1. (a) $\boxed{a_0 = 0.53 \cdot 10^{-8}\,\text{cm} = 0.53 \cdot 10^{-10}\,\text{m}}$

OK to memorize this. If not memorized,

$$\boxed{a_0 = \frac{1}{\alpha} \cdot \frac{\hbar}{m_e c}} = \frac{1}{\alpha} \frac{\hbar c}{m_e c^2}$$

$$= \frac{1}{(1/137)} \cdot \frac{197\ \text{MeV} \cdot \text{fm}}{0.511\ \text{MeV}}$$

$$= 53{,}000\ \text{fm} = 0.53 \cdot 10^5 \cdot 10^{-15}\ \text{m}$$

$$\boxed{a_0 = 0.53 \cdot 10^{-10}\ \text{m} = 0.53 \cdot 10^{-8}\ \text{cm}}$$

(b) OK to recall <u>scaling</u> of Bohr radius:

$$a = \frac{1}{\alpha Z} \cdot \frac{\hbar}{mc}$$

$$= \frac{1}{Z} \cdot \frac{m_e}{m} \cdot a_0$$

$$= \frac{1}{10} \cdot \frac{1}{200} \cdot 0.53 \cdot 10^{-10}\ \text{m}$$

$$= \frac{0.53}{2} \cdot 10^{-13}\ \text{m}$$

$$\boxed{a = 0.27 \cdot 10^{-13}\ \text{m} = 0.27 \cdot 10^{-11}\ \text{cm}}$$

(c) OK to recall:

$$B = -\frac{1}{2} \alpha^2 Z^2 m_e c^2$$

$$B + m_e c^2 = \left(1 - \frac{1}{2} \alpha^2 Z^2\right) m_e c^2$$

$$= \left(1 - \frac{1}{2} \left(\frac{1}{137}\right)^2 \cdot 100^2\right) \cdot 0.511$$

$$\boxed{B + m_e c^2 = 0{,}733 \cdot 0{,}511 = 0{,}38 \text{ MeV}}$$

(d) $\alpha = 1/137$

(e) $\hbar c = 197 \text{ eV} \cdot \text{nm}$

(f) $\lambda \nu = c \; ; \; \lambda = \frac{c}{\nu} = \frac{hc}{E} = \frac{2\pi \hbar c}{E}$

$$\lambda = \frac{2\pi \cdot 197 \text{ MeV} \cdot \text{fm}}{5}$$

$$= 250 \text{ fm} = 250 \cdot 10^{-15} \text{ m}$$

$$\boxed{\lambda = 2{,}5 \cdot 10^{-13} \text{ m}}$$

(g) $\frac{p^2}{2m} = T$

$$p = \sqrt{2mT}$$

$$\lambda = \frac{h}{p} = \frac{2\pi \hbar}{\sqrt{2mT}} = \frac{2\pi \hbar c}{\sqrt{2mc^2 T}}$$

$$= \frac{2\pi \cdot 197}{\sqrt{2 \cdot 3726 \cdot 5}}$$

$$\boxed{\lambda = 6.4 \text{ fm} = 6.4 \cdot 10^{-15} \text{ m}}$$

2. (a) Force $F = -\frac{dV}{dr} = -\gamma$

<u>Force balance:</u>

$$\gamma = \frac{mv^2}{r}$$

<u>Angular momentum:</u>

$$mvr = n\hbar$$

$$v = \frac{n\hbar}{mr}$$

$$\gamma = \frac{m}{r}\left(\frac{n\hbar}{mr}\right)^2 = \frac{n^2\hbar^2}{mr^3}$$

$$\boxed{r = \left(\frac{n^2\hbar^2}{m\gamma}\right)^{1/3} = \left(\frac{\hbar^2}{m\gamma}\right)^{1/3} \quad n=1}$$

$$v = \frac{n\hbar}{mr} = \frac{n\hbar}{m} \cdot \left(\frac{m\gamma}{n^2\hbar^2}\right)^{1/3}$$

$$\boxed{v = \left(\frac{n\hbar\gamma}{m^2}\right)^{1/3} = \left(\frac{\hbar\gamma}{m^2}\right)^{1/3} \quad n=1}$$

$$E = \frac{1}{2}mv^2 + \gamma r$$

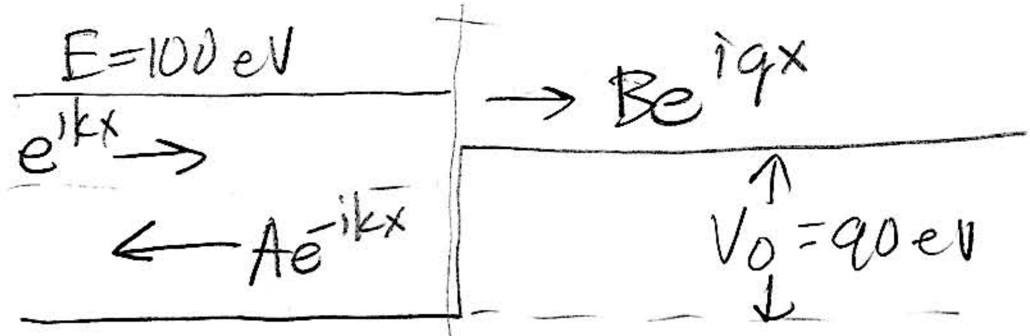
$$= \frac{1}{2}m \cdot \left(\frac{n\hbar\gamma}{m^2}\right)^{2/3} + \gamma \cdot \left(\frac{n^2\hbar^2}{m\gamma}\right)^{1/3}$$

$$\boxed{E = \frac{3}{2} \cdot \left(\frac{n^2\hbar^2\gamma^2}{m}\right)^{1/3} = \frac{3}{2}\left(\frac{\hbar^2\gamma^2}{m}\right)^{1/3} \quad n=1}$$

(b) $\boxed{\frac{r_2}{r_1} = \frac{2^{2/3}}{1^{2/3}} = 2^{2/3}}$

(c) $\boxed{\frac{E_2}{E_1} = \frac{2^{2/3}}{1^{2/3}} = 2^{2/3}}$

3.



$$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad q = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

$x=0$

$\psi(0)$: $\quad 1 + A = B$

$\psi'(0)$ $\quad k(1-A) = qB$

$$k(1-A) = q(1+A)$$

$$k - q = (k+q)\cdot A$$

$$A = \frac{k-q}{k+q}$$

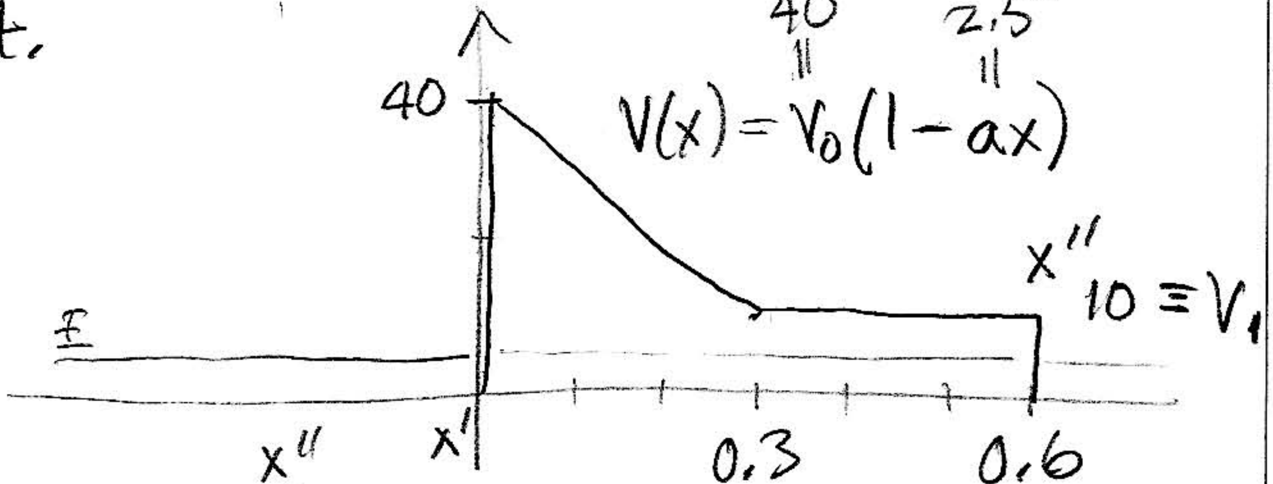
$$R = |A|^2 = \left|\frac{k-q}{k+q}\right|^2 = \left|\frac{\sqrt{E} - \sqrt{E-V_0}}{\sqrt{E} + \sqrt{E-V_0}}\right|^2$$

$$= \left|\frac{\sqrt{100} - \sqrt{10}}{\sqrt{100} + \sqrt{10}}\right|^2$$

$$\boxed{R = 0.27}$$

4.

$$V(x) = \overset{40}{\overset{\parallel}{V_0}}(1 - \overset{2.5}{\overset{\parallel}{a}}x)$$



$$\ln T = -2 \int_{x'}^{x''} \sqrt{\frac{2m}{\hbar^2}(V(x) - E)}\, dx$$

$$= -2 \int_0^{0.3} \sqrt{\frac{2m}{\hbar^2}(V_0(1-ax) - E)}\, dx - 2 \int_{0.3}^{0.6} \sqrt{\frac{2m}{\hbar^2}(V_1 - E)}$$

$$= -2\sqrt{\frac{2m}{\hbar^2}} \left[\int_0^{0.3} \sqrt{(V_0 - E) - V_0 a x}\, dx + \sqrt{V_1 - E}\,\Big|_{0.3}^{0.6} \right]$$

$$= -2\sqrt{\frac{2m}{\hbar^2}} \left[\frac{(V_0 - E - V_0 a x)^{3/2}}{-2/3\, V_0 a} \Bigg|_0^{0.3} + \sqrt{V_1 - E} \times 0.3 \right]$$

$$= -2 \frac{\sqrt{2mc^2}}{\hbar c} \underbrace{\left[\frac{-(40-5-30)^{3/2} + (40-5)^{3/2}}{\frac{2}{3} \cdot 40 \cdot 2.5} + 0.3 \cdot \sqrt{10-5} \right]}$$

$$= -2 \cdot \frac{\sqrt{2 \cdot 0.511 \cdot 10^6}}{197.3} \times 3.61 = -36.98$$

$$\ln T = -36.98$$

$$\boxed{T = e^{-36.98} = 8.7 \cdot 10^{-17}}$$

5. $$\frac{(\Delta p)^2}{2m} = E_1 = \frac{\hbar^2 \pi^2}{2ma^2}$$

$$(\Delta p) = \frac{\hbar \pi}{a}$$

$$\boxed{\Delta x \Delta p = \frac{a}{2} \cdot \frac{\hbar \pi}{a} = \frac{\pi}{2} \cdot \hbar}$$