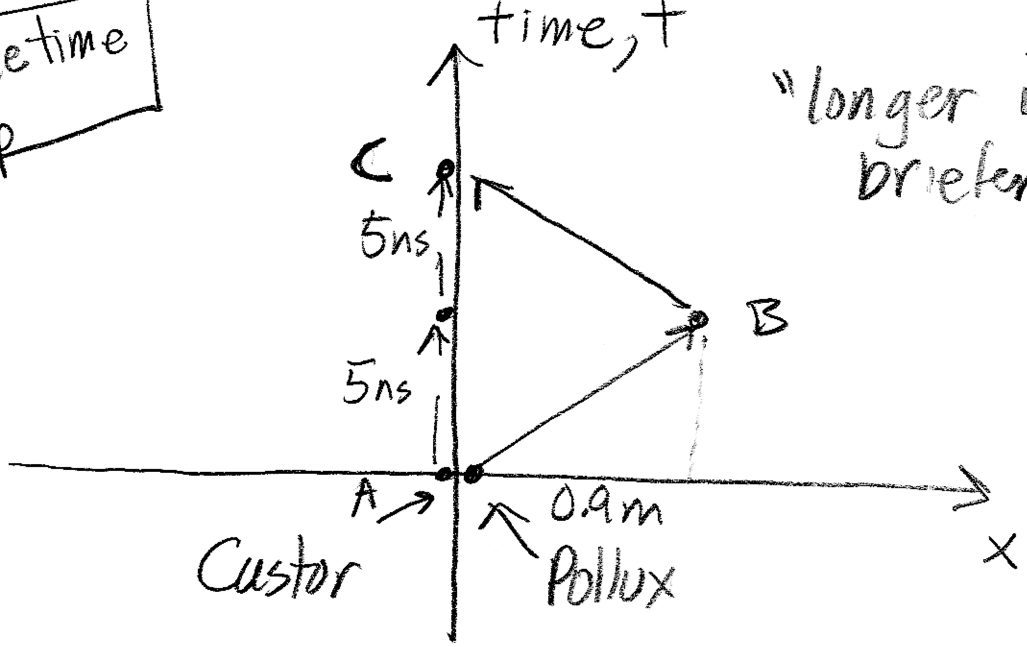
Spacetime Map



"longer is briefer"

Castor stays at $x=0$, goes from $A \to C$, that is, stays put, for 10ns

Pollux goes $A \to B$, $B \to C$; lots of acceleration at A/B/C.

How much time goes by on a clock that rides with Pollux?

⇒ look at intervals:

$A \to B$ $\quad \Delta t^2 - \frac{\Delta x^2}{c^2} = 5^2 - \left(\frac{0.9}{0.3}\right)^2 \text{ ns}^2$

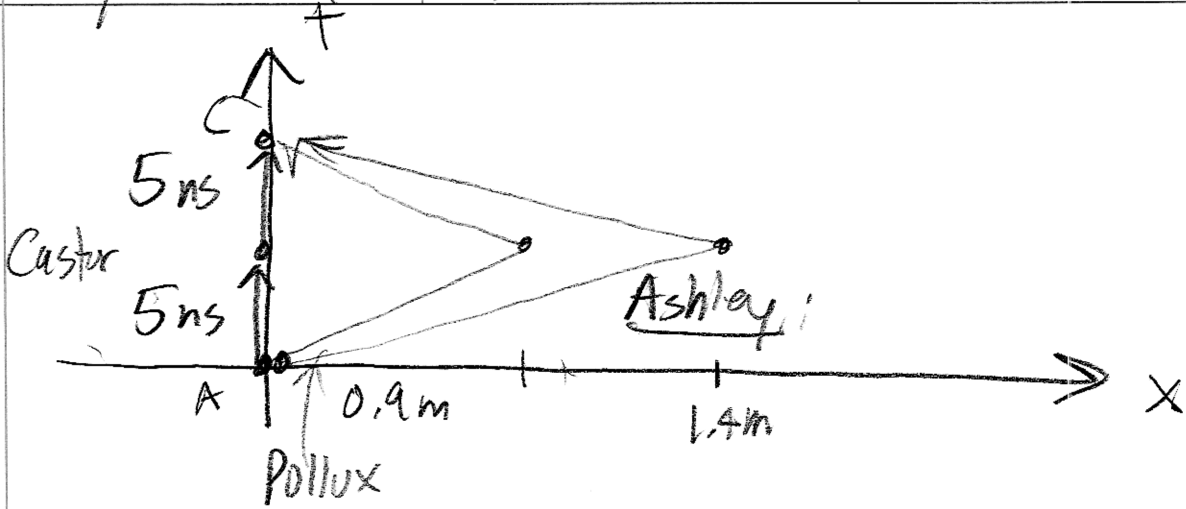
$(\text{Interval})^2 = 4^2 \text{ ns}$

$\tau = (\text{Interval}) = 4 \text{ ns}$

$B \to C$ also 4ns

Pollux: $A \to B \to C$, 8ns

Castor: $A \to C$, 10 ns



$0.3 \frac{m}{ns} \cdot 5\ ns = 1.5\ m$ = maximum distance. (can't exceed speed of light).

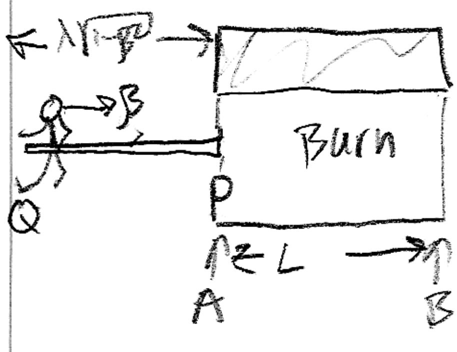
Ashley: $\Delta t^2 - \frac{\Delta x^2}{c^2} = 5^2 - \left(\frac{1.4}{0.3}\right)^2$

$$\Delta t^2 = 3.22\ ns^2$$

$$\boxed{\tau = \Delta t = 1.8\ ns}$$

Just can't get to (5ns, 2m), why? "spacelike"

The pole vaulter: Has a λ=20 meter pole, runs by at β such $\lambda\sqrt{1-\beta^2}$ L = 10 m. So we put them in a barn of length L=10 m!

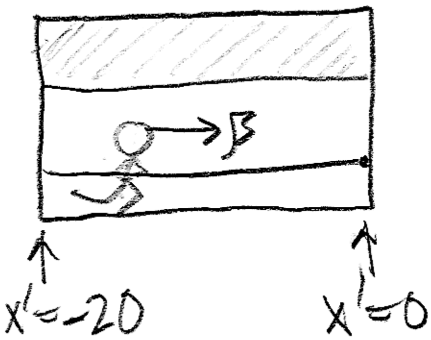


Event #1:
at t=0, point P of pole (tip) enters Barn at door A, which opens then.

$(\overset{ns}{t}, \overset{m}{x})$	Barn Rest Frame	Pole Rest Frame
Event #1: Pole Front Tip Enters Barn (door A opens)	(0,0) I	(0,0) I
Event #2: Pole Rear Tip Enters Barn (door A closes)	(38.5,0) II + III	(77.0, -20) III
Event #3: Pole Front Tip Exits Barn (door B opens)	(38.5, 10) II + III	(19.3, 0) II
Event #4: Pole Rear Tip Exits Barn (door B closes)	(77.0, 10) IV	(96.3, -20) IV

↑ order of events

Events #2 + #3



In barn frame, easy wait: $t = \frac{L}{\beta c}$ what is β?

$$20 \cdot \sqrt{1-\beta^2} = 10$$

$$1-\beta^2 = \left(\tfrac{1}{2}\right)^2 \Rightarrow \beta^2 = 1 - \tfrac{1}{4}$$

$$\beta = \frac{\sqrt{3}}{2}$$

$$t = \frac{L}{\beta c} = \frac{10}{\frac{\sqrt{3}}{2} \times 0.3} = 38.5 \text{ ns}$$

Event #2

In pole rest frame, $x' = -20$ m

$$t' = \frac{20 \text{ m}}{\beta c} = \frac{20}{\frac{\sqrt{3}}{2} \times 0.3} = 77.0 \text{ ns}$$

(βc: barn goes by at same speed)

Event #3 How long does it take foreshortened barn to go by? In pole rest frame.

$$t' = \frac{10 \cdot \frac{1}{2}}{\frac{\sqrt{3}}{2} \cdot 0.3} = 19.25\,\text{ns}$$

Event #4 Add the 19.25 ns it takes barn (Lorentz-contracted) to go by

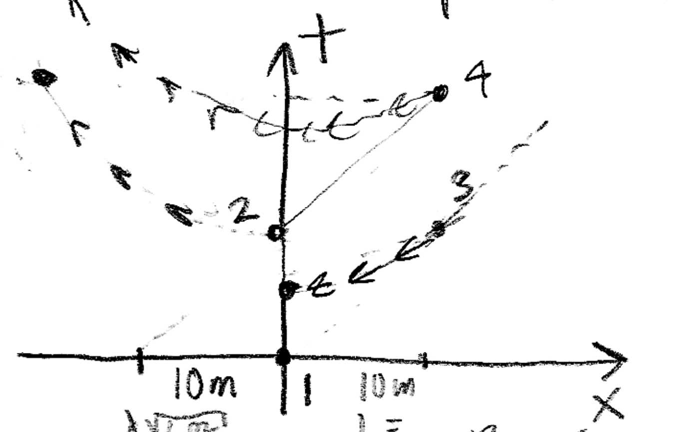
$$t' = 77 + 19.25 = 96.25\,\text{ns}$$

Note:

	$t^2 - \frac{x^2}{c^2}$		$t'^2 - \frac{x'^2}{c^2}$
Event #1	$0^2 - 0^2$	$=$	$0^2 - 0^2$
#2	$38.5^2 - 0^2$	$=$	$(77 = 2 \cdot 38.5)^2 - \left(\frac{-20}{0.3}\right)^2$
	1485	$=$	1485
#3	$38.5^2 - \left(\frac{10}{0.3}\right)^2$	$=$	$19.3^2 - 0^2$
	370	$=$	370
#4	$77^2 - \left(\frac{10}{0.3}\right)^2$	$=$	$96.3^2 - \left(\frac{20}{0.3}\right)^2$
	4814	$=$	4814

Never used a Lorentz Transformation!

Look at Spacetime!



Barn Frame