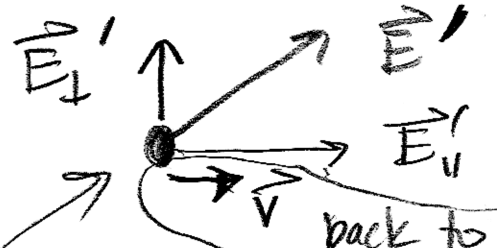
Frame



back to lab, unprimed frame.

q at rest, $t' = 0 = t$

$x' = 0$ initially, $\Delta x' \propto \Delta t'^2$

$\Delta E' \propto \Delta t'^2$

$$p_\parallel = \gamma\Big(p_\parallel' - \underbrace{\beta \tfrac{E'}{c}}\Big)$$

$$dp_\parallel = \gamma\, dp_\parallel' \qquad \text{neglig. contribution}$$

$$dp_\perp = dp_\perp'$$

$$t = \gamma\big(t' - \underbrace{\beta c x'}\big)$$

$$dt = \gamma\, dt'$$

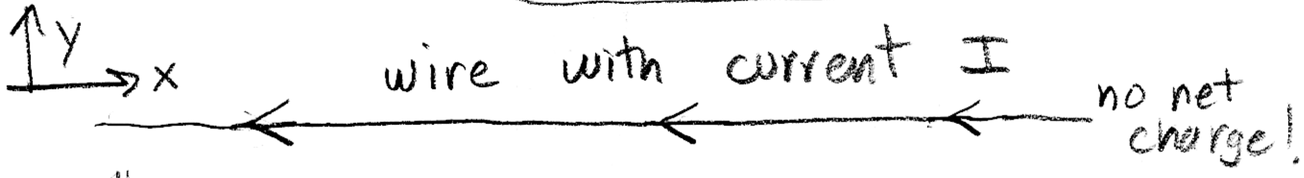
$$\frac{dp_\parallel}{dt} = \frac{\gamma\, dp_\parallel'}{\gamma\, dt'} = \frac{dp_\parallel'}{dt'} = qE_\parallel' = qE_\parallel$$

$$\frac{dp_\perp}{dt} = \frac{dp_\perp'}{\gamma\, dt'} = \frac{1}{\gamma}\frac{dp_\perp'}{dt'} = \frac{1}{\gamma}\cdot qE_\perp' = \frac{1}{\gamma} q \gamma E_\perp$$

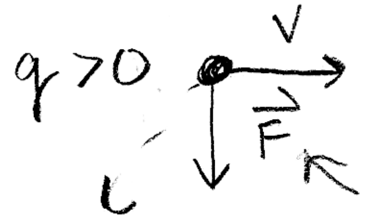
$= qE_\perp$ meaning $\frac{d\vec{p}}{dt} = q\vec{E}$, even if charge moving!

Origin of Magnetic Force

wire with current I



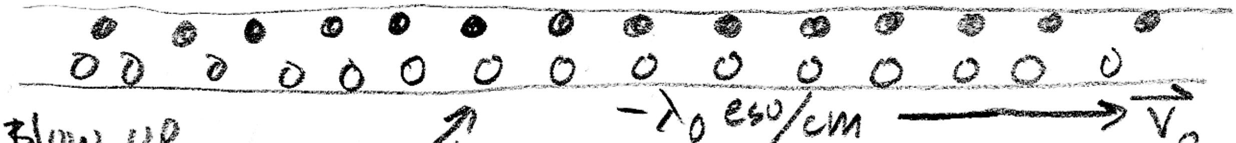
"Lab" frame.



(like a current ∥ to I).

feels repulsive force. (opposite currents repel)

Lab Frame: ions, + charge, immobile, λ_0 esu/cm



Blow up wire

$\beta_0 = \frac{V_0}{c}$

electrons moving, left to right, velocity $\vec{V}_0$

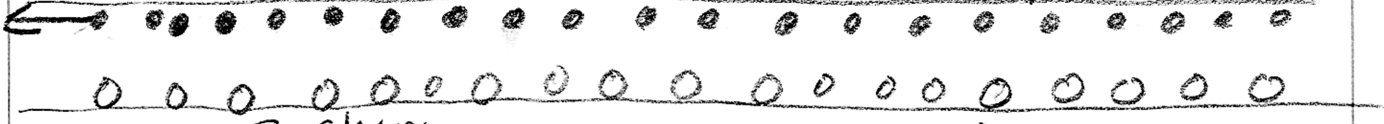
negative charge

current from right to left.

NO $\vec{E}$ field!! $\lambda_0 - \lambda_0 = 0$.

Boost to charge particle rest frame.

+ charge density $\lambda_0 \cdot \gamma > \lambda_0$, due to length contraction



− charge.

$\beta = \frac{V}{c}$

• q at rest

V_0' → what is γ_0'?

$$\beta_0' = \frac{\beta_0 - \beta}{1 - \beta\beta_0}$$

$$1 - \beta_0'^2 = 1 - \frac{(\beta_0 - \beta)^2}{(1 - \beta\beta_0)^2} = \frac{(1 - \beta\beta_0)^2 - (\beta_0 - \beta)^2}{(1 - \beta\beta_0)^2}$$

$$\frac{1}{\gamma_0'^2} = 1-\beta_0'^2 = \frac{1-2\beta\beta_0+\beta^2\beta_0^2-\beta_0^2+2\beta\beta_0-\beta^2}{(1-\beta\beta_0)^2}$$

$$\frac{1}{\gamma_0'^2} = \frac{1-\beta_0^2-\beta^2+\beta^2\beta_0^2}{(1-\beta\beta_0)^2} = \frac{(1-\beta^2)(1-\beta_0^2)}{(1-\beta\beta_0)^2}$$

$$\boxed{\gamma_0' = \frac{1-\beta\beta_0}{\sqrt{1-\beta^2}\sqrt{1-\beta_0^2}} = \gamma\gamma_0(1-\beta\beta_0)}$$

want the charge density of the negative charge in q's rest frame.

Route: (2 steps)

$$\underbrace{-\lambda_0 \frac{\text{esu}}{\text{cm}}}_{\substack{\text{wire rest} \\ \text{frame}}} \Rightarrow \underbrace{\frac{-\lambda_0}{\gamma_0}\frac{\text{esu}}{\text{cm}}}_{\substack{-\text{charge} \\ \text{rest frame}}} \Rightarrow \underbrace{-\frac{\lambda_0\gamma_0'}{\gamma_0}\frac{\text{esu}}{\text{cm}}}_{\substack{\text{q's rest} \\ \text{frame!}}}$$

already length contracted

$$\left.\begin{array}{l}-\text{charge}\\ \text{density}\end{array}\right\}\ \begin{array}{l}\text{q rest}\\ \text{frame}\end{array} = -\frac{\lambda_0}{\gamma_0}\cdot\gamma\gamma_0(1-\beta\beta_0)$$

$$= -\lambda_0\gamma(1-\beta\beta_0)$$

Net charge density:

$$\underbrace{+\ +\ -}_{\text{for } q \text{ at rest frame}} \quad = \gamma\lambda_0 - \lambda_0\gamma(1-\beta\beta_0)$$

$$\boxed{\lambda' = +\gamma\beta\beta_0\lambda_0}$$

note, linear in velocity of q!

In q's rest frame, there is a net charge density!

radial $E_r' = \frac{2\lambda'}{r'} = \frac{2\gamma\beta\beta_0\lambda_0}{r'}$

boost back to the lab frame.

$$F_r = \frac{dp_\perp}{dt} = \frac{1}{\gamma}\frac{dp_\perp'}{dt'} = \frac{1}{\gamma} q E_r' , \quad r = r' \ (\perp \text{ to boost})$$

$$F_r = \frac{2\beta\beta_0\lambda_0 q}{r}$$

$$F_y = -F_r = -\frac{2\beta\beta_0\lambda_0 q}{r}$$

$I = -\lambda_0 \cdot v_0 = -\lambda_0\beta_0 c \qquad \beta = v_x c$

$$\boxed{F_y = \frac{2I}{rc^2} q v_x}$$