$$1+x^2+x^4+\cdots+x^{N-1} = \frac{1-x^N}{1-x}$$

$$x = e^{-2i\phi} \qquad 5 = N-1$$

$$E \propto e^{(N-1)i\phi}\left(\frac{1-e^{-2iN\phi}}{1-e^{-2i\phi}}\right)\frac{\sin\left(\frac{\pi\sin\theta\, a}{\lambda}\right)}{\sin\theta}$$

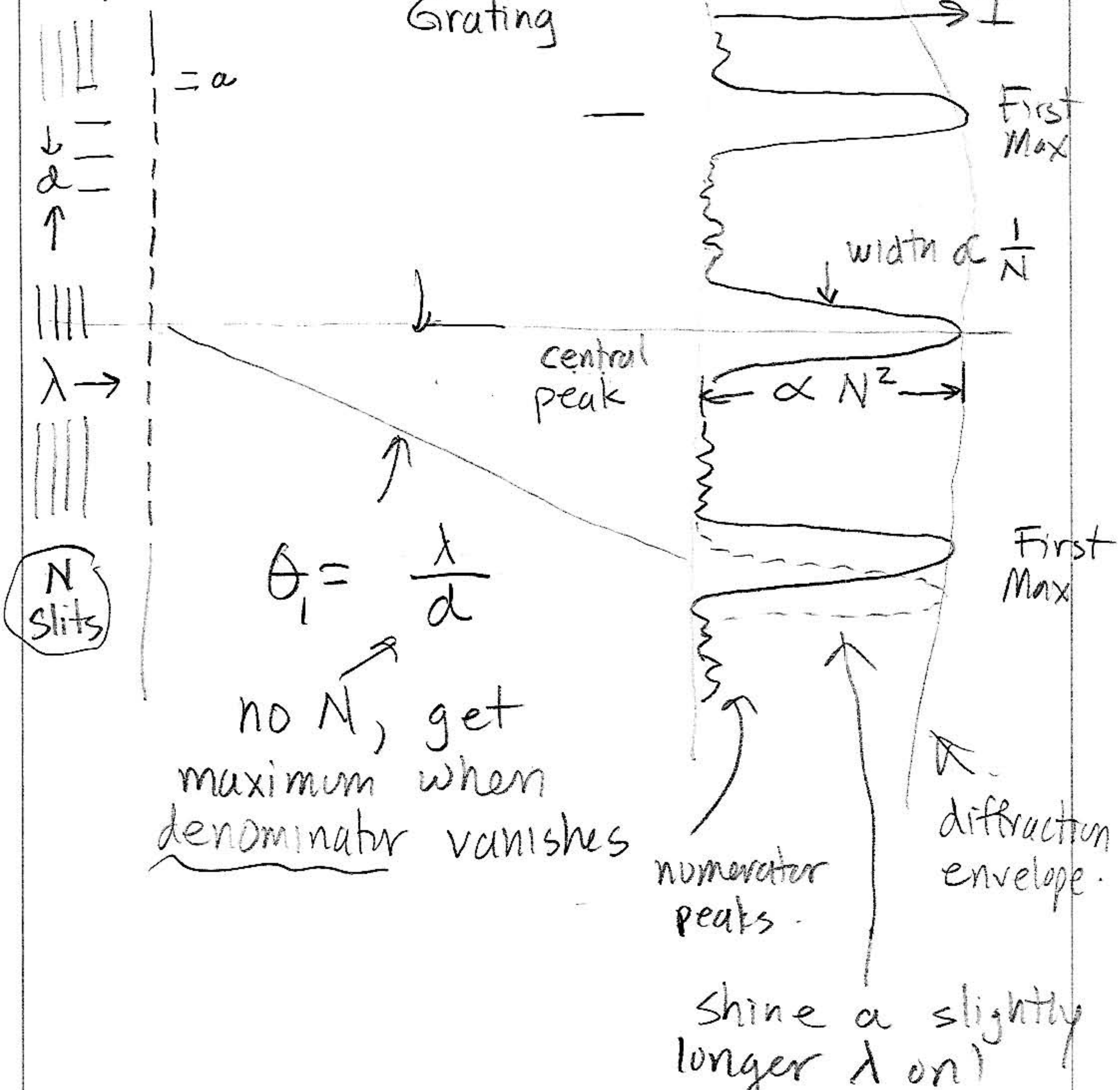
$$\propto \frac{\cancel{e^{-i\phi}}}{\cancel{e^{-i\phi}}}\left[\frac{e^{iN\phi}-e^{-iN\phi}}{e^{i\phi}-e^{-i\phi}}\right]\frac{\sin\left(\frac{\pi\sin\theta}{\lambda}a\right)}{\sin\theta}$$

$$E \propto \frac{\sin(N\phi)}{\sin\phi}\;\frac{\sin\left(\frac{\pi\sin\theta}{\lambda}a\right)}{\sin\theta}$$

$$E \propto \frac{\sin\left(N\frac{\pi\sin\theta}{\lambda}d\right)\sin\left(\frac{\pi\sin\theta}{\lambda}a\right)}{\sin\left(\frac{\pi\sin\theta}{\lambda}d\right)\;\underset{\substack{\uparrow\\ \text{added}\\ \text{here}}}{\frac{\pi a}{\lambda}}\sin\theta}$$

$$E^2 \propto \frac{\sin^2\left(N\frac{\pi\sin\theta}{\lambda}d\right)\;\sin^2\left(\frac{\pi\sin\theta}{\lambda}a\right)}{\left[\frac{\pi d}{\lambda}\,\frac{\pi a}{\lambda}\right]^2\sin^4\theta}$$

Grating



$$\theta_1 = \frac{\lambda}{d}$$

no N, get maximum when denominator vanishes

→ This is how you can distinguish slightly different wavelengths of light.

$$\to \Delta\lambda \propto \frac{1}{Nd} \Leftarrow$$ "uncertainty principle"

uncertainty in Δλ

$$\propto \frac{1}{\text{uncertainty in position}}$$

Quantum Mechanics

→ atoms ... discrete chunks of matter

GENERALIZED

↓

Quanta ↔ all matter, energy comes in packets

When does this matter?

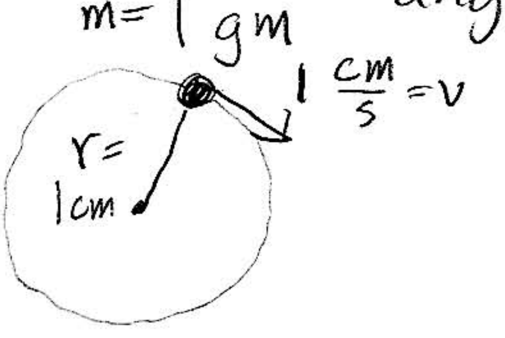
When a system has an "action" variable comparable in size to

$$\boxed{\begin{aligned} h &= 6.6 \cdot 10^{-27} \text{ erg-sec} \\ &= 6.6 \cdot 10^{-34} \text{ joule-sec.} \end{aligned}}$$

"Planck's Constant"

$\frac{cm^2 g}{s}$

Action: most familiar case is angular momentum



$L = mvr = 1 \frac{cm^2 g}{s}$

$\simeq 10^{26}$ times h!

When Action $\gg h$, "Classical" Physics pertains.

Action $\simeq h$, Classical fails

How: Bits of matter ("point particles") stop behaving like little pellets.

They start behaving like waves.

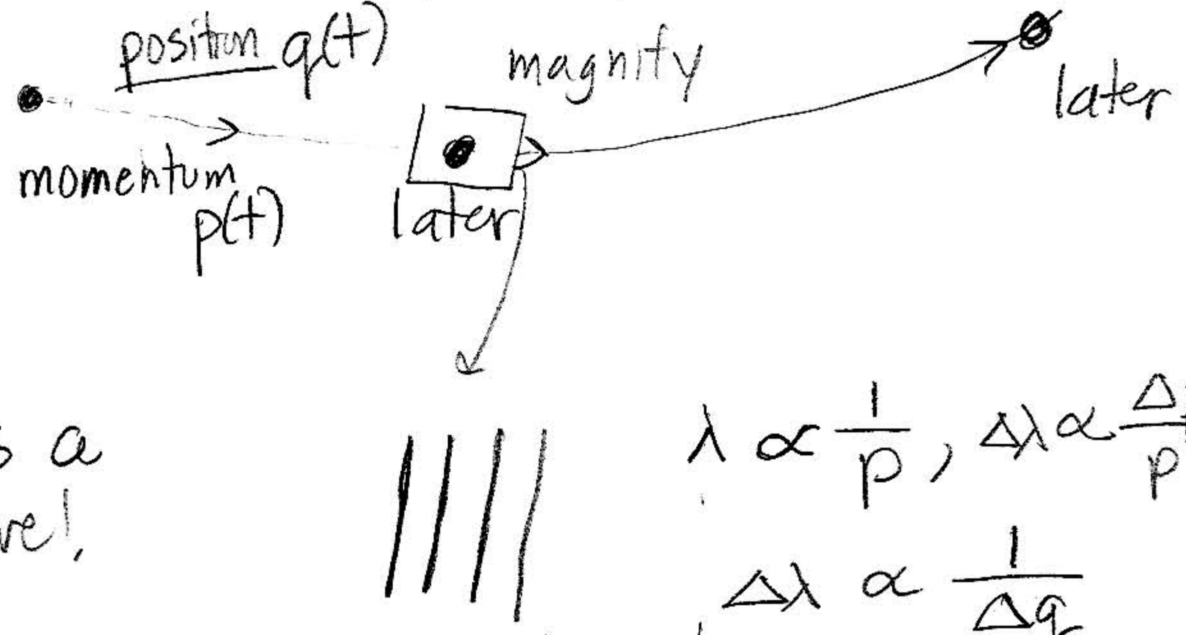
But, they also still behave like particles too!

Perplexing: $\boxed{\text{Wave particle duality}}$

→ sometimes wavelike
→ sometimes particle like.

Reason particle like:
- very fundamental (photon)
- boundary conditions on waves... "quantization" of energy levels.. eg wavelike electrons in atoms.

One consequence: "uncertainty principle"



its a wave!

$$\lambda \propto \frac{1}{p}, \quad \Delta\lambda \propto \frac{\Delta p}{p^2}$$

$$\underbrace{\Delta\lambda \propto \frac{1}{\Delta q}}_{\text{think of grating!}}$$

$$\frac{\Delta p}{p^2} \propto \frac{1}{\Delta q_H}$$

Heisenberg Uncertainty. $\boxed{\Delta p \Delta q \geq \frac{1}{2}\frac{h}{2\pi}}$

$\frac{h}{2\pi} \equiv \hbar$ "h-bar"

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100 years ago: puzzles

Size of atoms: Avogadro's Number

day to day life!

Blackbody: thermo ⟺ light

Photo electric Effect: light in quanta