Photon's → Counting Them

→ As particles, compton scattering aka, how to change frequency!

→ photon diffraction pattern. (Chap 4 of Wichmann).

Recall (p. 24 of Phys 25 notes)


$$\text{Energy Flux} = \frac{E_0^2 c}{8\pi}$$

$$= \left(\text{Energy}/\text{photon}\right) \cdot \frac{\#\ \text{photons}}{(\text{area}) \cdot \text{time}}.$$

$$\frac{E_0^2 c}{8\pi} = \hbar\omega \cdot f$$

$E_0 = 1$ statvolt/cm

1 Volt/meter

$\simeq$ (1/300 statvolt)/100 cm

$= 3.3 \cdot 10^{-5}$ statvolt/cm

suppose: $\hbar\omega \cong 1\ \text{eV} \simeq 1.6 \cdot 10^{-12}$ ergs

$$f = \frac{1^2 \cdot 3 \cdot 10^{10}}{1.6 \cdot 10^{-12} \cdot 8 \cdot \pi} \qquad \frac{(3.3 \cdot 10^{-5})^2 \cdot 3 \cdot 10^{10}}{1.6 \cdot 10^{-12} \cdot 8 \cdot \pi}$$

$$= 7.5 \cdot 10^{20} \ \frac{\text{photons}}{\text{cm}^2 \cdot \text{s}} \qquad 8.1 \cdot 10^{11} \ \frac{\text{photons}}{\text{cm}^2 \cdot \text{s}}$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

$$E_0 = 1 \ \frac{\text{statvolt}}{\text{cm}} \qquad E_0 = 1 \ \frac{\text{volt}}{\text{meter}}$$

$f = 1 \ \frac{\text{photon}}{\text{cm}^2 \cdot \text{s}}$ when $E_0 \simeq 3.7 \cdot 10^{-11} \ \frac{\text{statvolt}}{\text{cm}}$

$$\simeq 1 \ \frac{\mu V}{\text{meter}}$$

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Photon mass = 0 !

$$E = \hbar\omega = \sqrt{(cp)^2 + (\underbrace{mc^2}_{\to 0\,!})^2}$$

$$E = cp$$

$$\frac{E}{p} = \frac{m_0 \gamma \beta c^2}{m_0 \beta c} = \beta c = c$$

$$\boxed{\beta = 1 \quad \text{for photon!}}$$

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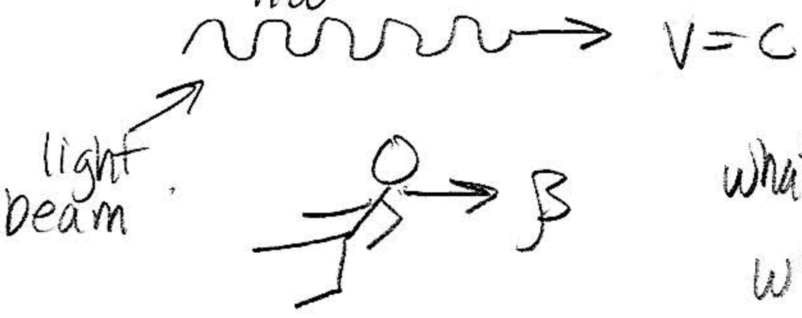


what frequency, $\omega'$, would you measure?

$$E' = \hbar\omega' = \gamma(E - \beta cp)$$

photon, $cp = E$!

$$\hbar\omega' = E\,\gamma(1-\beta)$$

$$\hbar\omega' = \hbar\omega\left[\frac{(1-\beta)}{\sqrt{1-\beta^2}} = \frac{1-\beta}{\sqrt{(1-\beta)(1+\beta)}}\right]$$

$$\boxed{\omega' = \omega \cdot \sqrt{\frac{1-\beta}{1+\beta}}}$$ "Doppler for photons"

run with: $\beta > 0$ $\quad \omega' < \omega$

into $\quad \beta < 0$ $\quad \omega' > \omega$

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Compton Scattering (1923)

$\gamma$ incident

$E = \hbar\omega$

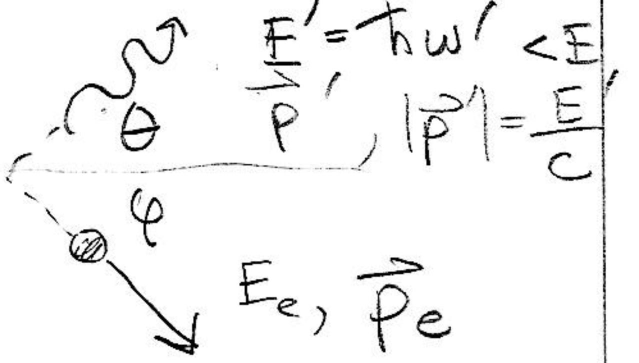
$p = \hbar\frac{\omega}{c} = \hbar k$

$\vec{p}$

Initial

$E_e = m_e c^2$

electron at rest



$$E + m_e c^2 = E' + E_e$$ energy (0th comp.)

$$\vec{p} = \vec{p}\,' + \underbrace{\vec{p}_e}_{\text{Eliminate}}$$

$$E_e = E + m_e c^2 - E'$$

$$\vec{p}_e = \vec{p} - \vec{p}\,'$$

$$E_e^2 - c^2|\vec{p}_e|^2 = m_e^2 c^4 = (E + m_e c^2 - E')^2 - c^2|\vec{p} - \vec{p}\,'|^2$$

$$\cancel{m_e^2 c^4} = (E - E')^2 + \cancel{(m_e c^2)^2} + 2(E - E')m_e c^2$$

$$- c^2\Big[\underbrace{|\vec{p}|^2}_{\frac{E^2}{c^2}} + \underbrace{|\vec{p}\,'|^2}_{\frac{E'^2}{c^2}} - 2\vec{p}\cdot\vec{p}\,'\Big]$$

$$\vec{p}\cdot\vec{p}\,' = \frac{E}{c}\frac{E'}{c}\cos\theta$$

$$0 = \overset{-2EE'}{+\cancel{2}(E - E')m_e c^2} + \cancel{2}EE'\cos\theta$$

$$-E m_e c^2 = E'(-m_e c^2 - E + E\cos\theta)$$

$$E' = \frac{E\, m_e c^2}{m_e c^2 + E(1-\cos\theta)} = \frac{E}{1 + \frac{E}{m_e c^2}(1-\cos\theta)}$$

$$\boxed{\omega' = \frac{\omega}{1 + \frac{\hbar\omega}{m_e c^2}(1-\cos\theta)}}$$