Photon Recap: $E = h\nu = \hbar\omega$

$$p = \frac{E}{c} = h\frac{\nu}{c} = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda} = \hbar k$$

- Blackbody
- Photoelectric Effect.
- Doppler ... $\nu' = \nu \cdot \sqrt{\frac{1-\beta}{1+\beta}}$
- Buildup of Interference Pattern.

Material Waves

Light Waves $\longleftrightarrow$ Particles (photons) Too

Electrons (particles) $\longleftrightarrow$ Waves Too? Yes!

Issue is the mass, which makes for complication.

Key: $E = \sqrt{(m_e c^2)^2 + (cp)^2}$

ω_F = "full" frequency

$$\boxed{\hbar\omega_F = \sqrt{(m_e c^2)^2 + (c\cdot\hbar k)^2}}$$

This is the generalization of photon wave concept to massive particle.

We'll work in non-relativistic approximation.

$$\hbar\omega_F \simeq \underbrace{m_e c^2}_{\text{gives frequency "offset"}} + \frac{\hbar^2 k^2}{2m_e}$$

$$\omega_F = \omega_0 + \omega$$

$$= \frac{m_e c^2}{\hbar}$$

then $\hbar\omega = \dfrac{\hbar^2 k^2}{2m_e}$

and $p = \hbar k = \dfrac{2\pi\hbar}{\lambda}$

$$\lambda = \frac{2\pi\hbar}{p}$$

T = kinetic energy = 10 keV
electron

$$\frac{p^2}{2m_e} = T \quad , \quad p = \sqrt{2m_e T}$$

$$cp = \sqrt{2m_e c^2 \cdot T} = \sqrt{2 \cdot .511 \cdot 10^6 \cdot 10 \cdot 10^3}$$

$$cp = 10^5 \text{ eV}$$

$$\lambda = \frac{2\pi \hbar c}{cp} \qquad \hbar c = 197.3 \text{ eV}\cdot\underbrace{\text{nm}}_{10^{-9}\text{ m}}$$

$$\lambda = \frac{2\pi \cdot 197.3}{10^5} \cdot 10^{-7} \text{ cm}$$

$$\lambda = \underbrace{1.2 \cdot 10^{-9} \text{ cm}}$$

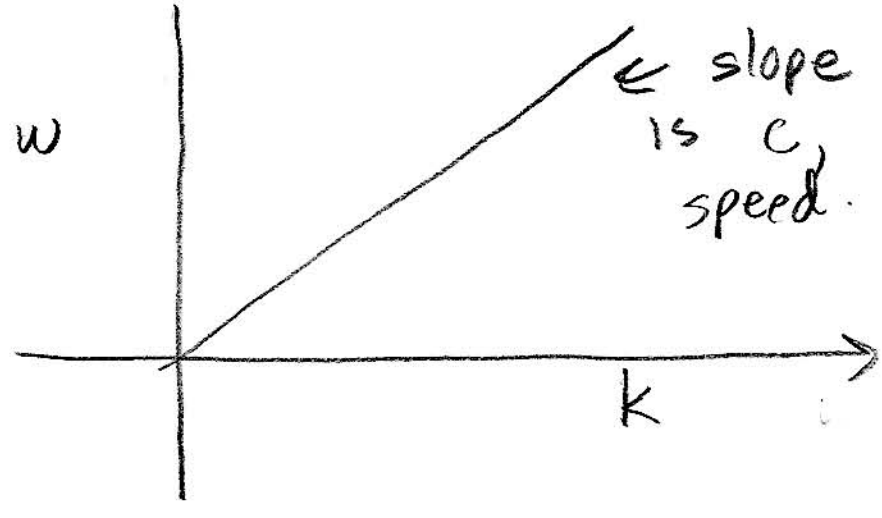
$\lesssim$ size of an atom

$\ll$ optical wavelength

relationship between
ω + k is called the
dispersion relation

Light (all massless fields) : $\frac{\omega}{k} = c$ (in vacuum)

$\omega = ck$

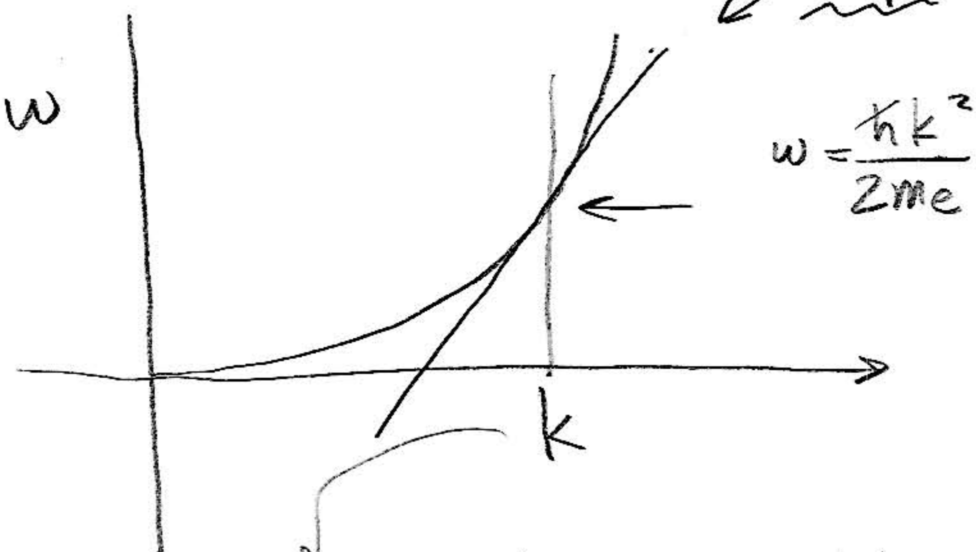


Non-relativistic, massive field...

$$\hbar\omega = \frac{\hbar^2 k^2}{2m_e}$$

$$\omega = \frac{\hbar k^2}{2m_e}$$

slope is velocity



now, $\frac{\omega}{k} = \frac{\hbar k^2}{2m_e}\frac{1}{k} = \frac{\hbar k}{2m_e} = \frac{p}{2m_e}$

$= \frac{1}{2} \cdot v \quad \left(v = \frac{p}{m_e}\right)$

aka, phase non-r

$$\frac{d\omega}{dk} = \frac{\hbar k}{m_e} = \frac{p}{m_e}.$$

$$\boxed{v = \frac{d\omega}{dk}} \quad \left(\text{not } \frac{\omega}{k}\right)!$$

aka "group velocity" → the real one)

Still (massive particles)

Electron Wave $\propto e^{i(\vec{k}\cdot\vec{x} - \omega t)}$

$\vec{k} = \frac{\vec{p}}{\hbar}$ $\qquad \omega = \frac{T}{\hbar}$ (kinetic)

But, due to

$$\omega = \frac{\hbar k^2}{2 m_e}$$

Electron wave does not satisfy the standard "wave equation"

What equation does it solve?

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→ pull down ω:

$$i\hbar \frac{\partial}{\partial t}\left(e^{i(\vec{k}\cdot\vec{x}-\omega t)}\right) = i\hbar(-i\omega)\, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$
$$= \hbar\omega\, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\frac{-\hbar^2}{2m_e}\nabla^2\left(e^{i(\vec{k}\cdot\vec{x}-\omega t)}\right) = \frac{-\hbar^2(i^2k^2)}{2m_e}\, e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$
$$= \left(\frac{\hbar^2k^2}{2m_e}\right) e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

so $\hbar\omega = \frac{\hbar^2k^2}{2m_e} \rightarrow \boxed{\omega = \frac{\hbar k^2}{2m_e}}$