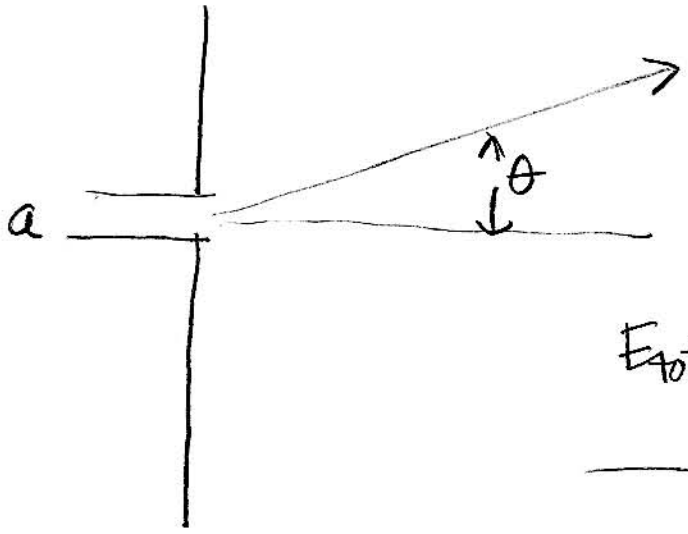
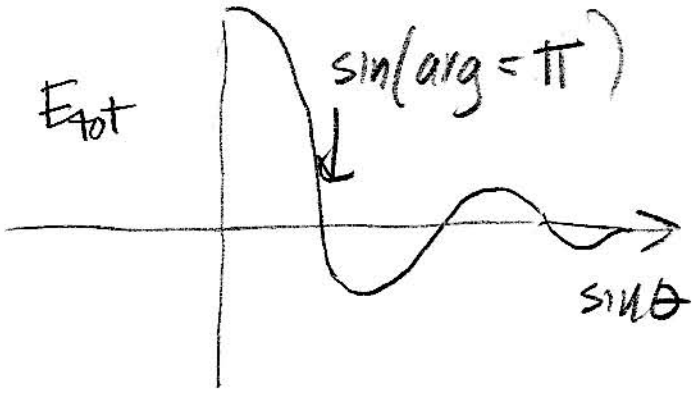
Diffraction Grating



$$E_{tot}(\theta) \propto \frac{\sin\left(\frac{\pi \sin\theta a}{\lambda}\right)}{\sin\theta}$$



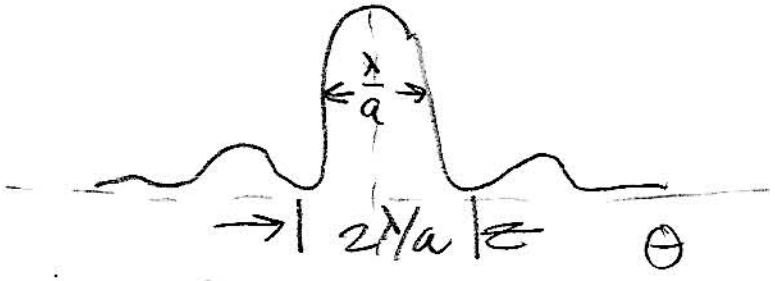
Hint: basis of uncertainty principle; localizing to a transverse "blurs out" the direction of the wave...

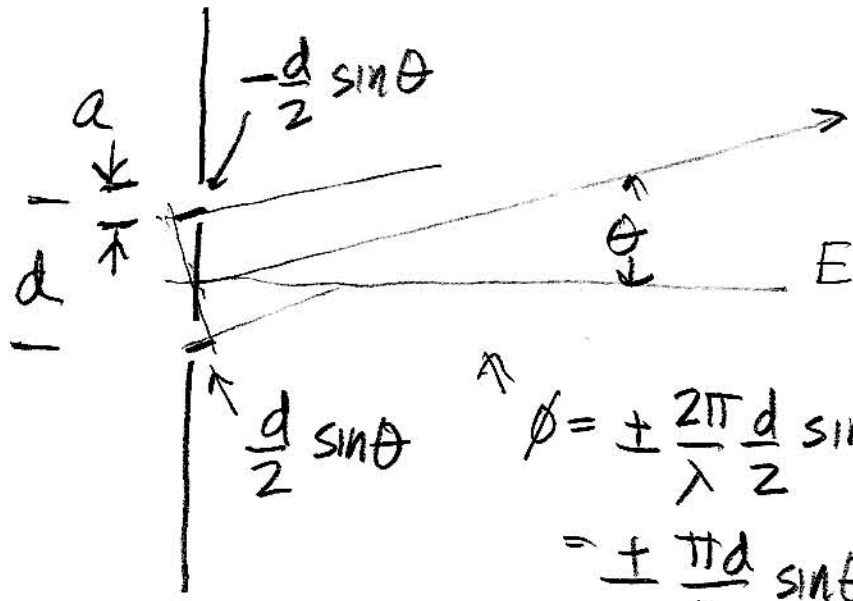
to approximately: $\frac{\pi \sin\theta a}{\lambda} = \pi$

$$\sin\theta = \frac{\lambda}{a} \; ; \; \pm \rightarrow \frac{2\lambda}{a}$$

What if... there is a combination of 2 (or more) slits?

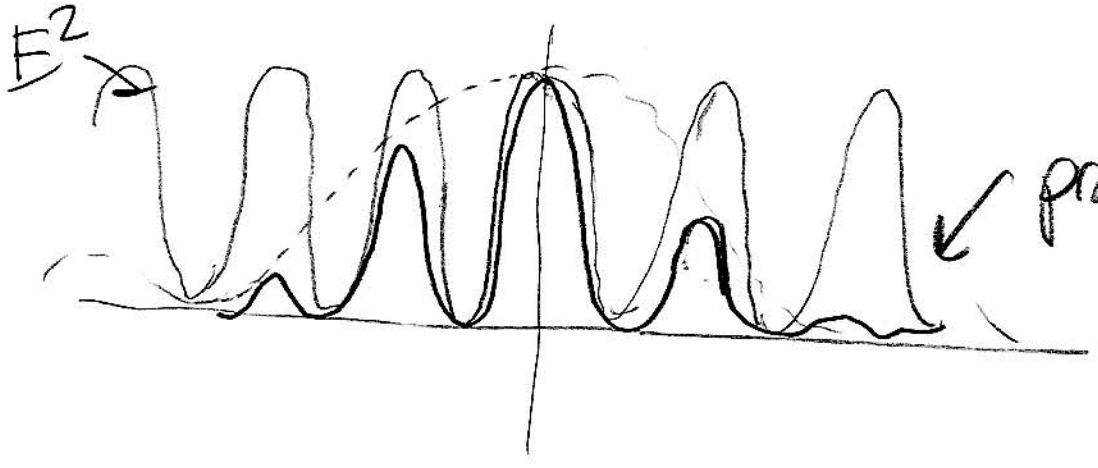
BTW: "λ/a" is approximate width in FWHM:



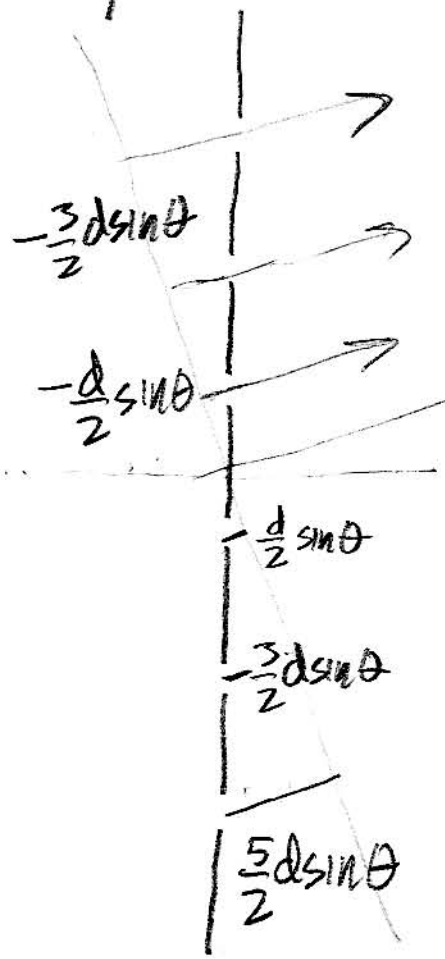


$$E \propto \cos\left(\frac{\pi d}{\lambda}\sin\theta\right) \frac{\sin\left(\frac{\pi \sin\theta a}{\lambda}\right)}{\sin\theta}$$

$d > a$, envelope



now add more slits.



$$E \propto \left(e^{5i\phi} + e^{3i\phi} + e^{i\phi} + e^{-i\phi} + e^{-3i\phi} + e^{-5i\phi}\right) \frac{\sin\left(\frac{\pi \sin\theta a}{\lambda}\right)}{\sin\theta}$$

$$\phi = \frac{\pi d}{\lambda}\sin\theta$$

$-(N-1)\cdot 2\phi$

$$E \propto e^{5i\phi}\left(1 + e^{-2i\phi} + e^{-4i\phi} + e^{-6i\phi} + e^{-10i\phi} + \dots\right) \times \frac{\sin\left(\frac{\pi \sin\theta a}{\lambda}\right)}{\sin\theta}$$

$5 = N-1$

$x = e^{-2i\phi}$

$1 + x + x^2 + \dots + x^{N-1}$ $\qquad x = e^{-i\phi}$

$1 + x + x^2 + \dots$ to infinity $= \frac{1}{1-x}$

$1 + x + x^2 + \dots + x^{N-1} = 1 + x + x^2 + \dots x^{N-1} + x^N + \dots$

$- x^N\left(1 + x + x^2 + \dots\right).$

$$1 + x^2 + x^4 + \cdots + x^{N-1} \qquad \frac{1 - x^N}{1 - x}$$

$$x = e^{-2i\phi} \qquad 5 = N - 1$$

$$E \propto e^{(N-1)i\phi} \left(\frac{1 - e^{-2iN\phi}}{1 - e^{-2i\phi}} \right) \frac{\sin\left(\frac{\pi \sin\theta}{\lambda} a\right)}{\sin\theta}$$

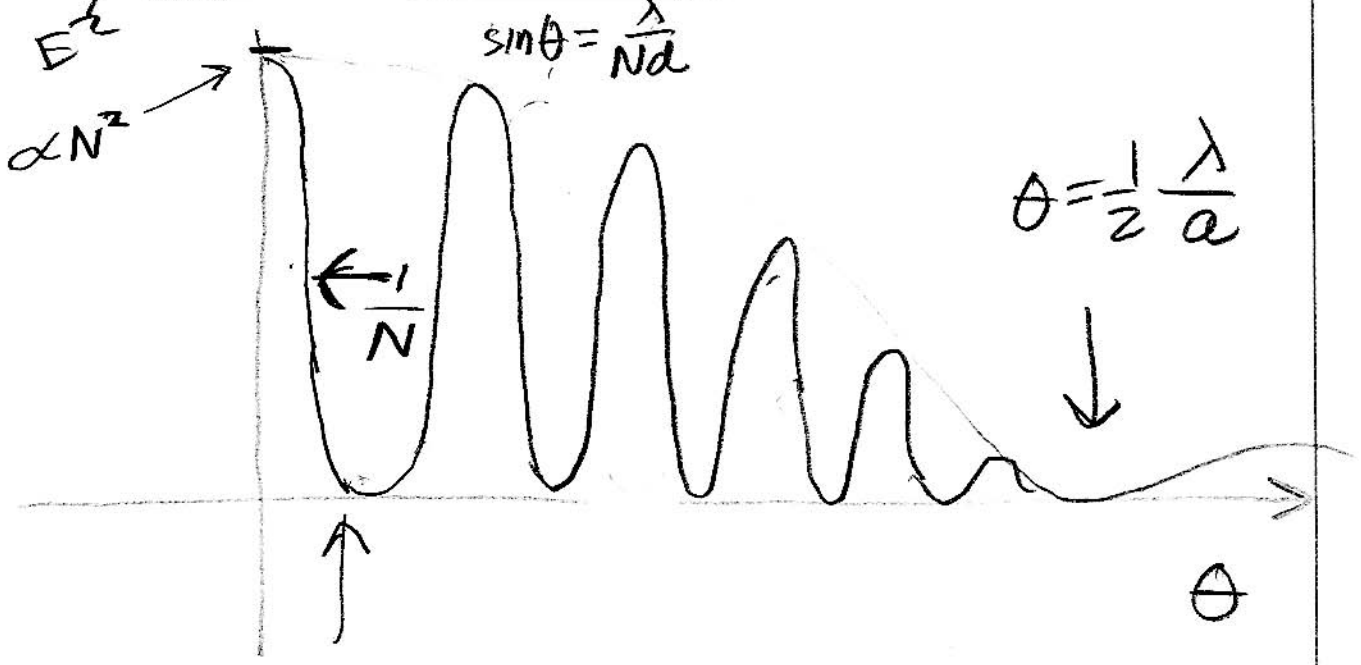
$$\propto \frac{e^{-i\phi}}{e^{-i\phi}} \left[\frac{e^{iN\phi} - e^{-iN\phi}}{e^{i\phi} - e^{-i\phi}} \right] \frac{\sin\left(\frac{\pi \sin\theta}{\lambda} a\right)}{\sin\theta}$$

$$E \propto \frac{\sin(N\phi)}{\sin\phi} \; \frac{\sin\left(\frac{\pi \sin\theta}{\lambda} a\right)}{\sin\theta}$$

$$E \propto \frac{\sin\left(N \frac{\pi \sin\theta}{\lambda} d\right) \sin\left(\frac{\pi \sin\theta}{\lambda} a\right)}{\sin\left(\frac{\pi \sin\theta}{\lambda} d\right) \quad \frac{\pi a}{\lambda} \sin\theta}$$

↑ added here

$$E^2 \propto \frac{\sin^2\left(N \frac{\pi \sin\theta}{\lambda} d\right) \sin^2\left(\frac{\pi \sin\theta}{\lambda} a\right)}{\left[\frac{\pi d}{\lambda} \; \frac{\pi a}{\lambda}\right]^2 \sin^4\theta}$$



$$\frac{N\pi \sin\theta}{\lambda} d = \frac{\pi}{2}$$

$$\sin\theta = \frac{1}{2}\frac{\lambda}{Nd}$$

$d > a$

N: maybe large!