

$$\ln \tau(s) = -47.6 - 74.5 + \ln \sqrt{\frac{1}{E}} + 341 \sqrt{\frac{1}{E}}$$

$$\ln \tau(s) = -122 + \ln \sqrt{\frac{1}{E}} + 341 \cdot \sqrt{\frac{1}{E}}$$

E in MeV

① $E = 1$ MeV

$$\ln \tau(s) = -122 + 341 = 218$$

$$\log_{10} \tau = \log_{10} e \cdot \ln \tau$$

$$= 0.434 \ln \tau = 95$$

$$\tau = 10^{95} \text{ s} \approx \frac{10^{95}}{3.14 \cdot 10^7} = 3 \cdot 10^{87} \text{ y!}$$

② $E = 5$ MeV

$$\log_{10} \tau(s) = 12.8 \Rightarrow \underline{\underline{1.9 \cdot 10^5 \text{ y!}}}$$

Check:

$$\ln \tau = \underbrace{\ln \left[2.1 \cdot 10^{-21} \text{ s} \sqrt{\frac{1}{E}} \right]}_{\substack{E=1, \\ E=5,}} - 74.5 + \underbrace{341 \sqrt{\frac{1}{E}}}_{\substack{340.6 \\ 152.3}}$$

$$E=1, = -122.1$$

$$340.6$$

$$E=5, = -122.9$$

$$152.3$$

$$\ln \tau \approx -122.5 + 341 \cdot \sqrt{\frac{1}{E}}$$

$$\log_{10} \tau \approx -53 + 148 \cdot \sqrt{\frac{1}{E}}$$

(see plot).

Quantization as an Eigenvalue Problem

Simplest application... particle in a (1-d) box...

Schrödinger Equation... (1-d)

$$\underbrace{\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} \Psi(x,t) + V(x) \Psi(x,t)} = i\hbar \frac{\partial}{\partial t} \Psi(x,t)$$

Total Energy:

Kinetic $\rightarrow \frac{-\hbar^2}{2m} \frac{d^2}{dx^2}$

Potential $\rightarrow V(x)$

"Eigenvalue"

$$\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} \Psi(x,t) + V(x) \Psi(x,t) = E \Psi(x,t) = i\hbar \frac{\partial}{\partial t} \Psi(x,t)$$

↑
just a multiple!

Time dependence super easy

In this case,

$$\Psi(x,t) = \psi(x) e^{\frac{-iEt}{\hbar}}$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \psi(x) \cdot \left(i\hbar \frac{-iE}{\hbar} \right) e^{\frac{-iEt}{\hbar}}$$

$$= E \psi(x) e^{\frac{-iEt}{\hbar}} = E \Psi(x,t)$$

but $\psi(0) = \psi(a) = 0$

$$Ae^{+0} + Be^{-0} = 0$$

$$B = -A$$

$$A(e^{ika} - e^{-ika}) = 0$$

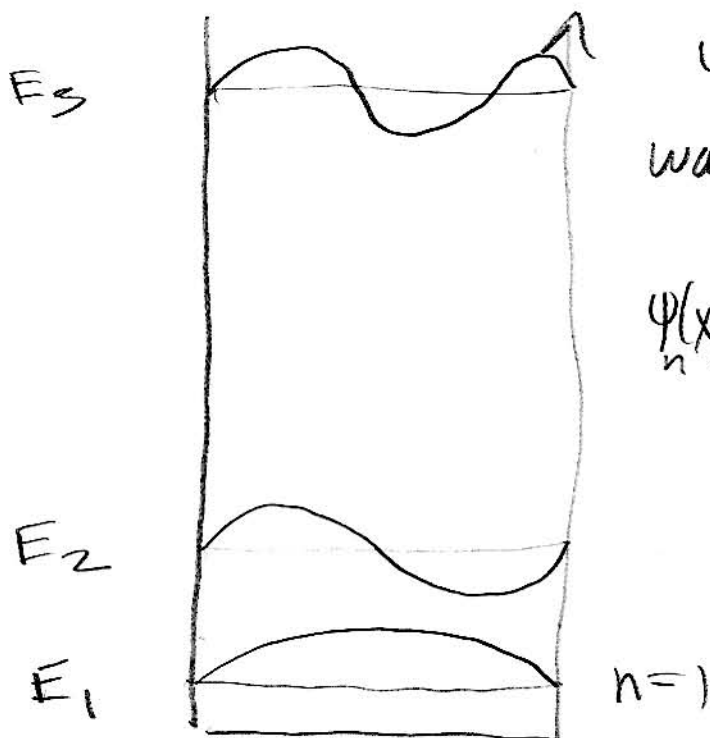
$$2Ai \sin(ka) = 0$$

$$ka = n\pi !$$

$$E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{a} \right)^2$$

$$= n^2 \underbrace{\frac{\hbar^2 \pi^2}{2ma^2}}_{E_1}$$

$$E_n = n^2 E_1$$



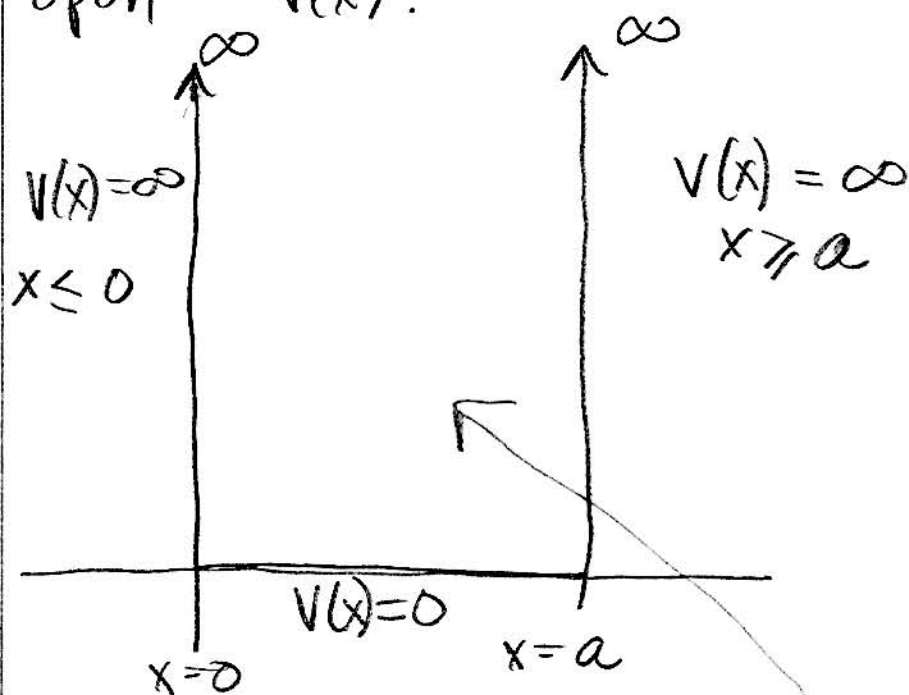
$$\psi(x,t)_a$$

want $\int_0^a dx |\psi(x,t)|^2 = 1$

$$\psi_n(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) e^{-i\frac{E_n t}{\hbar}}$$

$$\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) + V(x) \psi(x) = E \psi(x)$$

needs to be solved, all in space, for the time dependence to be appropriate. In general, this will depend upon $V(x)$:



here, $V(x) = 0$.

$$\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) = E \psi(x)$$

$$\psi(x) = A e^{\pm i k x}$$

then
$$\frac{-\hbar^2 (\pm i k)^2}{2m} A e^{\pm i k x} = E A e^{\pm i k x}$$

$$\left(\frac{p^2}{2m}\right) \quad \frac{\hbar^2 k^2}{2m} = E \quad \rightarrow \quad k = \sqrt{\frac{2m}{\hbar^2} E}$$