Schrödinger Equation

$V(x) = 0$

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{x},t) = -\frac{\hbar^2}{2m_e} \nabla^2 \Psi(\vec{x},t)$$

↑ one derivative ↗ 2 derivatives

not like photon wave equation

$\Psi(\vec{x},t) \rightarrow$ like electric field, but $|\Psi(\vec{x},t)|^2$ is the probability of finding the particle.

Notice : $\mathrm{Re}\left[e^{i(\vec{k}\cdot\vec{x} - \omega t)}\right]$

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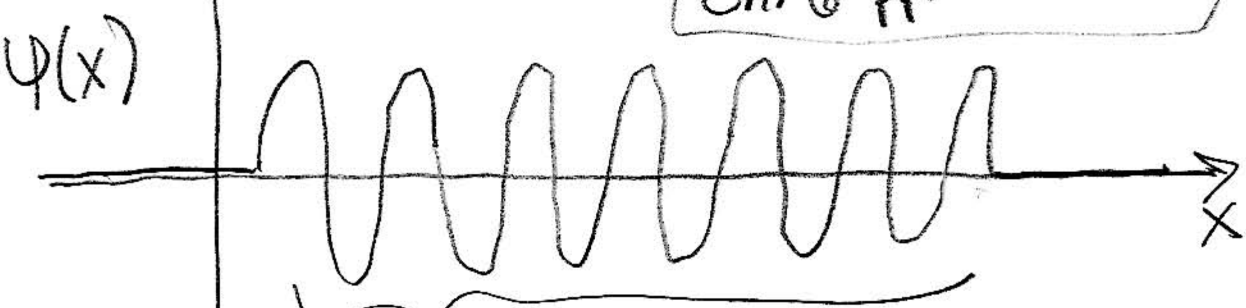
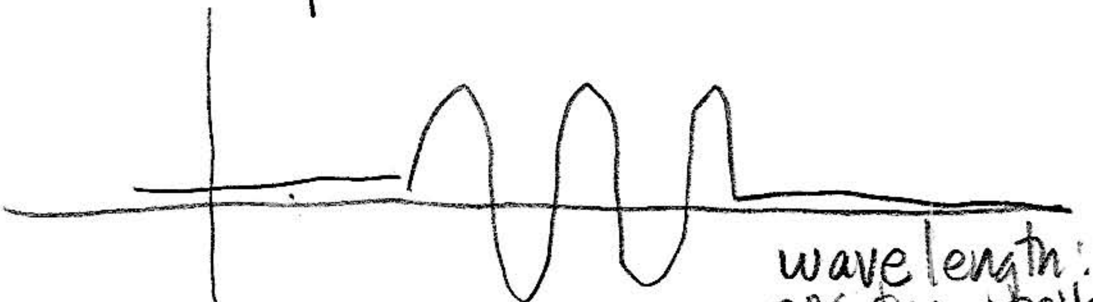


wavelength well defined.
position not



wavelength: poorer
position better

$$\frac{\Delta\lambda}{\lambda} \sim \frac{1}{n}$$ n = # of oscillations think of measuring with ruler

$$\Delta x \sim n\lambda$$

$$\lambda = \frac{h}{p} \quad , \quad \Delta\lambda = -\frac{h\,\Delta p}{p^2}$$

$$\left|\frac{\Delta\lambda}{\lambda}\right| = \frac{-\frac{h\,\Delta p}{p^2}}{\frac{h}{p}} = \left|-\frac{\Delta p}{p}\right|$$

$$\Delta x|\Delta p| \simeq (\cancel{n}\lambda)\left(p\cdot\frac{1}{\cancel{n}}\right)$$

$$\simeq \frac{h}{p}\cdot p$$

$$\boxed{\Delta x\,\Delta p \sim h}$$ ← beginning of uncertainty principle.

How uncertainty principle describes atomic stability

← wave function of electron.

$$\overline{p_x} = \overline{p_y} = \overline{p_z} = 0 \quad \left(\begin{array}{c}\text{electron not}\\ \text{running away}\end{array}\right)$$

$$\overline{(p_x - \overline{p_x})^2} = \overline{p_x^2} \quad \text{since } \overline{p_x} = 0$$

$$\overline{(\Delta p)^2} = \overline{(p_x - \overline{p_x})^2} + \overline{(p_y - \overline{p_y})^2} + \overline{(p_z - \overline{p_z})^2}$$

$$= \overline{p_x^2} + \overline{p_y^2} + \overline{p_z^2} = \overline{p^2}$$

$$\overline{(\Delta p)^2} = \overline{p^2}$$

similarly, $\overline{(\Delta r)^2} = \overline{r^2}$

$$\sqrt{\overline{(\Delta p^2)}}\sqrt{\overline{(\Delta r^2)}} \gtrsim \hbar$$

$$(\overline{p^2})^{1/2}(\overline{r^2})^{1/2} \gtrsim \hbar$$

$$\text{so} \quad (\overline{p^2}) \gtrsim \frac{\hbar^2}{\overline{r^2}} \quad \left(\begin{array}{c}\text{assume}\\ \text{equality}\end{array}\right)$$

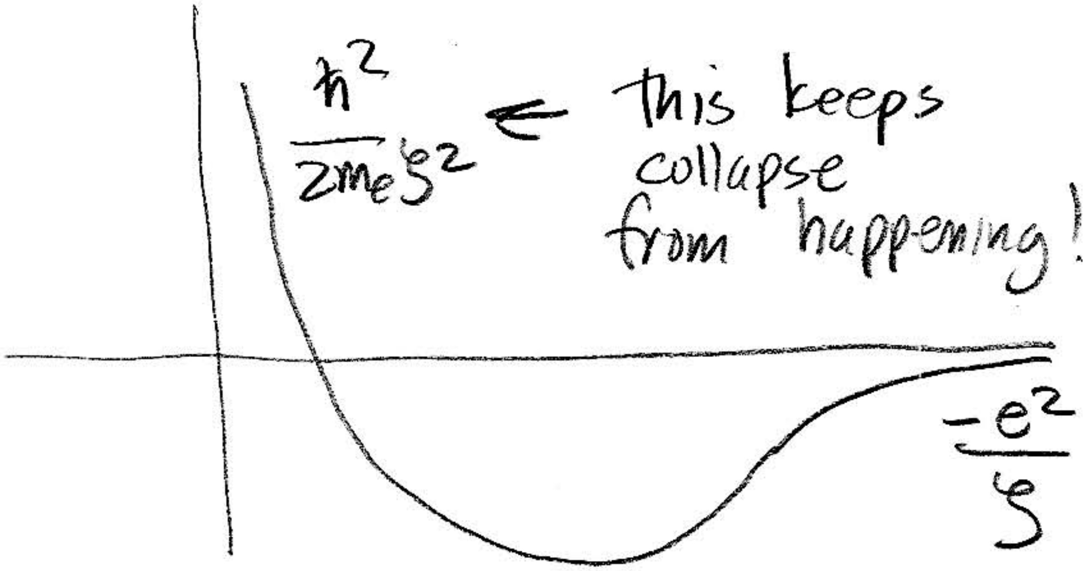
Average energy : Hydrogen atom $\simeq \dfrac{\overline{p^2}}{2m_e} - \dfrac{e^2}{\sqrt{\overline{r^2}}}$

$$\left[\text{SHO:} \simeq \frac{\overline{p^2}}{2m_e} + \frac{1}{2}k\overline{r^2}\right]$$

$$\overline{E} = \frac{\hbar^2}{2m_e\overline{r^2}} - \frac{e^2}{\sqrt{\overline{r^2}}} = \frac{\hbar^2}{2m_e\xi} - \frac{e^2}{\xi}$$

$$\xi \equiv (\overline{r^2})^{1/2}$$

$$\langle E \rangle = \frac{\hbar^2}{2m_e\xi^2} - \frac{e^2}{\xi}$$



$$\frac{\partial\langle E\rangle}{\partial\xi} = -\frac{2\hbar^2}{2m_e\xi^3} + \frac{e^2}{\xi^2} = 0$$

$$\frac{\hbar^2}{m_e} = e^2\xi$$

$$\xi = \frac{\hbar^2}{m_e e^2} = \frac{\hbar}{m_e c}\;\frac{\hbar c}{e^2}$$

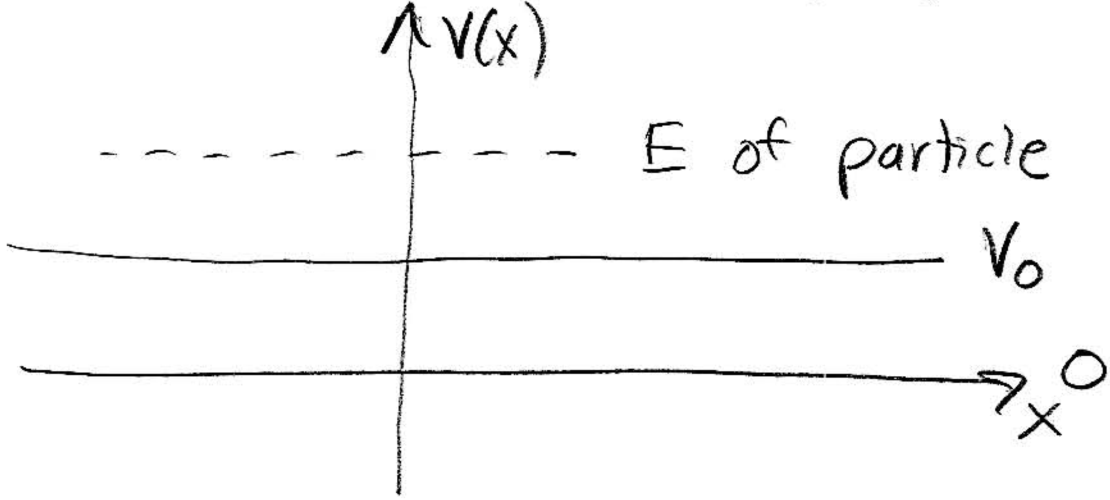
$$\xi = \frac{1}{\alpha}\cdot\lambda_e \quad !!$$

Qualitative value: squash an election, its kinetic energy explodes due to uncertainty principle.

Potential Step

Flat Potential

$V(x) = 0$, or, V_0



Solve:

$$\underset{\substack{\uparrow \\ \nabla^2 \text{ in} \\ \text{one dimension}}}{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}}\Psi(x,t) = -i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

Try $\Psi(x,t) = Ae^{\pm ikx - i\omega t}$

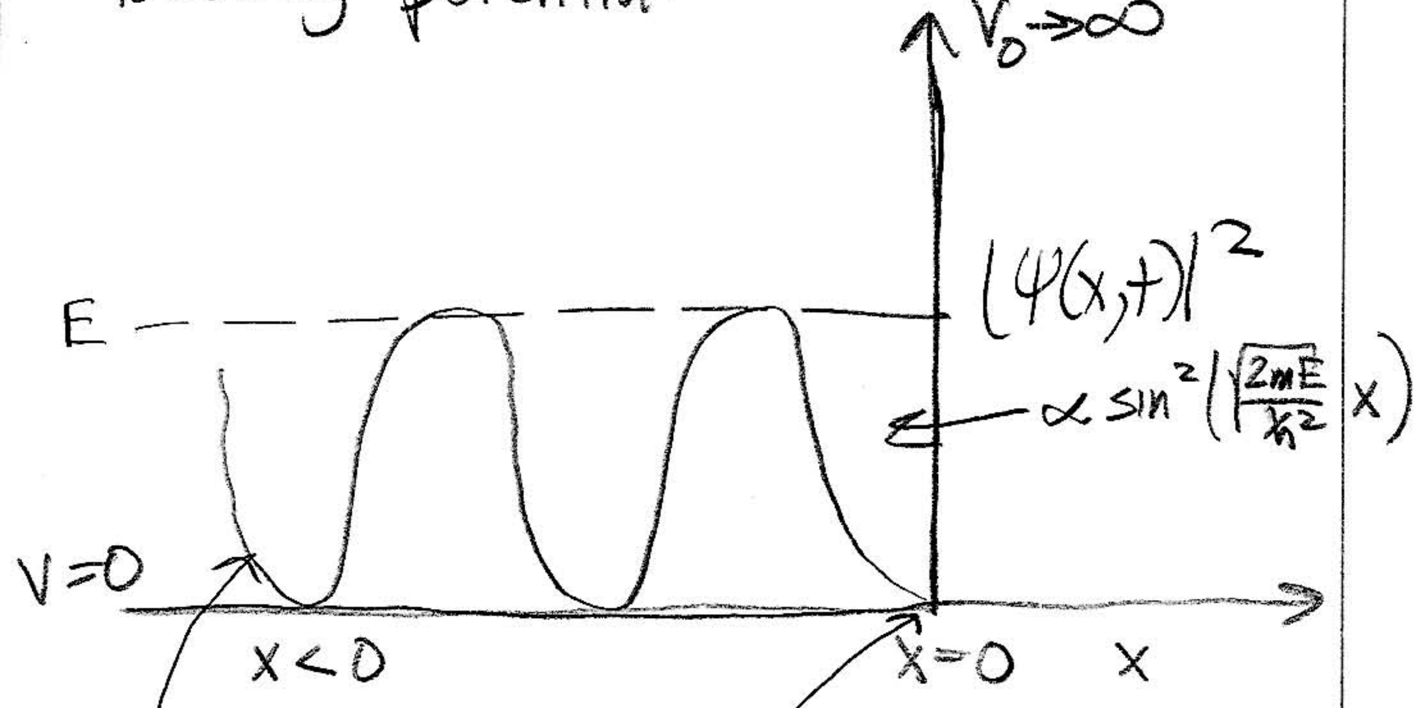
$$\underbrace{\frac{+\hbar^2 k^2}{2m_e}}_{\text{kinetic energy } = E - V_0} Ae^{\pm ikx - i\omega t} = \hbar\omega Ae^{\pm ikx - i\omega t}$$

$$k = \sqrt{\frac{2m}{\hbar^2}(E - V_0)} \quad \text{and } E > V_0$$

$$\omega = \frac{1}{\hbar}(E - V_0)$$

$$\Psi(x,t) = A e^{\pm i\sqrt{\frac{2m}{\hbar^2}(E-V_0)}\,x - i\frac{(E-V_0)}{\hbar}t}$$

Easy enough... Now try the following potential



$\Psi(x,t)$ must vanish at $x=0$

$$\Psi(x,t) = \left(A e^{+i\sqrt{\frac{2mE}{\hbar^2}}\,x} + B e^{-i\sqrt{\frac{2mE}{\hbar^2}}\,x}\right)\underset{\text{unimportant}}{\boxed{e^{\frac{-iEt}{\hbar}}}}$$

$$\Psi(0,t) = 0 = A + B$$

$$B = -A$$

$$\Psi(x,t) = \underset{\text{arbitrary}}{\boxed{2i\cdot A}}\;\underbrace{\sin\left(\sqrt{\frac{2mE}{\hbar^2}}\,x\right)}_{\text{interesting!}}$$