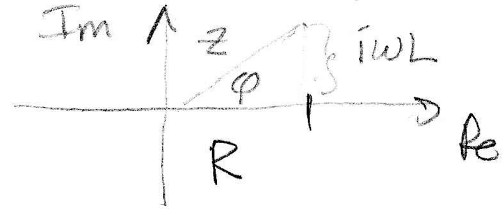
$$\frac{\mathcal{E}_0}{R + i\omega L} \quad \vdots$$



$$Z = \sqrt{R^2 + (\omega L)^2}\, e^{i\varphi} \qquad \tan\varphi = \frac{\omega L}{R}$$

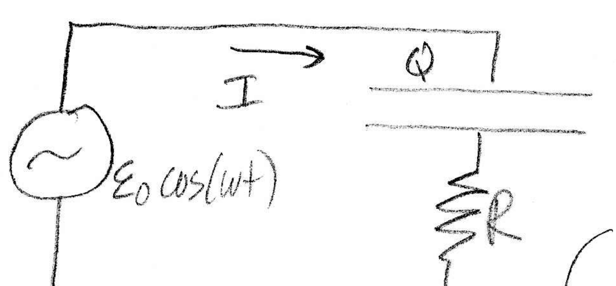
$$I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L)^2}}\, e^{-i\varphi}$$

$$I = I_0 e^{i(\omega t - \varphi)}$$

really, $\mathrm{Re}[I] = \frac{\mathcal{E}_0}{\sqrt{R^2+(\omega L)^2}} \cos(\omega t - \varphi)$

$$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right).$$

+ : must "get over" φ,
so shift of I is "forward"



here, $\frac{dQ}{dt} = I$

$$\frac{Q}{C} + IR = \mathcal{E}_0 \cos(\omega t)$$

try $I = I_0 e^{i\omega t}$

$$Q = \int I\,dt = \frac{I_0}{i\omega} e^{i\omega t}$$

$$\left(\frac{I_0}{i\omega C} + I_0 R\right) \cancel{e^{i\omega t}} = \mathcal{E}_0 \cancel{e^{i\omega t}}$$

$$I_0 = \frac{\mathcal{E}_0}{R + \frac{1}{i\omega C}} = \frac{\mathcal{E}_0}{R - \frac{i}{\omega C}}$$

− sign

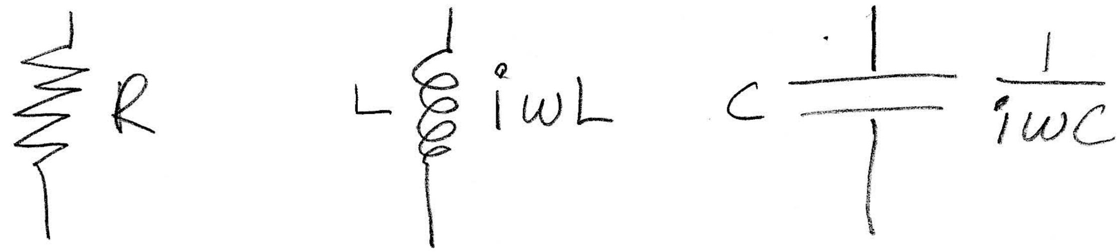
$$\omega L \rightarrow -\frac{1}{\omega C}$$

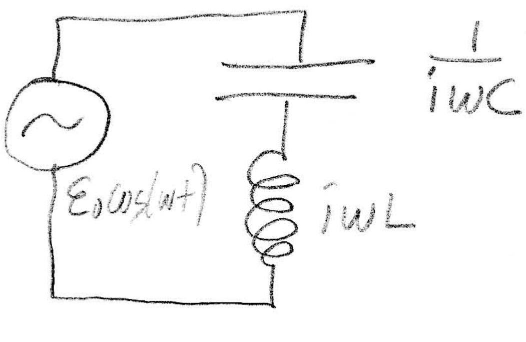
$$\mathrm{Re}[I] = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cos(\omega t + \varphi)$$

$$\varphi = \tan^{-1}\left(\frac{1}{\omega R C}\right)$$

"small" effect: small inductor, small ω
large capacitor, large ω

The preceding can be summarized with the concept of impedance, "generalized" resistance.



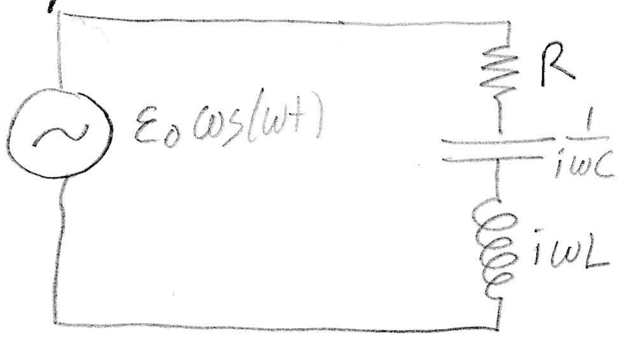


$$I_0 = \frac{\mathcal{E}_0}{\frac{1}{i\omega C} + i\omega L}$$

$$= \frac{\mathcal{E}_0}{i\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{\mathcal{E}_0}{\omega L - \frac{1}{\omega C}} \cdot \left(-i = e^{-i\pi/2}\right)$$

$$I = \frac{\mathcal{E}_0}{\omega L - \frac{1}{\omega C}} \cos\left(\omega t - \frac{\pi}{2}\right) = \frac{\mathcal{E}_0}{\omega L - \frac{1}{\omega C}} \sin(\omega t)$$



$$I_0 = \frac{\varepsilon_0}{R + \frac{1}{i\omega C} + i\omega L} = \frac{\varepsilon_0}{Z}$$

impedance

$= Y \cdot \varepsilon_0$

admittance $= \frac{1}{Z}$

$$|I_0| = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

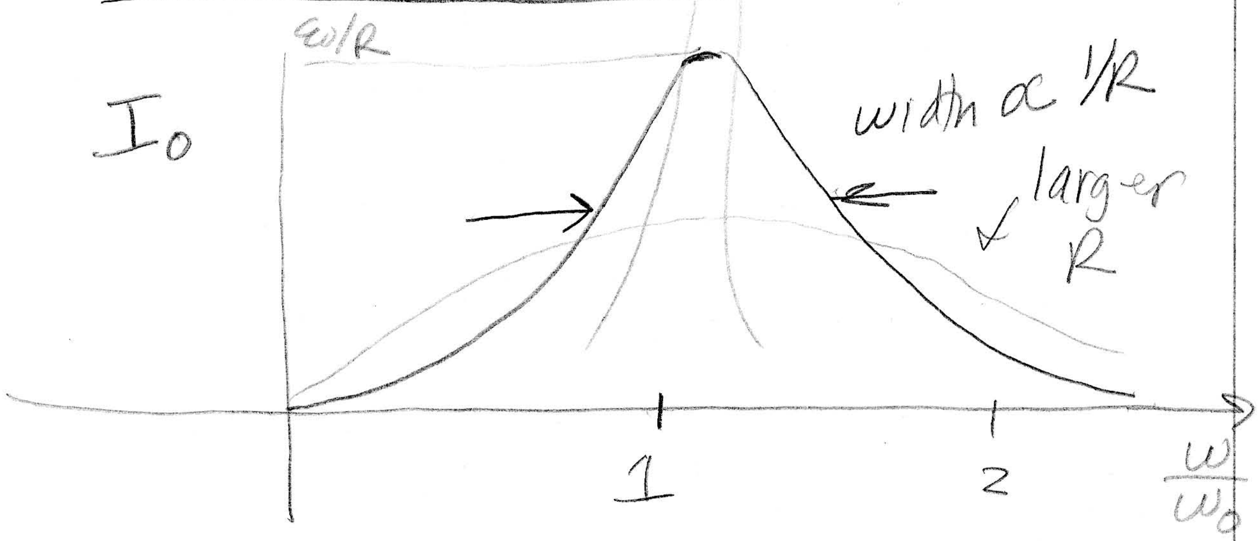
$|I_0|$ maximum when $\omega L - \frac{1}{\omega C} = 0$

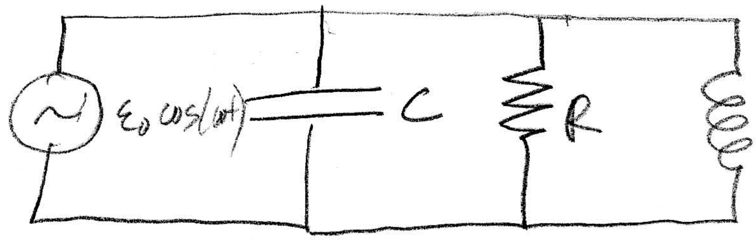
$$\omega^2 = \frac{1}{LC}$$

resonance at $\omega_0 = \frac{1}{\sqrt{LC}}$

at $\omega = \omega_0$ then $I_0 = \frac{\varepsilon_0}{R}$ ← as though

"Resonant Circuit"





in parallel!

$$Y = \frac{1}{Z} = \frac{1}{\left(\frac{1}{i\omega C}\right)} + \frac{1}{R} + \frac{1}{i\omega L}$$

$$Y = \frac{1}{R} + i\omega C - \frac{i}{\omega L}$$

$$I_0 = Y \cdot \mathcal{E}_0 = |Y| e^{i\phi}$$

$$|Y|^2 = \frac{1}{R^2} + \underbrace{\left(\omega C - \frac{1}{\omega L}\right)^2}$$

smallest now when

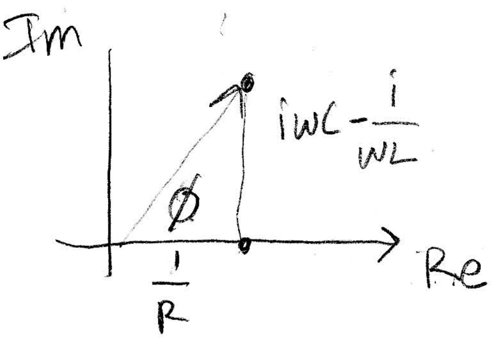
$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$\omega_0^2 = \frac{1}{LC} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

when $\omega = \omega_0$,

$$|Y| = \frac{1}{R}, \quad |I_0| = \frac{\mathcal{E}_0}{R}$$

Phase: (look at admittance)

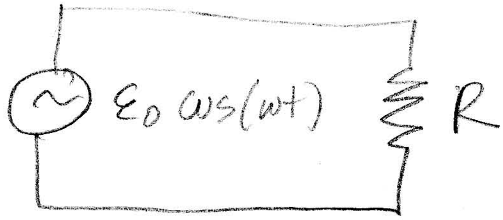


$$\phi = \tan^{-1}\left(\frac{\left(\omega C - \frac{1}{\omega L}\right)}{\frac{1}{R}}\right)$$

$$= \tan^{-1}\left(\omega RL - \frac{R}{\omega L}\right)$$

$R = 0$?

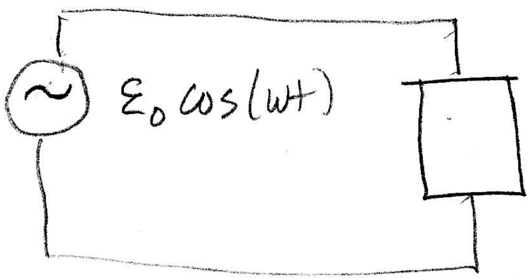
Power :



$$\underset{\substack{\uparrow \\ \text{instantaneous}}}{P} = \frac{\mathcal{E}_0^2}{R}\cos^2\omega t$$

$$\langle P \rangle = \frac{1}{2}\frac{\mathcal{E}_0^2}{R} = \frac{V_{rms}^2}{R}$$

$$V_{rms}^2 = \frac{1}{2}\mathcal{E}_0^2$$



$$Z = \frac{1}{Y} \qquad Y = |Y|e^{i\phi}$$

$$I_0 = Y\mathcal{E}_0 = |Y|e^{i\phi}\mathcal{E}_0$$

instantaneous power : $P = IV$

$$= Re[I]\cdot Re[V]$$

$$= Re[I_0 e^{i\omega t}]\,Re[\mathcal{E}_0 e^{i\omega t}]$$

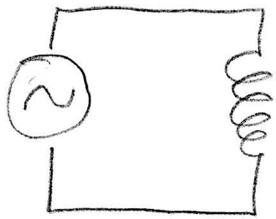
$$= Re[|Y|e^{i\phi}\mathcal{E}_0 e^{i\omega t}]\,Re[\mathcal{E}_0 e^{i\omega t}]$$

$$= |Y|\mathcal{E}_0^2\underbrace{\cos(\omega t+\phi)}_{\cos(\omega t)\cos\phi - \sin(\omega t)\sin\phi}\cos(\omega t)$$

$$\langle\cos^2(\omega t)\rangle = \frac{1}{2}$$

$$\langle\sin(\omega t)\cos(\omega t)\rangle = \langle\tfrac{1}{2}\sin(2\omega t)\rangle = 0$$

$$\boxed{\langle P \rangle = \frac{1}{2}|Y|\,\mathcal{E}_0^2 \cos\phi = \underbrace{I_{rms}}_{\frac{1}{\sqrt{2}}|Y|\mathcal{E}_0} \underbrace{V_{rms}}_{\frac{1}{\sqrt{2}}\mathcal{E}_0} \cos\phi}$$

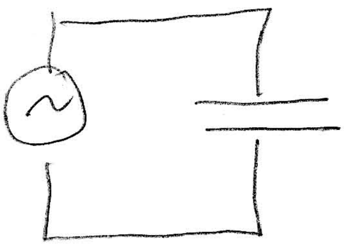


$i\omega L$

$$Y = \frac{1}{i\omega L} = \frac{-i}{\omega L} \qquad \phi = -\frac{\pi}{2}$$

$$\cos(\phi) = 0!$$

no power dissappated!



$\frac{1}{i\omega C}$

$$\cos(\phi) = 0$$

any combination of L + C ... no power dissipation!

Need R ...