Physics 25 Problem Set 7

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due Monday, May 22

Please make your work neat, clear, and easy to follow. It is hard to grade sloppy work accurately. Generally, make a clear diagram, and label quantities. Derive symbolic answers, and then plug in numbers after a symbolic answer is available.

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1. Quanta of the electromagnetic field (photons) have wavelength, frequency, and energy. One of those quantities is numerically specified below; numerically compute the other two.
   (a) A photon with a frequency in the 'High Frequency' or 'shortwave' radio band, coming from radio station KVOH in Simi Valley, with frequency 9.975 MHz.
   (b) A photon with a frequency in the middle of the FM radio band... 94 MHz.
   (c) A photon in the peak of the black body emission spectrum from a fevered child, with temperature of 104° F (use relationship 35a on page 24 of Wichmann to convert temperature in Kelvin to the peak wavelength).
   (d) A photon with just enough energy to eject an electron from a metal with work function of 5 eV.
   (e) A photon used in a dental x-ray, which has energy of 50 keV.
   (f) A photon from a nucleus, with energy of 2 MeV.
   (g) A photon that has come from a distant extreme star, with energy of 50 TeV.
   (h) The most energetic photons ever observed, which have been observed with the Fly's Eye, AGASA, and Auger experiments, which have energy of about 2×10^{18} eV.

2. An electron is initially infinitely far away from a nucleus with charge Ze. It falls into the nucleus, and ends up in the first Bohr orbit, which has been discussed in class. The energy it loses is converted to a photon; find the wavelength λ of the photon in natural atomic units, and then find the ratio λ/a_{0Z}, where a_{0Z} is the radius of the first Bohr orbit, and express that ratio in terms of Z, the fine structure constant α, and other fundamental constants like integers and π. For $Z = 1$, make a figure approximately to scale, showing the size of the Bohr atom compared to λ. Does the electric field from the photon vary much across the extent of the atom? Sometimes several photons of smaller energy are emitted when the electron falls in; will these photons have wavelengths that are shorter or longer than the one depicted in the figure?

3. We found the binding energy of a Bohr atom, $R_{\infty Z} = (1/2)Z^2\alpha^2 m_e c^2$ (equation 25b on page 60, when $Z = 1$). Although this equation was derived in the non-relativistic limit, let's use it to explore the physical phenomenon that can happen with Z gets very large.
   (a) If Z is just large enough, the binding energy of one electron might actually exceed $2m_e c^2$. Physically, a process of *pair creation* becomes possible when when Z is this large. What happens is an electron, initially infinitely far away, gets captured, and there is sufficient energy made available by the capture to create one *new* electron and an *anti-electron*, aka

a positron, which has the same rest mass as the electron, but opposite charge. If these two new particles are created, what happens next? Can the process repeat itself (assuming Z was just big enough so the binding energy of one electron just barely exceeded $2m_e c^2$)?

(b) Next, imagine two equal masses m bound together by *gravity*, instead of the electrostatic force. Don't worry about the reduced mass, instead, explain why it is approximately OK to replace the constants $Z\alpha$ of the Bohr atom with the constants $G_N m^2/(\hbar c)$, where G_N is the gravitational constant. How large must m be for the binding energy to exceed $2mc^2$? Numerically evaluate this m in grams; this is approximately the 'Planck Mass', where quantum mechanical effects become important for gravity. Can the process repeat this time?

4. Wichman, Chapter 2, page 87, problem 5.
