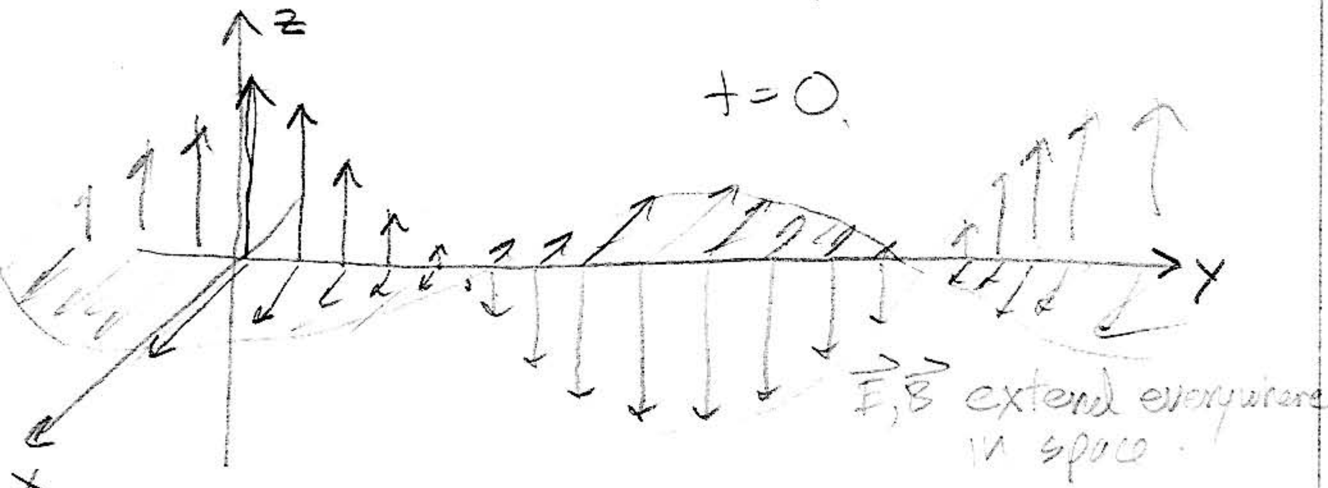
→ And let there be light

$$\rho = \vec{J} = 0 \quad \text{(free space)}$$

$$\text{curl}\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\text{curl}\,\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

Ansatz: $\vec{E} = \hat{z} E_0 \cos(y - vt)$




$$\vec{B} = \hat{x} B_0 \cos(y - vt)$$

Try now

$$\text{curl}\,\vec{E} : \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_0 \cos(y-vt) \end{vmatrix} = \hat{x} \cdot \frac{\partial}{\partial y}\left(E_0 \cos(y-vt)\right) \cancel{- \hat{y}\,\frac{\partial}{\partial x}\left(E_0 \cos(y-vt)\right)} \rightarrow 0$$

$$= -\hat{x} E_0 \sin(y - vt)$$

$$\text{curl}\,\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_0 \cos(y-vt) & 0 & 0 \end{vmatrix} = \begin{array}{l} \hat{x} \cdot 0 \\ -\hat{y} \cdot \left(-\frac{\partial}{\partial z} B_0 \cos(y-vt)\right) \\ + \hat{z}\left(-\frac{\partial}{\partial y} B_0 \cos(y-vt)\right) = +\hat{z} B_0 \sin(y-vt) \end{array}$$

$$-\frac{1}{c}\frac{\partial}{\partial t}\vec{B} = -\frac{1}{c}\frac{\partial}{\partial t}\hat{x}B_0\cos(y-vt)$$

$$= -\frac{v}{c}\hat{x}B_0\sin(y-vt)$$

$$\text{curl }\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$-\hat{x}E_0\sin(y-vt) = -\frac{v}{c}\hat{x}B_0\sin(y-vt)$$

$$E_0 = \frac{v}{c}B_0$$

$$\text{curl }\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

$$\frac{\partial\vec{E}}{\partial t} = \frac{\partial}{\partial t}\left(\hat{z}E_0\cos(y-vt)\right) = +v\hat{z}E_0\sin(y-vt)$$

$$-\hat{z}B_0\sin(y-vt) = +\frac{v}{c}\hat{z}E_0\sin(y-vt)$$

$$B_0 = \frac{v}{c}E_0$$

$$\cancel{E_0} = \frac{v}{c}\cdot\frac{v}{c}\cancel{E_0}$$

$$\left(\frac{v}{c}\right)^2 = 1 \qquad \boxed{v = \pm c}$$

## Wave Equation

$$\rho = \vec{J} = 0$$

$$\text{curl } \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \qquad \text{curl } \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = 0$$

take curl of both sides

$$\text{curl curl} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix} \pm \frac{1}{c}\frac{\partial}{\partial t}\begin{pmatrix} \text{curl } \vec{B} \\ \text{curl } \vec{E} \end{pmatrix} = 0$$

$$\text{curl curl } \vec{C} = \text{grad}(\text{div } \vec{C}) - \nabla^2 \vec{C}$$

div $\vec{B}$ = div $\vec{E} = 0$ — Laplacian

$$(\nabla^2 C_x, \nabla^2 C_y, \nabla^2 C_z)$$

$$\text{curl } \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \qquad \text{curl } \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$-\nabla^2 \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix} = 0$$

Wave equation! $c$ = speed of propagation

Each component of $\vec{E}, \vec{B}$ must satisfy, and maxwell's equation too.

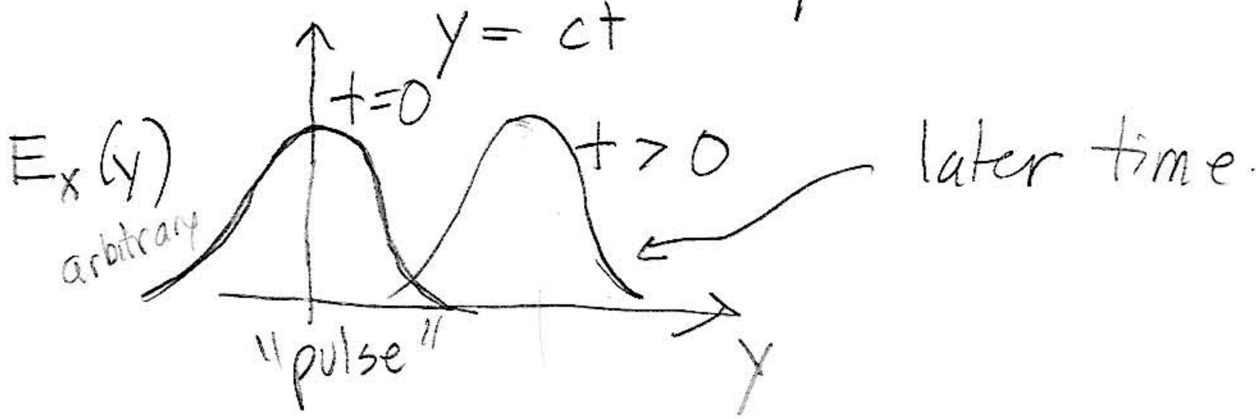
$$\underbrace{\nabla^2}\, E_x - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E_x = 0$$

easier if $\nabla^2$ were only $\frac{\partial^2}{\partial x^2}$ or $\frac{\partial^2}{\partial y^2}$ or $\frac{\partial^2}{\partial z^2}$

Suppose propagation in y direction

$$\vec{E}(x,t) = \hat{x}\,\underbrace{E_x(y-ct)} + \hat{y}\,\underbrace{E_y(y-ct)} + \hat{z}\,\underbrace{E_z(y-ct)}$$

$\underset{\sim}{\text{all}}$ travelling in y-direction




→ wave equation satisfied.

$\vec{\nabla}\cdot\vec{E} = 0$ Maxwell Equation

$$\underbrace{\frac{\partial E_x(y-ct)}{\partial x}}_{0 \text{ trivial}} + \underbrace{\frac{\partial E_y(y-ct)}{\partial y}}_{\text{non trivial}} + \underbrace{\frac{\partial E_z(y-ct)}{\partial z}}_{0 \text{ trivial}} = 0$$

means: $E_y$ not a wave.

$E_y$ = constant (or zero)

Travelling wave in y-direction has only x, z components of $\vec{E}$ (and $\vec{B}$) ... in free space.

$\vec{\nabla}\cdot\vec{B}=0$ "Transverse Vector Wave"