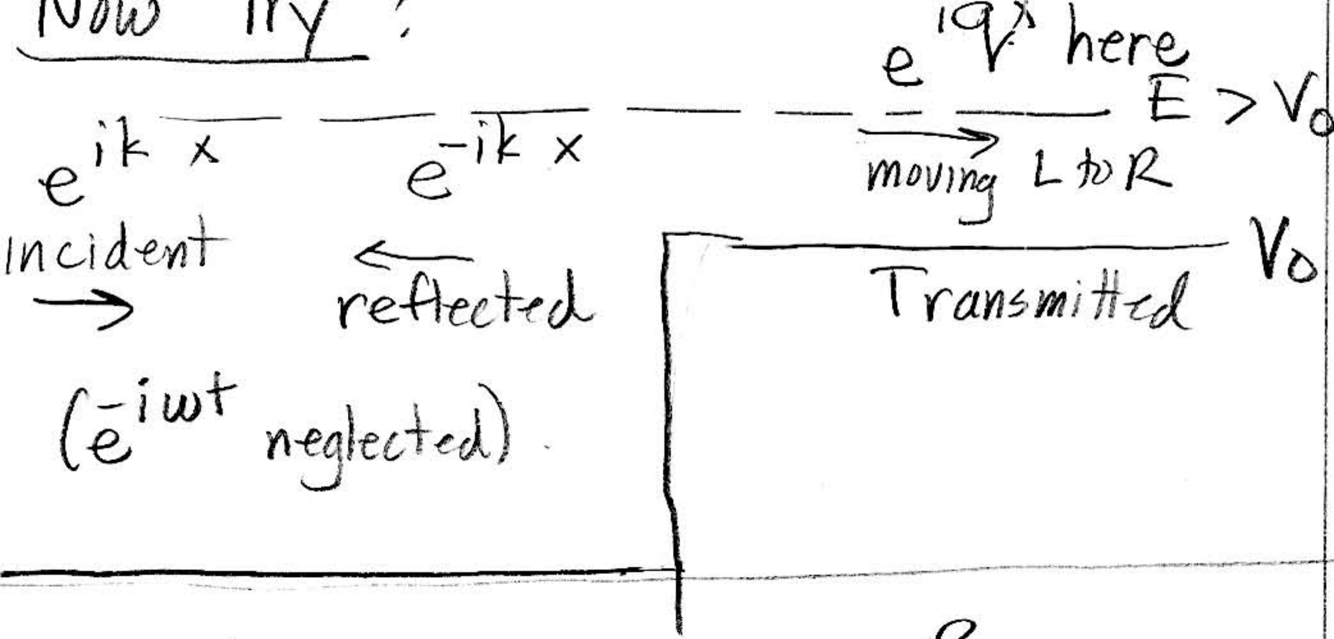
One interpretation:

$e^{+ikx - i\omega t}$ is incident from left to right

$e^{-ikx - i\omega t}$ is what bounces back off the wall

Now try:



Generalization:

$$\underbrace{\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\Psi = E\Psi}_{\text{this same as}} = \underbrace{i\hbar\frac{\partial}{\partial t}\Psi}_{\text{this part not obvious, but true!}}$$

this same as

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi = (E - V(x))\Psi$$

this part not obvious, but true!

Consequence: $e^{iEt/\hbar}$ never matters here!

and ....

L region: $\psi_L(x) = Ae^{ikx} + Be^{-ikx}$

R region $\psi_R(x) = Ce^{iqx}$

$$k = \sqrt{\frac{2m}{\hbar^2}E} \qquad q = \sqrt{\frac{2m}{\hbar^2}(E - V_0)}$$

$x = 0$: ψ must be <u>continuous</u>

$$\psi_L(0) = \psi_R(0)$$

$$A + B = C$$

ψ' must also be continuous

$$ik(A - B) = iqC$$

$$k(A - B) = q(A + B)$$

$$(k - q)A = (k + q)B$$

$$\frac{B}{A} = \frac{k - q}{k + q}$$

$$\underline{\text{Reflection Probability}} = \left|\frac{B}{A}\right|^2 = \left|\frac{k-q}{k+q}\right|^2$$

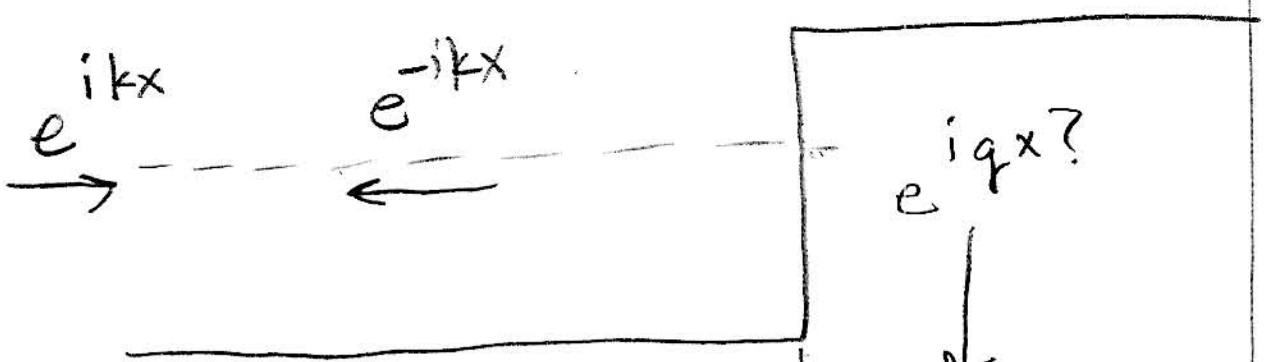
$$\boxed{R = \left|\frac{\sqrt{E} - \sqrt{E-V_0}}{\sqrt{E} + \sqrt{E-V_0}}\right|^2}$$

Transmission: $T = 1 - R \neq \left|\frac{C}{A}\right|^2$

must discuss probability current to understand why!

What happens when $E < V_0$?

The most fun ...



$x=0$

$$k = \sqrt{\frac{2mE}{\hbar^2}} \qquad q = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} < 0$$

$$= i\sqrt{\frac{2m(V_0-E)}{\hbar^2}} = i|q|$$

$\psi(0)$ continuous: $Ae^{ik\cdot 0} + Be^{-ik0} = Ce^{-|q|\cdot 0}$

$$A + B = C$$

$\psi'(0)$ continuous

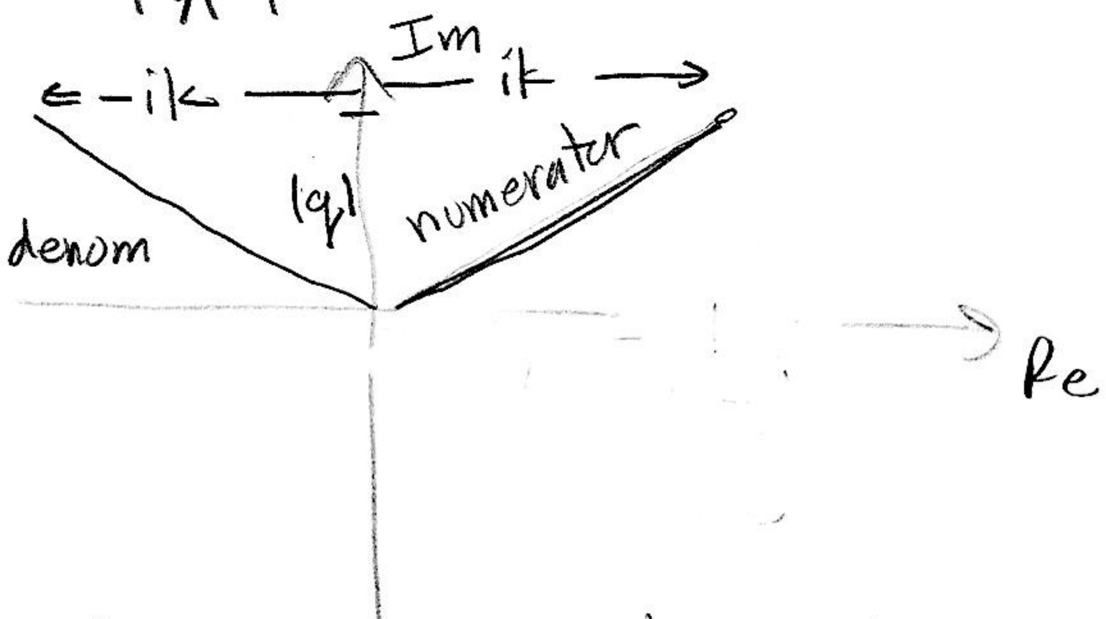
$$ik(A-B) = -|q|\cdot C$$

$$ik(A-B) = -|q|(A+B)$$

$$(|q| + ik)A = (-|q| + ik)B$$

$$\frac{B}{A} = \frac{ik + |q|}{ik - |q|}$$

$$\left|\frac{B}{A}\right| = 1 \ !$$



num + denom same lengths!

$$\frac{C}{A} = 1 + \frac{B}{A}$$

$$= \frac{ik - |q| + |q| + ik}{ik - |q|}$$

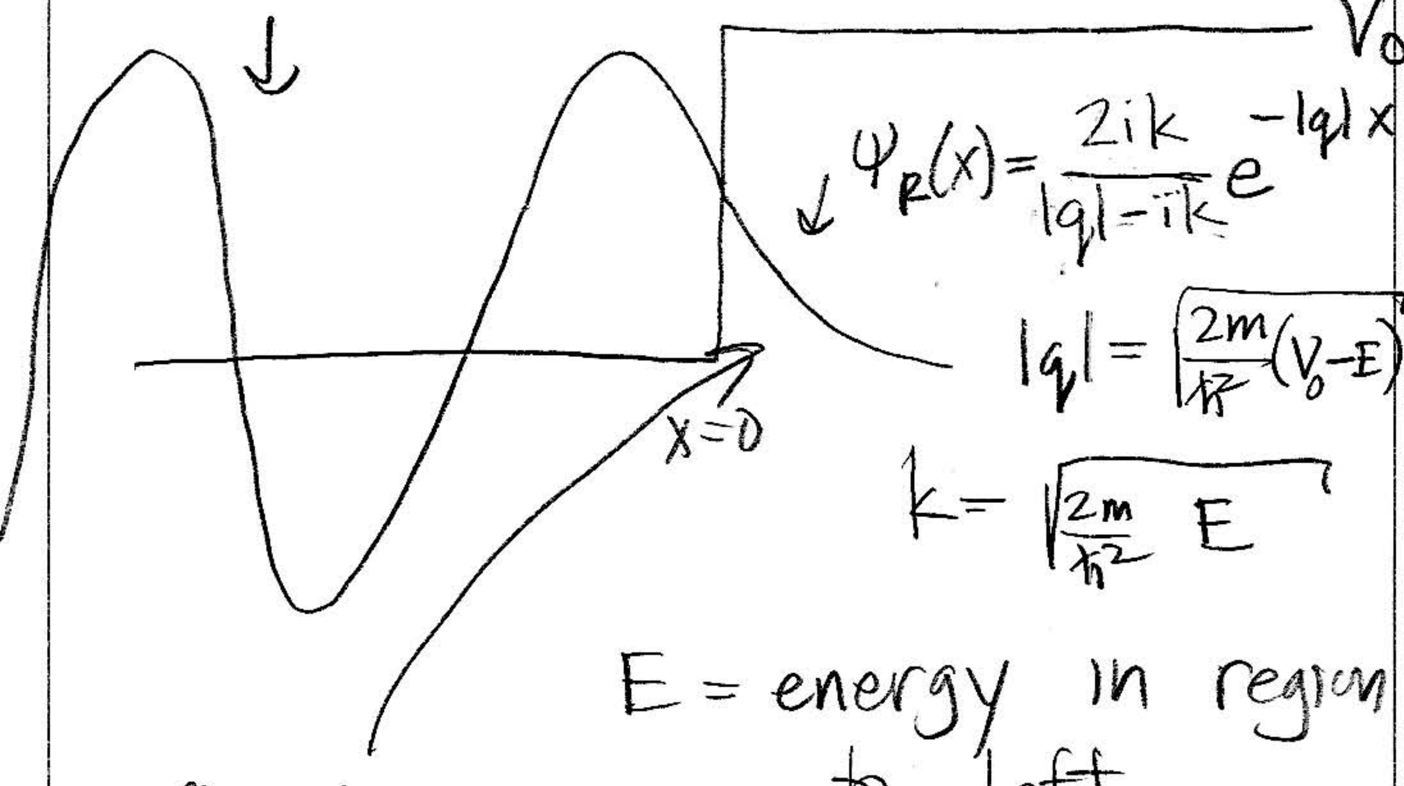
$$\boxed{\frac{C}{A} = \frac{2ik}{|q| - ik}} \neq 0\,!$$

unless $|q| = 0$

when $E = V_0$

take $A = 1$

$\psi_L(x)$



$$\psi_R(x) = \frac{2ik}{|q| - ik} e^{-|q|x}$$

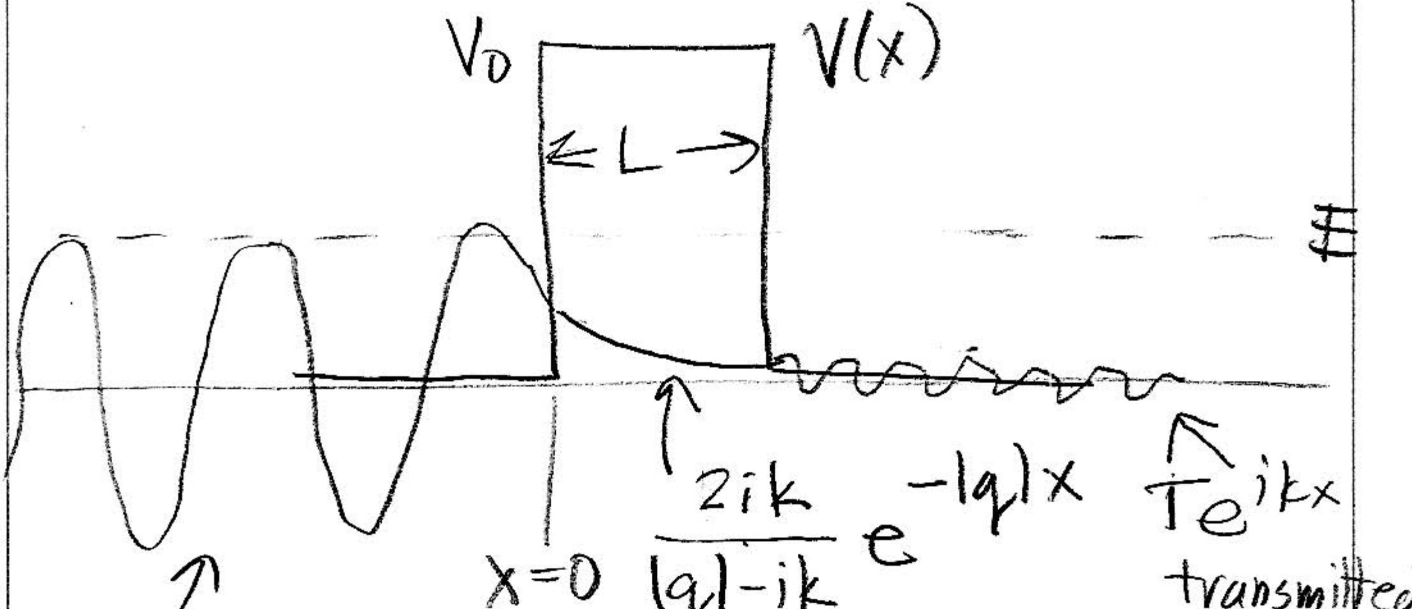
$$|q| = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$$

$$k = \sqrt{\frac{2m}{\hbar^2} E}$$

E = energy in region to left.

Quantum Mechanical Tunneling!

Particles Can Sneak Through Walls!



e^{ikx} incident

e^{-ikx} reflected

$k=\sqrt{\frac{2mE}{\hbar^2}}$

$q=\sqrt{\frac{2m(V_0-E)}{\hbar^2}}$

$$T \simeq \frac{2ik}{|q|-ik}\, e^{-|q|L}$$