Physics 25 Practice Final
3 hours, 3 Pages
Final: 8am, Wed., June 11, 1640 Broida

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Sunday, June 8

Write your answers in blue books. Calculators and two pages of notes allowed. No textbooks allowed. Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Make it clear what you think is known, and what is unknown and to be solved for. Except for extremely simple problems, derive symbolic answers, and then plug in numbers (if necessary) after a symbolic answer is available. **Put a box around your final answer... otherwise we may be confused about which answer you really mean, and you could lose credit.**

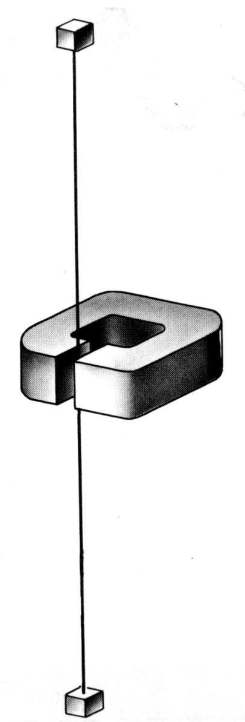

Figure 1: For use in Problem 1.

1. As shown in Fig. 1, a taut wire passes through the gap of a small magnet, where the field strength is 10,000 gauss. The length of the wire in the gap is 1 cm. Calculate the amplitude of the induced alternating voltage when the wire is vibrating at its fundamental frequency of 1000 Hz with an amplitude of 0.1 cm.

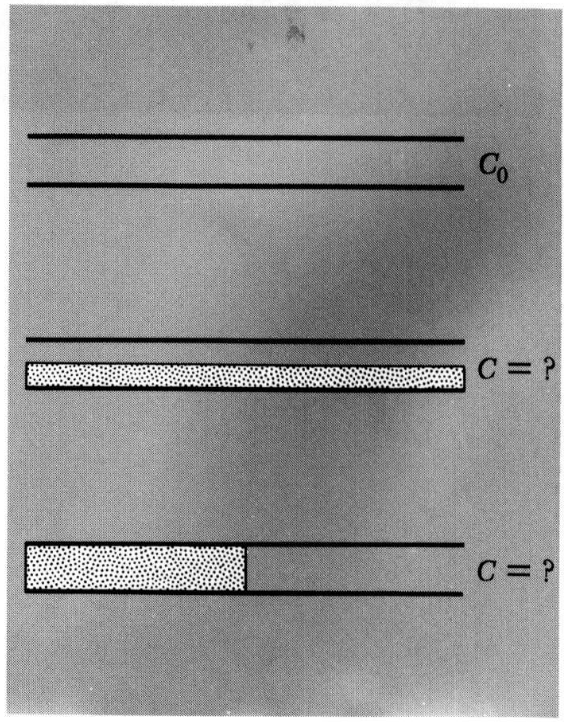

Figure 2: For use in Problem 2.

2. Fig. 2 shows three capacitors of the same area and plate separation. Call the capacitance of the vacuum capacitor C_0. Each of the other two is half-filled with a dielectric, with the same dielectric constant ϵ, but differently deployed, as shown. Find the capacitance of each of these two capacitors. (Neglect edge effects).

3. You know that the *total* energy (including rest energy) of a particle of rest mass m is $E = \sqrt{(mc^2)^2 + (cp)^2}$, where c is the speed of light and p is the momentum. Evaluate the phase velocity and group velocity when the rest mass is small, but not negligible, compared to the total energy or the momentum; expand the square root to lowest order in the mass m. Use the quantization conditions relating the *total* E to the circular frequency ω and p to the wavenumber k. Keep one term in the dispersion relation beyond the simple expression that would apply for a photon that has zero rest mass. Express your final results for the phase and group velocities in terms of mc^2 and cp.

4. In this problem, look into a Bohr atom, described in the non-relativistic approximation, that consists of two masses, each of mass m, which attract purely through the *gravitational* force.

   (a) Find the radius r of the lowest Bohr orbit in terms of $\hbar$, G, and m.

   (b) Find the binding energy in terms of the same quantities.

   (c) Find the value of m that results in a binding energy that exceeds $2mc^2$ in terms of the same quantities and the speed of light, c. This mass m is (approximately) known as the *Planck* mass, and describes a point mass so large that a pair of them, when stuck together, have zero rest energy and thus mass due to binding energy.

   (d) Evaluate that m numerically; you can use $G = 6.67 \times 10^{-11}\mathrm{m}^3/(\mathrm{kg\ s}^2)$, $\hbar = 1.05 \times 10^{-34}\,\mathrm{m}^2\mathrm{kg/s}$, $c = 3.00 \times 10^8$ m/s, and evaluate m in kilograms. Or, if you are comfortable, you can use $G/(\hbar c^5) = 6.71 \times 10^{-39}\,1/\mathrm{GeV}^2$, and you can evaluate mc^2 in GeV, which is 10^9eV.

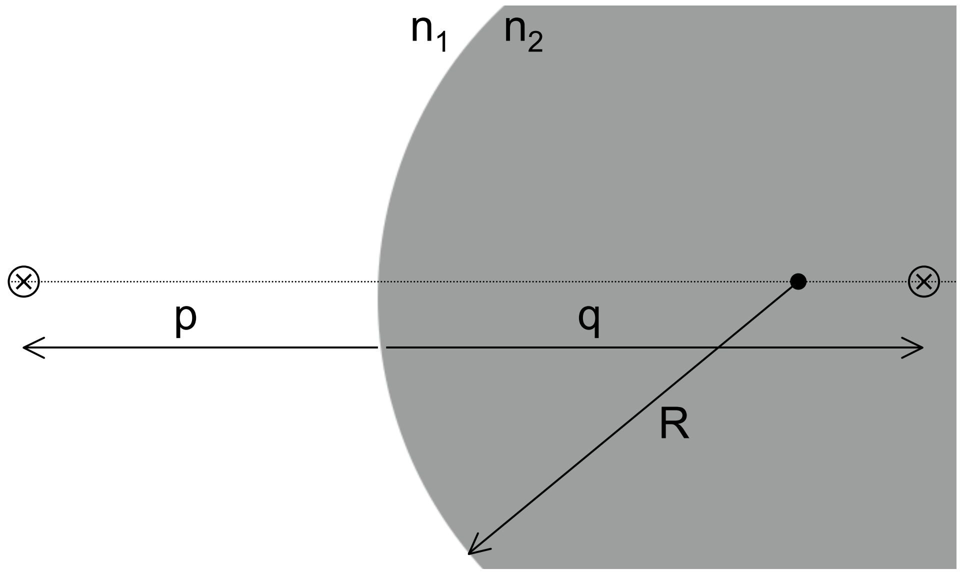

Figure 3: For use in Problem 5.

5. Two regions of different index of refraction, n_1 and n_2 are separated by a spherical surface of radius R, which is centered in the region with index n_2, as shown in Fig. 3. An object is displaced by p to the left of the boundary; the image is formed at a displacement q to the right of the boundary. The displacements p and q are related through an equation:

$$\frac{n_1}{p} + \frac{A}{q} = \frac{1}{f}$$

Find the constants A and f (or $1/f$) in terms of the other quantities. Assume all angles are extremely small.

6. A *proton* with mass $mc^2 = 938\,\mathrm{MeV}$ (where $1\,\mathrm{MeV}{=}10^6\,\mathrm{eV}$) is in an infinite square well in one dimension that extends from $x = -a = -10^{-11}$ meters to $x = a = 10^{-11}$ meters.

   (a) Numerically give the energy and wavenumber of the ground state.

   (b) Numerically give the energy and wavenumber of the first excited state.

   (c) The normalized ground state is described by the wavefunction $\psi_1(x)$, and the first excited state is described by $\psi_2(x)$. At the time $t = 0$ the proton is in the wavefunction:

   $$\psi(x,0) = \frac{1}{\sqrt{2}}\psi_1(x) + \frac{1}{\sqrt{2}}\psi_2(x)$$

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      i. Give the probability $P(x,0)$ in terms of $\psi_1(x)$ and $\psi_2(x)$ (no need to give the specific functions).

      ii. Give the probability $P(x,t)$ in terms of the same quantities, as well as time, the ground state energy E_1, the first excited state energy E_2, $\hbar$, and time t.

      iii. The probability $P(x,t)$ is periodic with period T. Numerically evaluate T.