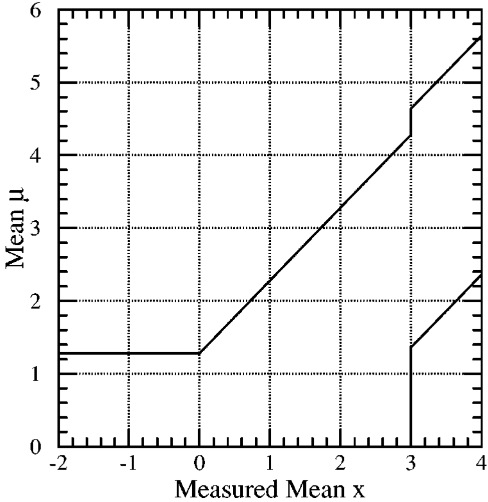


FIG. 4. Plot of confidence belts implicitly used for 90% C.L. confidence intervals (vertical intervals between the belts) quoted by flip-flopping physicist X, described in the text. They are not valid confidence belts, since they can cover the true value at a frequency less than the stated confidence level. For $1.36 < \mu < 4.28$, the coverage (probability contained in the horizontal acceptance interval) is 85%.

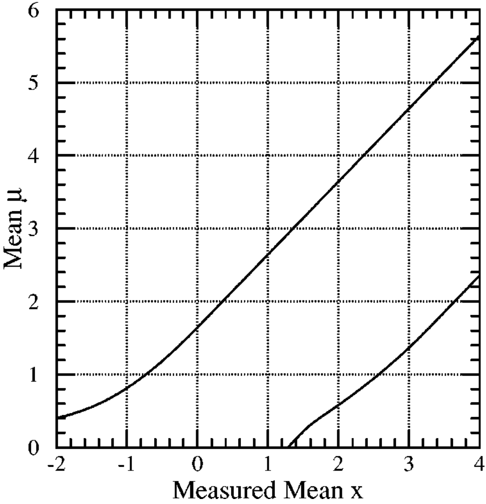


FIG. 10. Plot of our 90% confidence intervals for the mean of a Gaussian, constrained to be non-negative, described in the text.