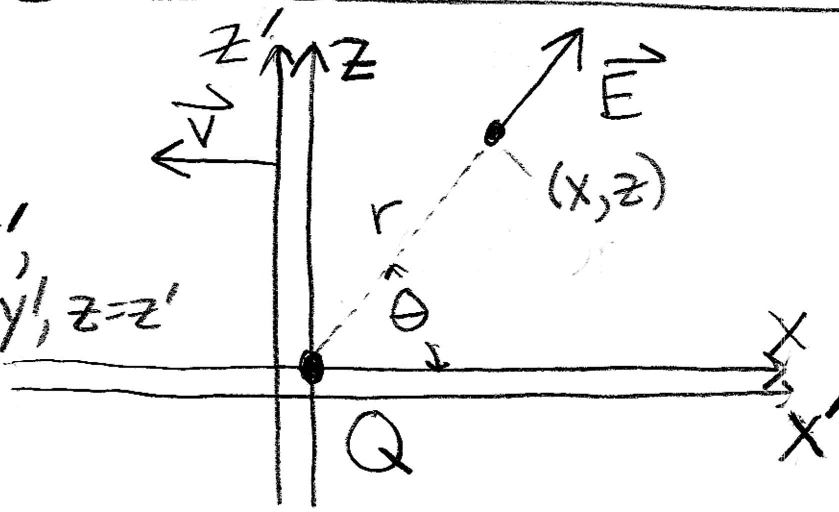
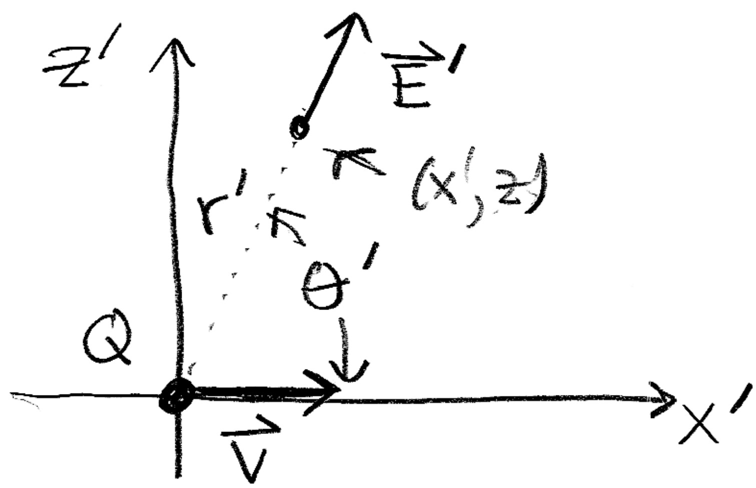
Field of a Point Charge That is Moving

at $t=t'$,
$x=x'$, $y=y'$, $z=z'$





unprimed frame

$$E_x = \frac{Q}{r^2} \times \cos\theta = \frac{Q}{r^2} \cdot \frac{x}{r} = \frac{Qx}{r^3}$$

$$= \frac{Qx}{(x^2+z^2)^{3/2}}$$

$$E_z = \frac{Q}{r^2} \cdot \sin\theta = \frac{Q}{r^2} \cdot \frac{z}{r} = \frac{Qz}{r^3}$$

$$= \frac{Qz}{(x^2+z^2)^{3/2}}$$

$$x = \gamma(x' - \beta c t') = \gamma x' \quad \text{at } t = t' = 0$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' - \beta \frac{x'}{c}\right) = -\gamma\beta\frac{x'}{c} \quad \text{at } t = t' = 0$$

$$E'_x = E_x = \frac{Qx}{(x^2+z^2)^{3/2}} = \frac{\gamma Q x'}{((\gamma x')^2 + z'^2)^{3/2}}$$

$\parallel$ to direction of motion

$$E'_z = \gamma E_z = \frac{\gamma Q z}{(x^2+z^2)^{3/2}} = \frac{\gamma Q z'}{((\gamma x')^2 + z'^2)^{3/2}}$$

$\perp$ to direction of motion

<u>note</u>: $\left.\dfrac{E'_x}{E'_z} = \dfrac{x'}{z'}\right\}$ $\vec{E}'$ still points along radii!

How strong is it?

$$E'^2_x + E'^2_z = \frac{\gamma^2 Q^2 (x'^2 + z'^2)}{(\gamma^2 x'^2 + z'^2)^3}$$

$$= \frac{\gamma^2}{\gamma^6} \frac{Q^2 (x'^2 + z'^2)}{\left(x'^2 + \frac{1}{\gamma^2} z'^2\right)^3} \qquad \frac{1}{\gamma^2} = 1 - \beta^2$$

$$= \frac{1}{\gamma^4} \frac{(x'^2 + z'^2)}{(x'^2+z'^2)^3} \cdot \frac{Q^2}{\left(1 - \beta^2 \frac{z'^2}{x'^2+z'^2}\right)^3}$$

$$E'^2 = \frac{1}{r'^4} \cdot \frac{Q^2(1-\beta^2)^2}{(1-\beta^2\sin^2\theta')^3}$$

$$\boxed{E' = \frac{Q}{r'^2} \cdot \frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}}$$

1) $\theta' = 0 \rightarrow E' = \frac{Q}{r'^2}$

2) $\theta' = \frac{\pi}{2} \rightarrow E' = \frac{Q}{r'^2} \cdot \frac{1-\beta^2}{(1-\beta^2)^{3/2}} = \frac{Q}{r'^2} \cdot \frac{1}{\sqrt{1-\beta^2}}$

$= \gamma \cdot \frac{Q}{r'^2}$

Looks like: