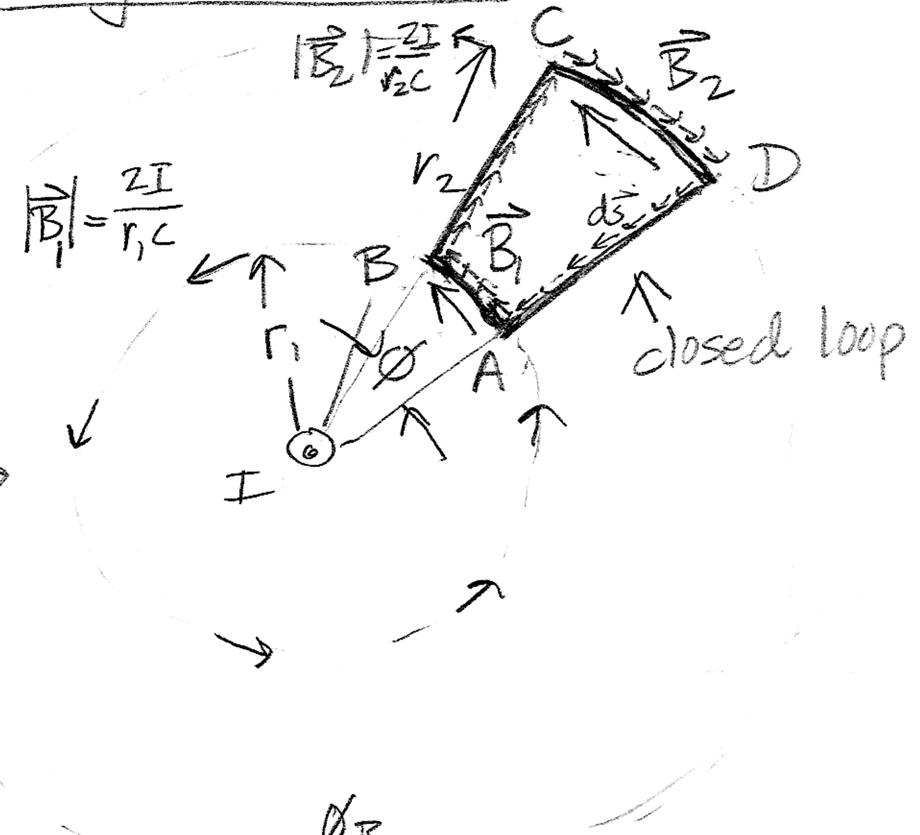
Line Integrals of $\vec{B}$



direction of $\vec{B}$: use right hand, put thumb in

$$\int_A^B \vec{B}_1 \cdot d\vec{s} = = \int_{\phi_A}^{\phi_B} \left(\frac{2I}{r_1 c}\right)(r_1\, d\phi) = \frac{2I}{c}\phi$$

$$\int_B^C \vec{B} \cdot d\vec{s} = 0 \qquad (\vec{B} \perp \text{ to } d\vec{s})$$

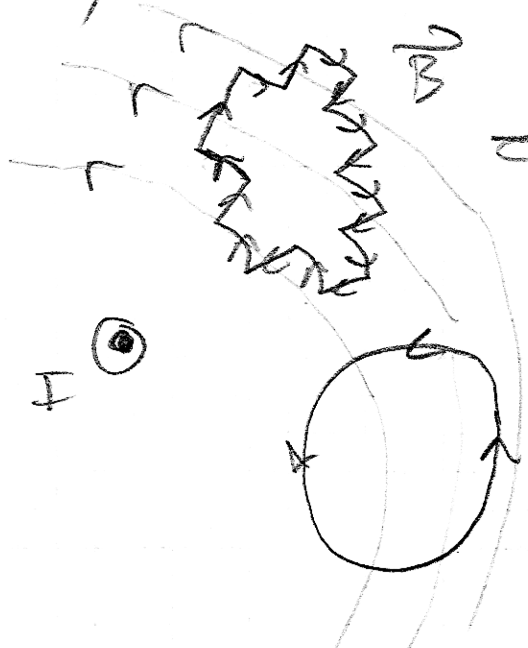
$$\int_C^D \vec{B}_2 \cdot d\vec{s} = \int_{\phi_C}^{\phi_D} \left(\frac{2I}{r_2 c}\right)(r_2\, d\phi) = \int_{\phi_B}^{\phi_A} \frac{2I}{c} d\phi$$

$$= -\frac{2I}{c}\phi$$

$$\int_D^A \vec{B} \cdot d\vec{s} = 0$$

$$\oint \vec{B} \cdot d\vec{s} = 0$$

<u>Claim</u>: can take a more peculiar path, like:



$\leftarrow \oint \vec{B} \cdot d\vec{s} = 0$

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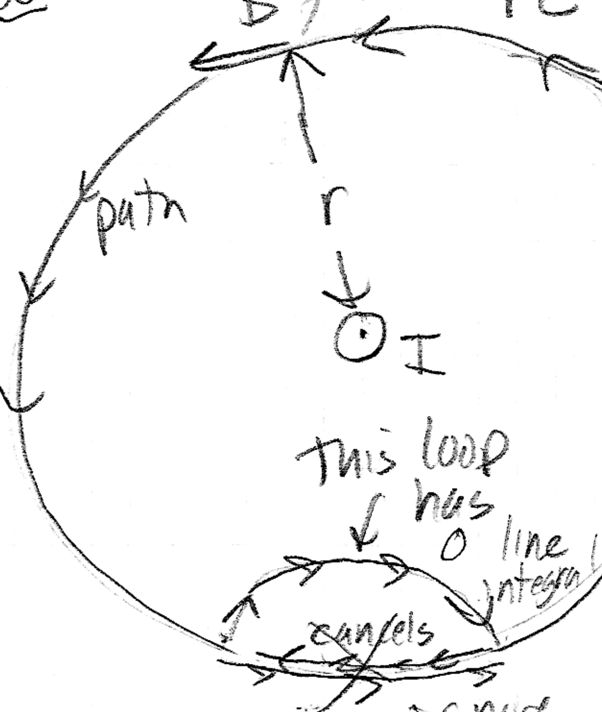
Now include I <u>inside</u> loop!

make loop a <u>circle</u>

$\vec{B}$, $|\vec{B}| = \frac{2I}{rc}$



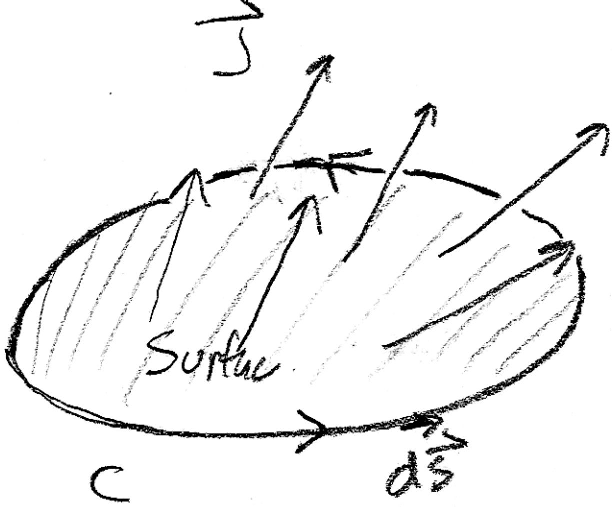
$$\boxed{\begin{aligned} &\oint \vec{B} \cdot d\vec{s} \\ &= 2\pi r \cdot \frac{2I}{rc} \\ &= \frac{4\pi}{c} I \end{aligned}}$$

<u>now</u>: distort path

$$\oint \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} \cdot (\text{current enclosed by path})$$

More generally,

$$I = \int_{\text{Surface}} \vec{J} \cdot d\vec{A}$$



$$\oint_{\text{Curve}} \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} \int_{\text{Surface}} \vec{J} \cdot d\vec{A}$$

speed of light

recall Gauss' Law:

$$\underbrace{\int_{\text{Surface}} \vec{E} \cdot d\vec{s}}_{\downarrow} = 4\pi \int_{\text{Volume}} \rho \, dV$$

$$\int_{\text{Volume}} (\text{div} \vec{E}) \cdot dV = 4\pi \int \rho \, dV$$

$$\boxed{\operatorname{div} \vec{E} = 4\pi\rho}$$

$$\oint_{\text{Curve}} \vec{B} \cdot d\vec{s} = \int_{\text{surface}} (\operatorname{curl} \vec{B}) \cdot d\vec{A}$$

← p. 68, Chap. 2

curl $\vec{B}$:
cartesian only

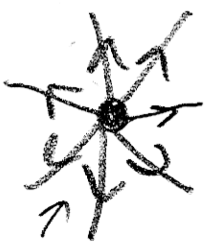
$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{x} \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_y & B_z \end{vmatrix} - \hat{y} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ B_x & B_z \end{vmatrix} + \hat{z} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ B_x & B_y \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) - \hat{y}\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right) + \hat{z}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)$$

Then $$\boxed{\operatorname{curl} \vec{B} = \frac{4\pi}{c} \vec{J}}$$

No sources of $\vec{B}$ field:



$\vec{B}$ NEVER

$$\int_S \vec{B} \cdot d\vec{s} = 0$$

$$\boxed{\operatorname{div} \vec{B} = 0}$$