Microscopic Quantities

$h = 6.6 \cdot 10^{-27}$ erg-sec ??

$$\rightarrow \quad \underset{\substack{\nearrow \\ \text{Avogadro's} \\ \text{Number}}}{N_0}\ \underset{\substack{\uparrow \\ \text{mass of} \\ \text{proton}}}{M_p} \simeq 1 \text{ gm}$$

$$M_p \simeq M_n$$

so

Z = # protons

N = # neutrons

$A = N + Z$

$Z \rightarrow$ chemical element, properties

$N \rightarrow$ not much, but they stabilize the protons electrostatically

$Z \rightarrow$ name

1	H
2	He
3	Li

$A \rightarrow$ "isotope"

^1H = hydrogen

^2H = deuterium

^3H = tritium

$$N_0\, m(^3H) = 3 \text{ gm}$$

$$N_0\, m(^A\underbrace{\underline{E}l}_{\substack{\text{Element}\\ \text{mass A}}}) = A \text{ gm}.$$

$$N_0 \underset{\uparrow}{k} = 8.314 \cdot 10^7 \frac{\text{erg}}{\text{K mole}}$$

Boltzmann Constant
$1.38 \cdot 10^{-16} \frac{\text{erg}}{\text{K}}$

$$F = N_0\, e = 96{,}487 \frac{\text{coulomb}}{\text{mole}}$$

"Faraday of charge"

$$\omega = 2\pi\nu$$

$$\underbrace{h\nu}_{\substack{\text{energy}\\ \text{of a}\\ \text{quantum of}\\ \text{light}}} = h \cdot \frac{\omega}{2\pi} \equiv \hbar\omega$$

$$\boxed{\hbar = \frac{h}{2\pi}}$$

$$\bar{\lambda} = \frac{\lambda}{2\pi}$$

$$\text{so } \lambda\nu = \bar{\lambda}\omega = c$$

$$\tilde{\nu} \equiv \text{wave number} \equiv \frac{1}{\lambda}$$

note: $h\nu = \frac{hc}{\lambda} = hc\tilde{\nu}$

Energy

Energy of an electron falling through 1 volt of potential very useful unit...

$$eV = 1.6 \cdot 10^{-12}\ \text{erg}$$

"Natural units" for atomic work

→ m_e governs most atomic (not nuclear) energy.

→ c speed of light

→ $\hbar$ energy · time.

1 unit of mass = m_e

momentum = $m_e c$

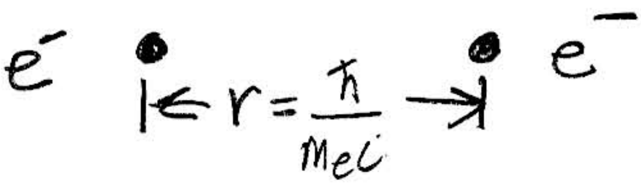
energy = $m_e c^2$ $(0.511 \cdot 10^6\ \text{eV})$

time = $\frac{\hbar}{m_e c^2}$ $(1.29 \cdot 10^{-21}\ \text{s})$

length = $\frac{\hbar}{m_e c}$ $(3.86 \cdot 10^{-11}\ \text{cm})$

freq = $\frac{1}{\text{time}}$ = $\frac{m_e c^2}{\hbar}$ $(7.76 \cdot 10^{20}\ \frac{1}{s})$

In these units ....



how much energy?

$$= \frac{e^2}{\left(\frac{\hbar}{\cancel{m_e} c}\right)} \cdot \frac{1}{m_e c^2}$$

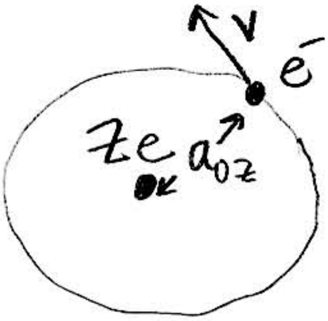
(p.57)

$$= \frac{e^2}{\hbar c} \cong \frac{1}{137}$$

Electrostatic energy in atoms tends to be way smaller than the rest energy of an electron.

This constant $\frac{e^2}{\hbar c} \equiv \alpha$ "fine structure constant"

Bohr Radius



Last time:

$$m_e v \cdot a_{0Z} = \hbar$$

$$a_{0Z} = \frac{1}{Z} \cdot \frac{\hbar^2}{m_e e^2}$$

$$= \frac{1}{Z} \cdot \frac{\hbar}{m_e c} \cdot \frac{\hbar c}{e^2}$$

$$\frac{\hbar}{m_e c} \equiv \text{compton wavelength of the.. electron}$$

"First Bohr Radius" $a_{0Z} = \frac{1}{Z} \cdot \bar{\lambda}_e \cdot \frac{1}{\alpha}$

$$a_{0Z} \cong \underset{\uparrow}{\underline{137}} \cdot \frac{\bar{\lambda}_e}{Z}$$

atoms are big on a fundamental scale!

How about v...

$$m_e v_{0Z} a_{0Z} = \hbar$$

$$v_{0Z} = \frac{\hbar}{m_e a_{0Z}} = \frac{\hbar}{m_e} \cdot \frac{1}{\left(\frac{1}{Z} \cdot \frac{\hbar}{m_e c} \cdot \frac{1}{\alpha}\right)}$$

$$\boxed{\begin{aligned} v_{0Z} &= \alpha Z \cdot c \\ v_{0Z} &\cong \frac{Z}{137} \cdot c \end{aligned}}$$

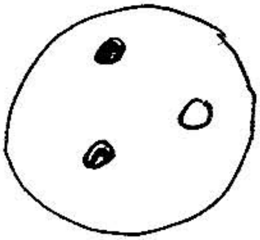
electrons in atoms move slowly compared to the speed of light.

→ Electromagnetism is weak

→ Inside the proton and neutron are quarks:

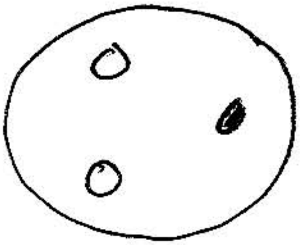
- ● up quark, charge $\frac{2}{3}e$
- ○ down quark, charge $-\frac{1}{3}e$

proton:



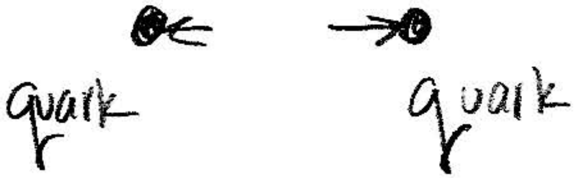
$$q = \left(\frac{2}{3} + \frac{2}{3} - \frac{1}{3}\right)e$$
$$= e$$

neutron:



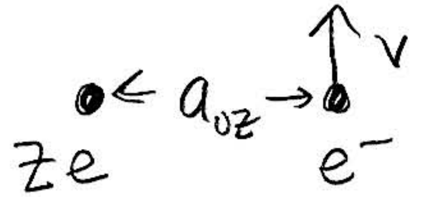
$$q = \left(-\frac{1}{3} - \frac{1}{3} + \frac{2}{3}\right)e$$
$$= 0$$

new force, much stronger than electromagnetism



$$v_{quark} \sim c \quad !!!$$

Binding Energy



$$E_{total} = -\frac{Ze^2}{a_{0Z}} + \frac{1}{2} m_e V_{0Z}^2$$

$$= -\frac{Ze^2}{\frac{1}{Z}\frac{\hbar}{m_e c}\cdot\frac{1}{\alpha}} + \frac{1}{2} m_e \alpha^2 Z^2 c^2$$

$$= -Z^2 \cdot \alpha \cdot \frac{e^2}{\hbar c} \cdot m_e c^2 + \frac{1}{2}\alpha^2 Z^2 m_e c^2$$

$$\boxed{E_{total} = -\frac{1}{2}\underset{\underset{\left(\frac{1}{137}\right)^2}{\uparrow}}{\alpha^2}\, \underset{\nwarrow}{Z^2}\, m_e c^2}$$

when $Z=1$, then

$$E_{total} \simeq -\frac{1}{2}\left(\frac{1}{137}\right)^2 \cdot (511{,}000)\ \text{eV}$$

$$\boxed{E_{total} \simeq -13.6\ \text{eV}}$$

→ binding energy of electron in Hydrogen Atom