α-decay → Applied Tunneling

$$^{238}_{92}U \to \, ^{234}_{90}Th + \, ^{4}_{2}He$$

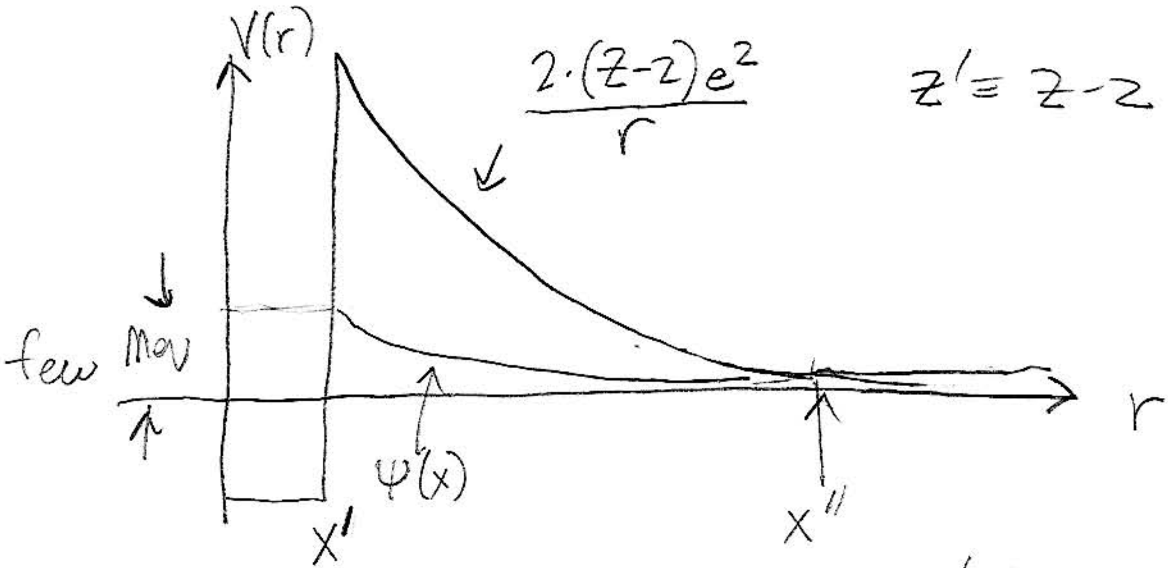
AKA, α-particle.

$\tau_{1/2} = 4.5 \cdot 10^9 y$

fraction of U's that survive time $t = 2^{-t/\tau_{1/2}}$

Typically, $E \approx$ few MeV

$V(x)$ for α-particle.



$Z' = Z-2$

$x' = R = r_0 A^{1/3}$

$r_0 = 1.3 \cdot 10^{-13}$ cm

$E = \frac{2Z'e^2}{x''}$

$x'' = \frac{2Z'e^2}{E}$

$x'' \to R_c = \frac{2Z'e^2}{E}$

$$\ln T = -2 \int_R^{R_c} dr \sqrt{\frac{2m}{\hbar^2} \underbrace{\left(\frac{2Z'e^2}{r} - E\right)}_{\text{vanishes when } r = R_c}}$$

use $x = \frac{r}{R_c}$

$$\ln T = -2 \int_{R/R_c}^{1} R_c\,dx \sqrt{\frac{2mE}{\hbar^2}\left(\frac{R_c}{r} - 1\right)}$$

$$= -2 \cdot \frac{2Z'e^2}{E} \cdot \sqrt{\frac{2mE}{\hbar^2}} \int_{R/R_c}^{1} dx \sqrt{\frac{1}{x} - 1}$$

$$= -\frac{4e^2}{\hbar c} Z' \sqrt{\frac{2mc^2}{E}} \cdot \int_{R/R_c}^{1} dx \sqrt{\frac{1}{x} - 1}$$

$R/R_c \ll 1$, turns out, so,

$$\int_{R/R_c}^{1} dx \sqrt{\frac{1}{x} - 1} = \int_0^1 dx \sqrt{\frac{1}{x} - 1} - \int_0^{R/R_c} dx \sqrt{\frac{1}{x} - 1}$$

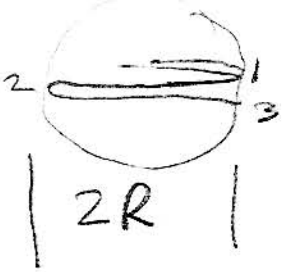
$$\cong \underbrace{\int_0^1 dx \sqrt{\frac{1}{x} - 1}} - \int_0^{R/R_c} dx \sqrt{\frac{1}{x}}$$

$$\simeq \frac{\pi}{2} - 2\sqrt{R/R_c}$$

$$\boxed{\ln T \simeq -\frac{2\pi e^2 Z'}{\hbar c}\sqrt{\frac{2mc^2}{E}} + 8\sqrt{\frac{e^2}{\hbar c} Z' \cdot \frac{mc^2}{\hbar c} \cdot R}}$$

Turn this into a mean life

Think of α bouncing back + forth inside nucleus. Takes $\sim 1/T$ bounces before it escapes



Bounce every $2R$

V = speed $\Rightarrow \frac{1}{2}mv^2 \simeq E$

$$V = \sqrt{\frac{2E}{m}}$$

$$\tau_0 = \text{time for 1 bounce} = \frac{2R}{V}$$

$$= 2R\sqrt{\frac{m}{2E}}$$

bounces in $\tau \Rightarrow \frac{\tau}{\tau_0} = \frac{1}{2R}\sqrt{\frac{2E}{m}} = \frac{1}{T}$

$$\tau = \tau_0 \cdot \frac{1}{T} = 2R\sqrt{\frac{m}{2E}} \cdot \frac{1}{T}$$

$$\ln\tau = \ln\tau_0 - \ln T$$

$$\boxed{\ln\tau = \ln\left(2R\sqrt{\frac{m}{2E}}\right) - 8\sqrt{\frac{e^2 Z}{\hbar c} \cdot \frac{mc^2}{\hbar c} \cdot R} + \frac{2\pi e^2 Z}{\hbar c}\sqrt{\frac{2mc^2}{E}}}$$

Calculate $E \sim 5$ MeV

$$\tau_0 = 2R\sqrt{\frac{m}{2E}} = 2\frac{R}{c}\sqrt{\frac{mc^2}{2E}}$$

mc^2: for α-particle, 4.002602 amu.
1 amu = 931.5 MeV

$$m_\alpha c^2 = (4.003)\cdot(931.5) = 3728 \text{ MeV}$$

$E = 1$ MeV ⇐ Typical Estimate!

$$R = (1.3\cdot 10^{-13})\, A^{1/3} \text{ cm} \simeq 7.3\cdot 10^{-13} \text{ cm}$$

$A: \simeq 226$ ⇐ Typical

$$R \simeq 7.3\cdot 10^{-13} \text{ cm}$$

$$\tau_0 = 2\cdot \frac{7.3\cdot 10^{-13}\text{ cm}}{3\cdot 10^{10}\frac{\text{cm}}{\text{s}}}\cdot \sqrt{\frac{3728}{2\cdot 1\cdot (E/1)}}$$

$$\boxed{\tau_0 \simeq (2.1\cdot 10^{-21}\text{ s})\cdot \sqrt{\frac{1}{E(\text{MeV})}}}$$

$$8\cdot\sqrt{\frac{e^2}{\hbar c} z' \frac{mc^2}{\hbar c} R} = 8\cdot\sqrt{\frac{1}{137} 86\cdot \frac{3728}{197.3}\cdot 7.3}$$

try $z' = 86$; $\hbar c = 197.3$ MeV·fm fm $= 10^{-13}$ cm $= 10^{-15}$ m
(typical)

$$\boxed{= 74.5 \quad \text{(dimensionless)}}$$

$$\frac{2\pi e^2}{\hbar c} z' \sqrt{\frac{2mc^2}{E}} = 2\pi\cdot\frac{1}{137}\cdot 86\cdot\sqrt{\frac{2\cdot 3728}{1\cdot (E/1\text{MeV})}}$$

$$= 341\sqrt{\frac{1}{E(\text{MeV})}}$$

$$\ln \tau = \ln\left[2.1\cdot 10^{-21}\text{s}\sqrt{\frac{1}{E}}\right] - 74.5 + 341\sqrt{\frac{1}{E(\text{MeV})}}$$