Origin of Magnetic Force

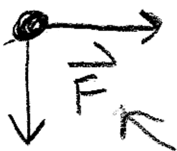
y, x

wire with current I

no net charge!

"Lab" frame.

$q > 0$



v, $\vec{F}$ ← feels repulsive force.

(like a current $\parallel$ to I).

(opposite currents repel)

Lab Frame: ions, + charge, immobile, λ_0 esu/cm

$-\lambda_0$ esu/cm $\longrightarrow \vec{v}_0$

Blow up wire

electrons moving, left to right, velocity $\vec{v}_0$

$\beta_0 = \frac{v_0}{c}$

$\vec{v}$, F

negative charge

current from right to left.

NO $\vec{E}$ field!! $\lambda_0 - \lambda_0 = 0$.

Boost to charge particle rest frame.

$-\vec{v}$ + charge density $\lambda_0 \cdot \gamma > \lambda_0$, due to length contraction

− charge.

$\beta = \frac{v}{c}$ • q at rest $\longrightarrow v_0'$ → what is γ_0'?

$$\beta_0' = \frac{\beta_0 - \beta}{1 - \beta\beta_0}$$

$$1 - \beta_0'^2 = 1 - \frac{(\beta_0 - \beta)^2}{(1-\beta\beta_0)^2} = \frac{(1-\beta\beta_0)^2 - (\beta_0-\beta)^2}{(1-\beta\beta_0)^2}$$

$$\frac{1}{\gamma_0'^2} = 1-\beta_0'^2 = \frac{1-2\beta\beta_0+\beta^2\beta_0^2-\beta_0^2+2\beta\beta_0-\beta^2}{(1-\beta\beta_0)^2}$$

$$\frac{1}{\gamma_0'^2} = \frac{1-\beta_0^2-\beta^2+\beta^2\beta_0^2}{(1-\beta\beta_0)^2} = \frac{(1-\beta^2)(1-\beta_0^2)}{(1-\beta\beta_0)^2}$$

$$\boxed{\gamma_0' = \frac{1-\beta\beta_0}{\sqrt{1-\beta^2}\sqrt{1-\beta_0^2}} = \gamma\gamma_0(1-\beta\beta_0)}$$

want the charge density of the negative charge in q's rest frame.

Route: (2 steps)

$$\underbrace{-\lambda_0 \frac{\text{esu}}{\text{cm}}}_{\substack{\text{wire rest}\\ \text{frame}}} \Rightarrow \underbrace{\frac{-\lambda_0}{\gamma_0}\frac{\text{esu}}{\text{cm}}}_{\substack{-\text{ charge}\\ \text{rest frame}}} \Rightarrow \underbrace{-\frac{\lambda_0\gamma_0'}{\gamma_0}\frac{\text{esu}}{\text{cm}}}_{\substack{\text{q's rest}\\ \text{frame!}}}$$

already length contracted

$$\left.\begin{array}{l}-\text{ charge}\\ \text{density}\end{array}\right\}\ \text{q rest frame} = -\frac{\lambda_0}{\gamma_0}\cdot\gamma\gamma_0(1-\beta\beta_0)$$

$$= -\lambda_0\gamma(1-\beta\beta_0)$$

Net charge density:

$$\underbrace{+ \quad + \quad -}_{\text{for } q' \text{ at rest frame}} = \gamma\lambda_0 - \lambda_0\gamma(1-\beta\beta_0)$$

$$\boxed{\lambda' = +\gamma\beta\beta_0\lambda_0}$$

note, linear in velocity of q'

In q''s rest frame, there is a net charge density!

radial $\quad E_r' = \dfrac{2\lambda'}{r'} = \dfrac{2\gamma\beta\beta_0\lambda_0}{r'}$

boost back to the lab frame.

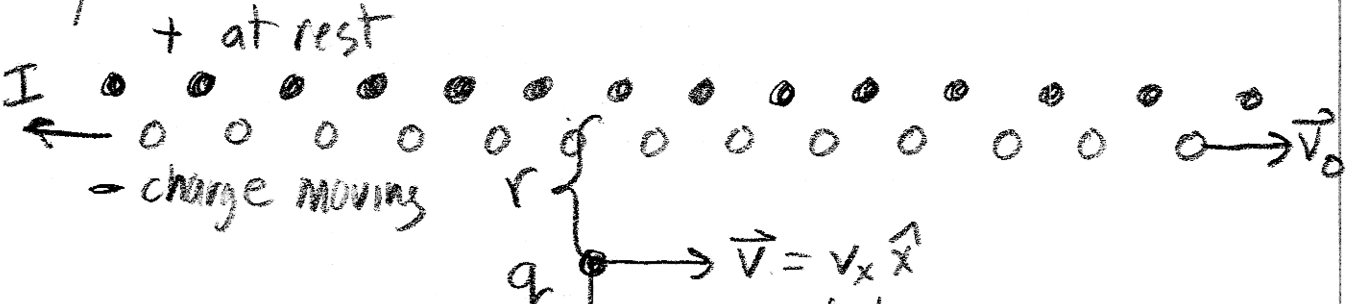
$$F_r = \frac{dp_\perp}{dt} = \frac{1}{\gamma}\frac{dp_\perp'}{dt'} = \frac{1}{\gamma} q E_r' \,, \quad \underset{(\perp \text{ to boost})}{r = r'}$$

$$F_r = \frac{2\beta\beta_0\lambda_0 q}{r}$$

$$F_y = -F_r = -\frac{2\beta\beta_0\lambda_0 q}{r}$$

$I = -\lambda_0 \cdot v_0 = -\lambda_0\beta_0 c \qquad \beta = v_x\, c$

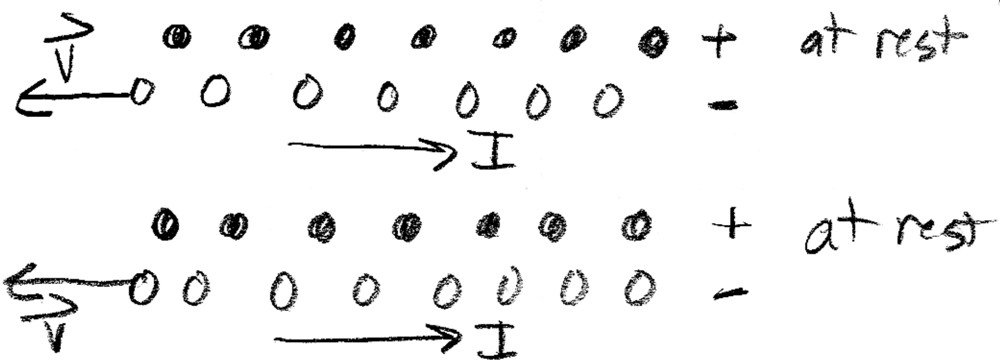
$$\boxed{F_y = \frac{2I}{rc^2} q v_x}$$



$$F_y = \frac{-2|I|}{rc^2} q v_x = \frac{2I}{rc^2} q v_x$$

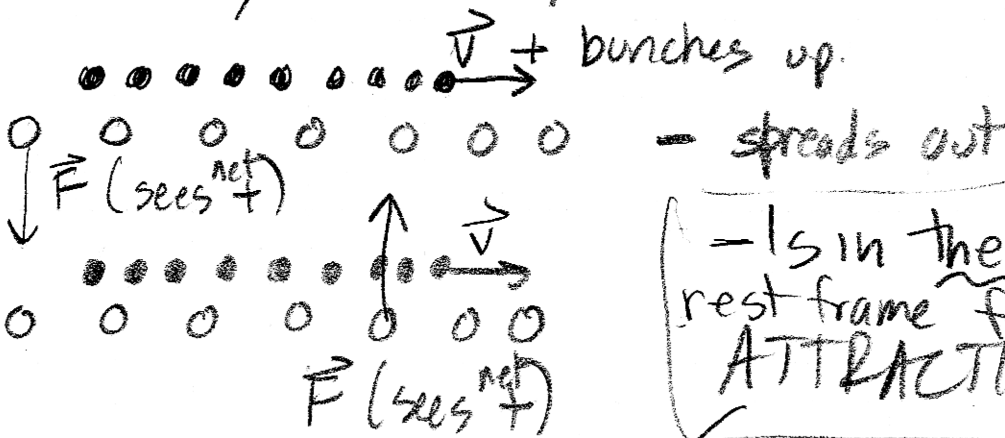
↑ I negative

Pair of wires: reason through attractions.



note: now in + rest frame, in that frame, no force on + charge!

"boost" or "jump" to − charge rest frame, to analyze that



−'s in their rest frame feel ATTRACTION.

net attraction, when we boost back to original frame

What about charge heading toward wire?