$$\vec{E} = \hat{x}E_x(y-ct) + \hat{z}E_z(y-ct)$$

$\vec{\nabla}\times\vec{E}$:

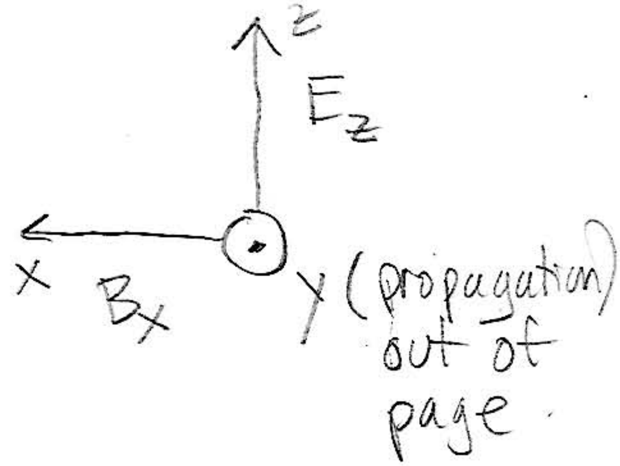
$$= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(y-ct) & 0 & E_z(y-ct) \end{vmatrix}$$

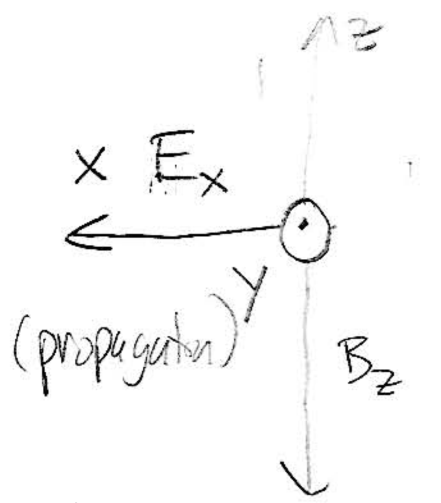
$$= \hat{x}\frac{\partial E_z(y-ct)}{\partial y} - \hat{z}\frac{\partial E_x(y-ct)}{\partial y}$$

$$= -\hat{x}\frac{1}{c}\frac{\partial B_x}{\partial t}(y-ct) - \hat{z}\frac{1}{c}\frac{\partial B_z}{\partial t}(y-ct)$$

$$E_z'(y-ct) = -\frac{1}{c}(-c)B_x'(y-ct) \qquad -E_x' = -\frac{1}{c}(-c)B_z'$$

$$E_z = B_x \qquad -E_x = B_z$$





$\vec{E}\times\vec{B}$ = direction of propagation

Two Polarizations

$\cos(y-ct) \rightarrow$ generalize to

$$\cos\left[\frac{\pm 2\pi}{\lambda}(y-ct)\right] \quad \lambda = \text{wavelength}$$

$$= \cos(\pm k_y y - \omega t) \quad k_y = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi c}{\lambda} = 2\pi\nu$$

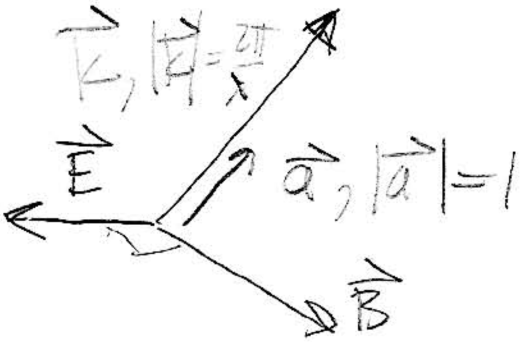
generalize furter:

$$k_y y \rightarrow \vec{k}\cdot\vec{r} = \frac{2\pi}{\lambda}(a_x x + a_y y + a_z z)$$

$$\vec{k} = \frac{2\pi}{\lambda}\underbrace{(a_x, a_y, a_z)}_{|a_x|^2+|a_y|^2+|a_z|^2=1} \qquad \vec{r} = (x, y, z)$$

unit vector

Welford p.5



direction of $\vec{E}\times\vec{B}$ is $\vec{k}$!

$$\begin{pmatrix}\vec{E}\\ \vec{B}\end{pmatrix} = \mathrm{Re}\left[\begin{pmatrix}\vec{E}_0\\ \vec{B}_0\end{pmatrix} e^{\pm i(\vec{k}\cdot\vec{r}-\omega t)}\right]$$

Some fun: 2 polarizations of $\vec{E}_0, \vec{B}_0$

could have $\vec{E}_0 = z_1\vec{E}_1 + z_2\vec{E}_2$

when complex → circular polarization

Energy In EM Wave

density $\frac{1}{8\pi}(E^2 + B^2)$ $\quad \frac{\text{ergs}}{\text{cm}^3}$

$$E = B = E_0 \cos\left(\pm \vec{k}\cdot\vec{r} - \omega t\right)$$

$$E^2 + B^2 = 2E_0^2 \cos^2\left(\pm \vec{k}\cdot\vec{r} - \omega t\right)$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x) = \tfrac{1}{2}(1 + \cos^2 x - \sin^2 x)$$

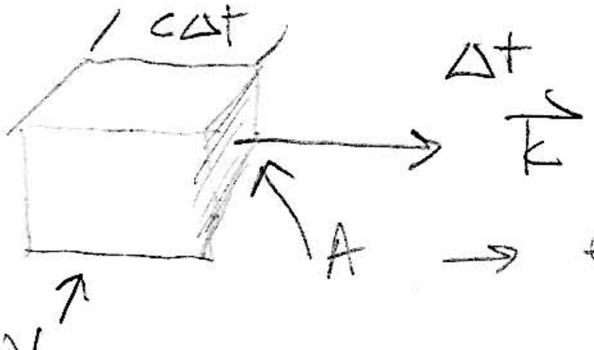
$$= \tfrac{1}{2}(2\cos^2 x) = \cos^2 x$$

$$E^2 + B^2 = E_0^2\left(1 + \underbrace{\cos 2(\vec{k}\cdot\vec{r} - \omega t)}_{}\right)$$

averages to zero over time and/or space

$$\langle E^2 + B^2 \rangle = E_0^2$$

$$\langle \text{energy density} \rangle = \frac{E_0^2}{8\pi}$$



A → energy that flows across this surface, $\perp$ to direction of motion.

$$\text{Energy} = \frac{E_0^2}{8\pi} \cdot V = \frac{E_0^2}{8\pi} \cdot A \cdot c\Delta t$$

$$\text{Power Flux} = \frac{\text{Energy}}{A\,\Delta t} = \frac{E_0^2 c}{8\pi}$$

Light

Electromagnetic waves

$\lambda \simeq 550\ nm$ is peak of eye's sensitivity

(see EM spectrum figure)

$\simeq \frac{1}{50}$ Thickness of human hair (usually too small to measure)

$$\nu = \frac{c}{\lambda} = \frac{3 \cdot 10^{8}\ m}{550 \cdot 10^{-9}\ m} \simeq \frac{1}{200} \cdot 10^{17} \frac{1}{s}$$

$\nu \simeq 5 \cdot 10^{14}\ 1/s$ (too fast to measure)

→ look at time averaged quantities,

usually $\langle Re[e^{\pm i(\vec{k}\cdot\vec{r}-\omega t)}] \rangle = \frac{1}{2}$

→ exception: interference, beats

Crucial Concept "beam of light"

→ one wavelength (monochromatic)

(next page)