Physics 25 Problem Set 10

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due Wednesday, June 11 at the final (8am)

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

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1. Design an infinite square well that traps an electron, where the difference in energy between the ground and first excited states is just right to emit a photon with a wavelength of 4.27 μm. This photon wavelength is just right to be absorbed by CO_2, and absorption of this photon is therefore just right to monitor atmospheric CO_2 concentrations, for example, due to global warming.

   (a) What should be the width of the square well, in nanometers?

   (b) What are the energies of the ground and first excited states, in eV?

   (c) What are the normalized wavefunctions of the first and second excited states?

   (d) Suppose there is an initial state where at $t = 0$ the probability of finding the electron in the ground state is 1/2, and the probability of finding the electron in the first excited state is 1/2. Derive an expression for $P(x, t)$, the probability of finding the electron at x as a function of time. Find the period T for $P(x, t)$ to return to $P(x, 0)$, and plot $P(x, t)$ for $t = 0, T/4, T/2, 3T/4, T$. Assume the complex phase between the first and second excited states' amplitudes is 0.

2. In reality, an infinite square well is impractical, but trapping an electron in a potential that is infinite for $x < 0$, 0 when $0 \leq x \leq a$, and $= V_0$ when $x > a$ is more practical. In this problem, design a potential of this type where the energy between the ground and first excited state is just right to emit a photon that has wavelength 4.27 μm, appropriate for absorption by CO_2.

   (a) Make a plot of the potential.

   (b) Assume that:

   $$\frac{2m_0V_0a^2}{\hbar^2} = (2\pi)^2$$

   How many bound states are present in this potential and why? Hint: look at Equation 5.16, and Figure 5-10, but look out... the potential here is not quite the same as that in Figure 5-9, but some of the bound states of this problem end up having the same characteristics of those in Figure 5-9.

   (c) Determine the characteristics of the first two bound states using Equation 5.16 for α_1 and α_2, where α is defined on pages 167 and 168. Ans: $\alpha_1 = 2.698$, $\alpha_2 = 5.284$. Of course, if you describe a successful technique to arrive at these numbers, you'll get full credit.

   (d) Numerically evaluate a (in nm) and V_0 (in eV).

   (e) For the ground state, what is the probability of finding the electron with x satisfying $0 \leq x \leq a$?
