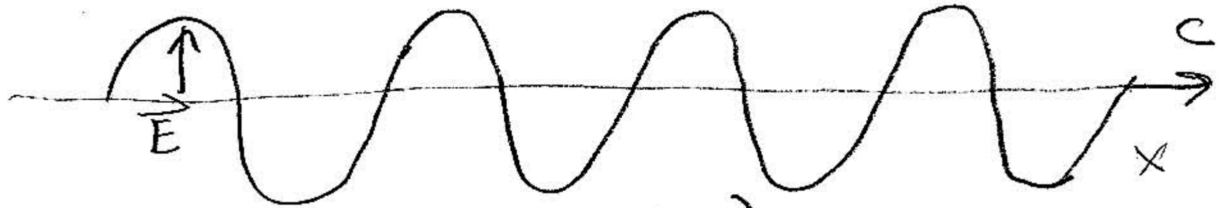
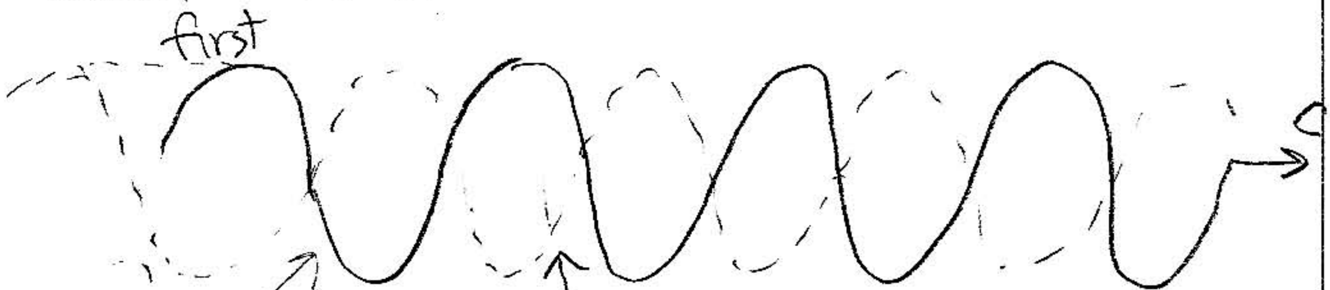
Interference

e.m. wave:



$$\vec{E} = \vec{E}_0 e^{i(\omega t - k_x x)}$$

Imagine 2 waves on top of each other.



$\vec{E}_0 e^{i(\omega t - k_x x)}$ $\quad$ $\vec{E}_0 e^{i(\omega t - k_x x \pm \pi)}$

add together: $= E_0 (1 + e^{\pm i\pi}) e^{i(\omega t - k_x x)}$

$= 0!$

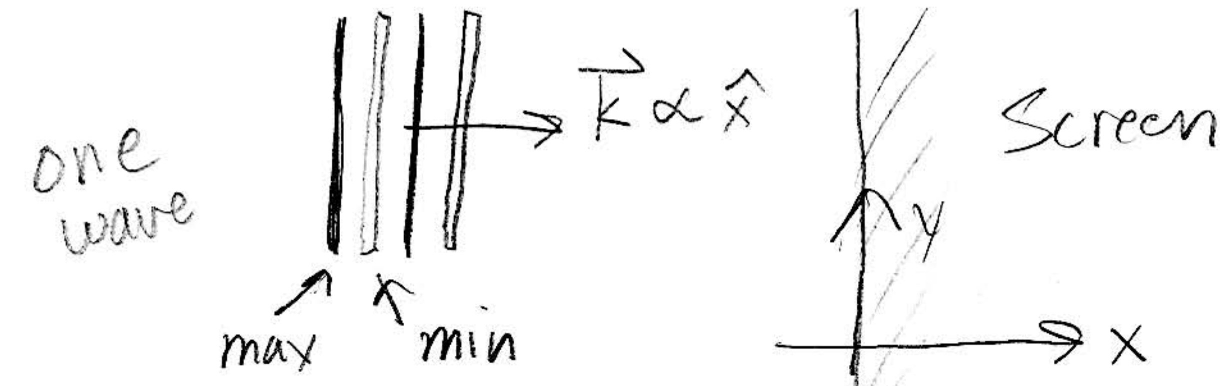
<u>Ponder</u>: second wave not same amplitude... other amplitud.

$$\vec{E}_{total} \propto (E_{01} + E_{02} e^{\pm i\phi})$$

<u>Intensity</u> $\propto |E_{01} + E_{02} e^{\pm i\phi}|^2$

$$= \underbrace{(E_{01} + E_{02} \cos\phi)^2}_{\cos\phi = -1 \text{ destructive}} + \underbrace{(E_{02} \sin\phi)^2}_{\sin\phi = 0}$$

More Fun: 2 dimensions

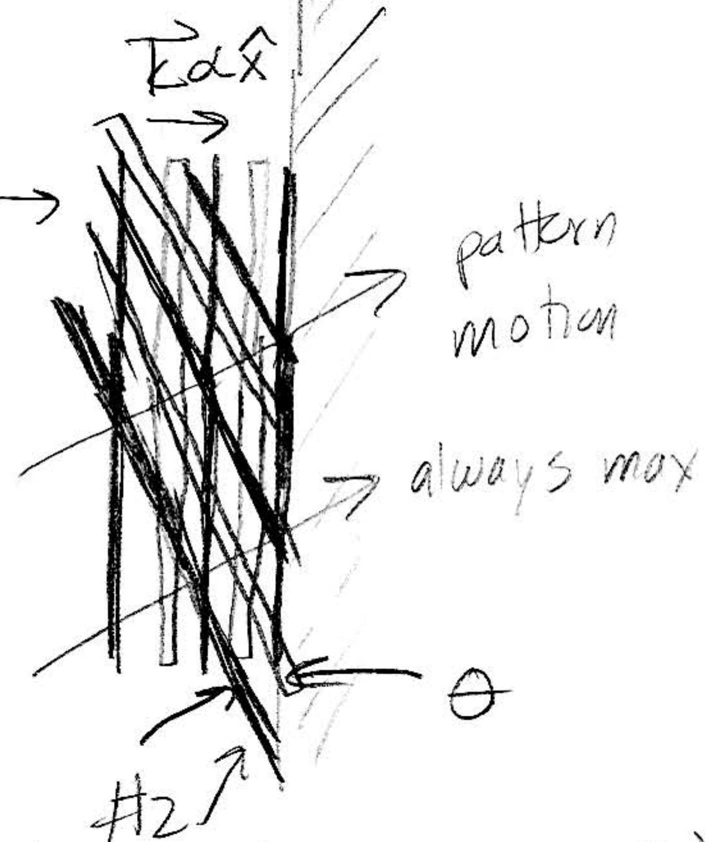


#1 →

$$\vec{E}_0 e^{i(\omega t - kx)}$$

$k = \frac{2\pi}{\lambda}$

assume same strength



#2

$$\vec{E}_0 e^{i\left(\omega t - \frac{2\pi}{\lambda}(\cos\theta\, x + \sin\theta\, y)\right)}$$

$$\vec{E}_{tot} = \vec{E}_0 e^{i\omega t}\left(e^{-\frac{2\pi}{\lambda} i (\cos\theta\, x + \sin\theta\, y)} + e^{\frac{2\pi}{\lambda} i x}\right)$$

take $x = 0$

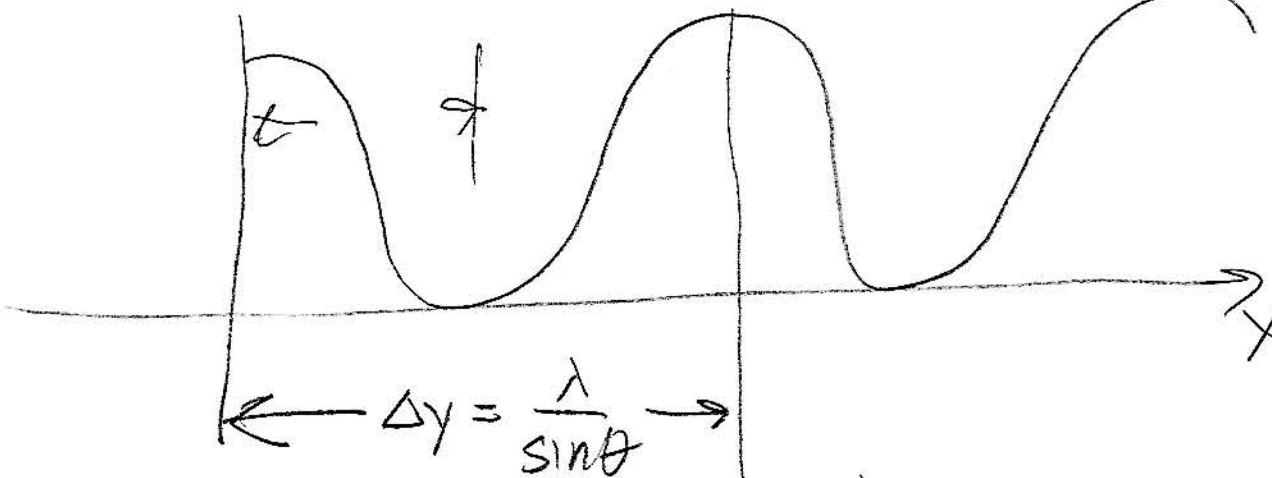
$$|\vec{E}_{tot}|^2 = E_0^2 \left| e^{-\frac{2\pi}{\lambda} i y \sin\theta} + 1 \right|^2$$

$$|e^{i\phi}+1|^2 = |e^{i\phi/2}||e^{i\phi/2} - e^{-i\phi/2}|^2$$

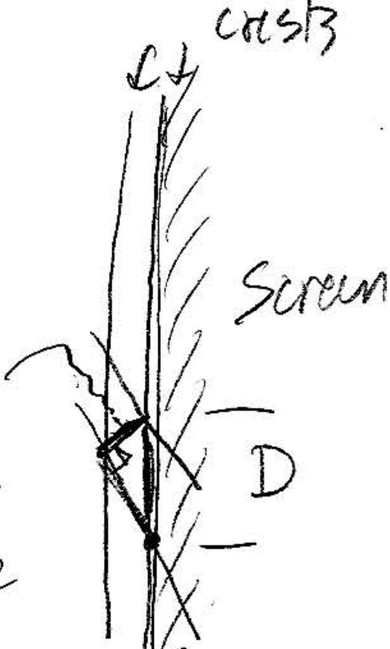
$$= \cos^2\frac{\phi}{2}$$

$$|E_{tot}|^2 = E_0^2\cos^2\left(\frac{\pi}{\lambda}y\sin\theta\right)$$

$$= E_0^2\cos^2\left(\pi\cdot\frac{y}{(\lambda/\sin\theta)}\right)$$



Interpretation:



when λ
constructive
interference

$\sin\theta = \frac{\lambda}{D}$

$D = \frac{\lambda}{\sin\theta}$

destructive when $\lambda/2$, $\sin\theta = \frac{\lambda}{2D}$, $D = \frac{\lambda}{2\sin\theta}$