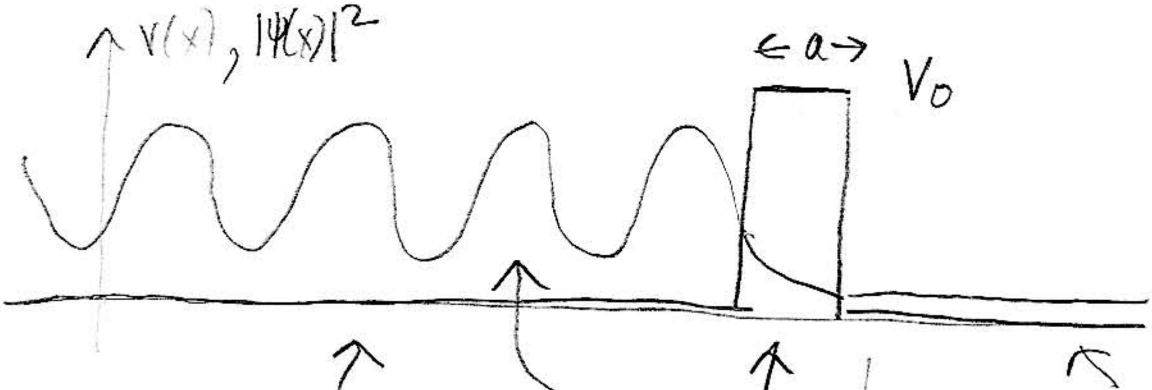
Transmission Through Complex Barriers

Simple Barrier



$$\psi(x) = e^{ikx} + Ae^{-ikx}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}} \quad (\text{S.E.})$$

E = kinetic energy of particle

e^{ikx} : describes incident wave.

Ae^{-ikx} : describes reflected wave... if $V_0 \to \infty$ $|A| \to 1$, but $|A| < 1$

$$P(x) = |\psi(x)|^2$$

$$= (e^{ikx} + Ae^{-ikx})(e^{-ikx} + A^* e^{ikx})$$

$$= 1 + |A|^2 + \underbrace{Ae^{-2ikx} + A^* e^{2ikx}}$$

$$= 1 + |A|^2 + \underbrace{2\text{Re}[Ae^{2ikx}]}_{\text{parts of } \cos 2kx,\ \sin(2kx)}$$

$$\simeq e^{-qx}$$

$$q = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

$$e^{-qa} \cdot e^{+ikx}$$

$$P(x) \simeq e^{-2qa}$$

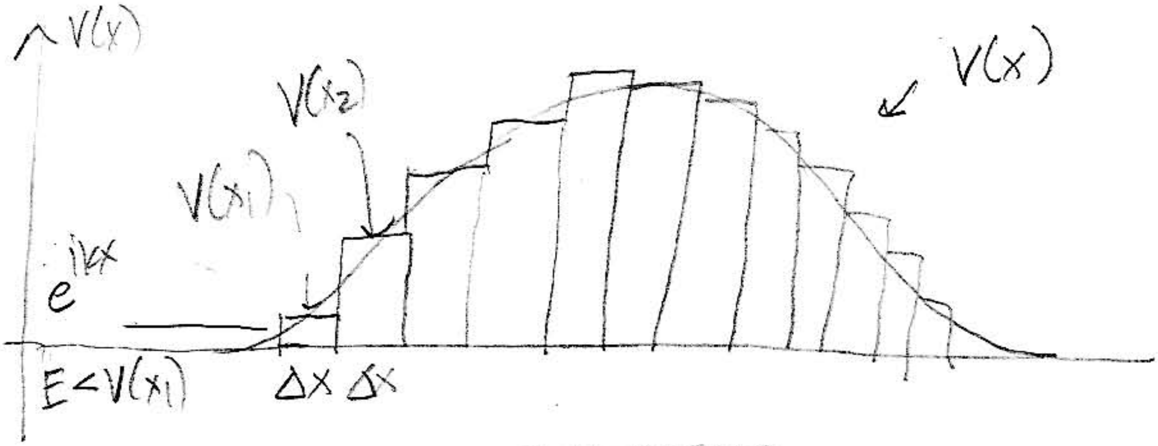
"Transmission Coefficient"

$$T = e^{-2qa}$$

$$\ln T = -2qa = -2\sqrt{\frac{2m(V_0-E)}{\hbar^2}}\, a$$

Tunnelling

Build Up a Complex Barrier



$$\ln T_1 \cong -2 \sqrt{\frac{2m(V(x_1)-E)}{\hbar^2}}\ \Delta x$$

$$\ln T_2 \cong -2 \sqrt{\frac{2m(V(x_2)-E)}{\hbar^2}}\ \Delta x$$

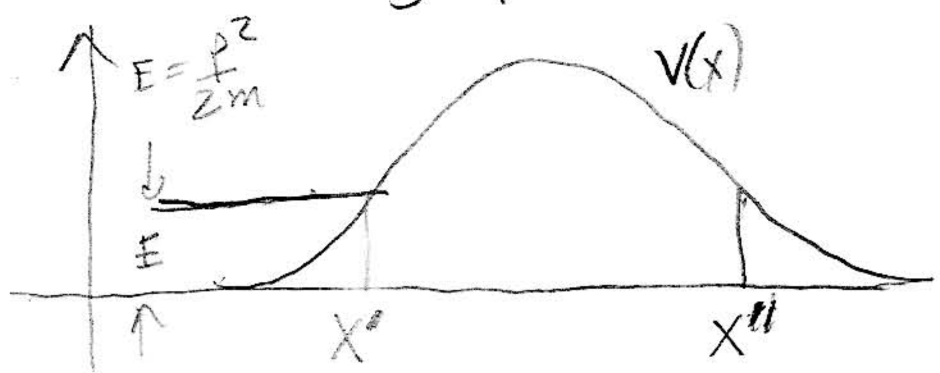
$$T_1 T_2 \ \ldots \qquad \ln T_1 T_2 = \ln T_1 + \ln T_2$$

$$T = T_1 T_2 T_3 \ldots T_N \ \ldots \qquad \ln T = \sum_{n=1}^{N} \left(-2 \sqrt{\frac{2m(V(x_n)-E)}{\hbar^2}}\right) \Delta x$$

$$\ln T \simeq -2 \int_{x'}^{x''} dx \sqrt{\frac{2m(V(x)-E)}{\hbar^2}}$$

<u>careful</u>: $V(x') \equiv E = V(x'')$

"Turning points"



α-decay → Applied Tunneling

$$^{238}_{92}U \rightarrow \, ^{234}_{90}Th + \, ^{4}_{2}He$$

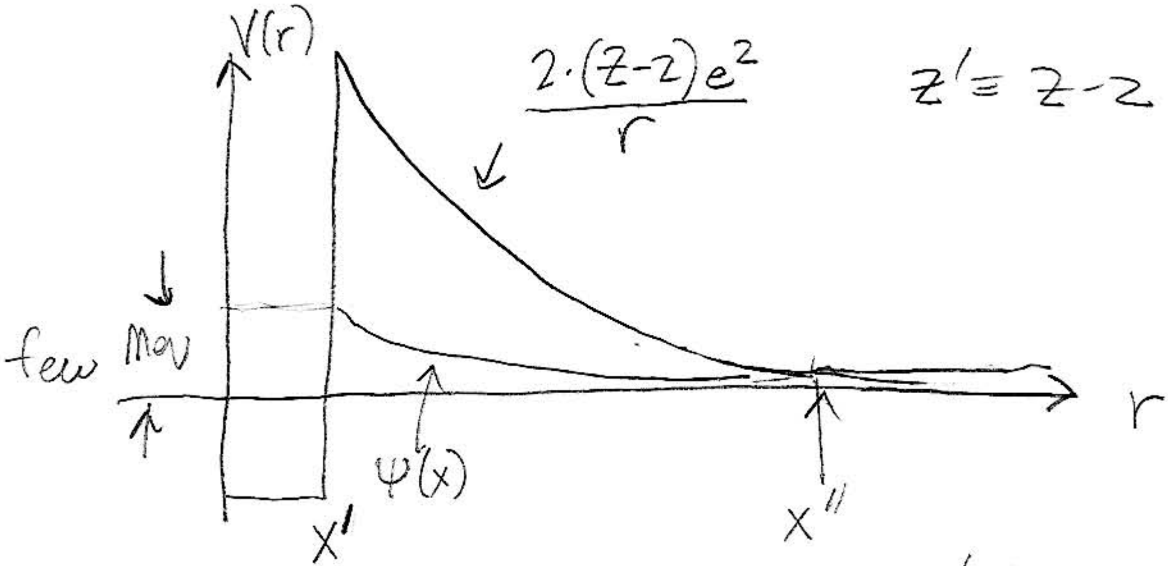
AKA, α-particle.

$\tau_{1/2} = 4.5 \cdot 10^9 y$

fraction of U's that survive time $t = 2^{-t/\tau_{1/2}}$

Typically, $E \approx$ few MeV

V(x) for α- particle.



$Z' = Z-2$

$x' = R = r_0 A^{1/3}$

$r_0 = 1.3 \cdot 10^{-13}$ cm

$$E = \frac{2Z'e^2}{x''}$$

$$x'' = \frac{2Z'e^2}{E}$$

$$x'' \rightarrow R_c = \frac{2Z'e^2}{E}$$

$$\ln T = -2 \int_R^{R_c} dr \sqrt{\frac{2m}{\hbar^2} \underbrace{\left(\frac{2Z'e^2}{r} - E\right)}_{\text{vanishes when } r = R_c}}$$

use $x = \frac{r}{R_c}$