Now turn the "clock" to be parallel to the direction of motion.

#2 looking at their own clock.

$\Delta t_2 = \frac{2L}{c}$ (independent of

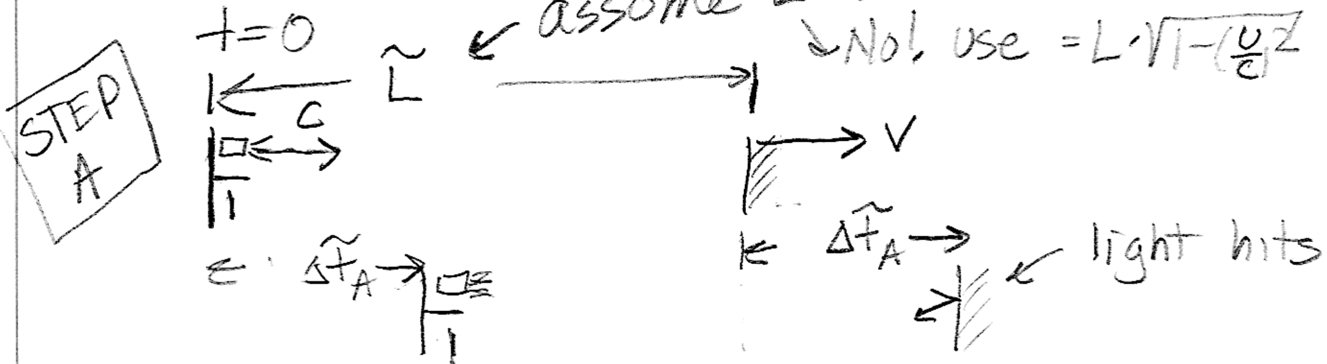
#1 looking at #2's clock... answer should not depend on orientation!

$$\Delta\tilde{t} = \Delta t_2 \times \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = \frac{2L}{c}\frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$

Try working this out from the ground up... Time interval, "lab frame", (?) moving emitter to mirror, $\Delta\tilde{t}_A$

$\leftarrow$ assume L same!

No! use $= L\cdot\sqrt{1-\left(\frac{u}{c}\right)^2} = \tilde{L}$



emission to hitting mirror

$$c\cdot\Delta\tilde{t}_A = \tilde{L} + u\Delta\tilde{t}_A$$

$$(c-u)\Delta\tilde{t}_A = \tilde{L}$$

$$\Delta\tilde{t}_A = \frac{\tilde{L}}{c-u}$$

Call the time from getting to the mirror back to the detector $\Delta\tilde{t}_B$

$$u\Delta\tilde{t}_B + c\Delta\tilde{t}_B = \tilde{L}$$

$$\Delta\tilde{t}_B = \frac{\tilde{L}}{c+u} \quad \left[\sqrt{1-\left(\tfrac{u}{c}\right)^2}\right]$$

Apparently, $\Delta\tilde{t} = \Delta\tilde{t}_A + \Delta\tilde{t}_B = \frac{\tilde{L}}{c-u} + \frac{\tilde{L}}{c+u}$

$$= \frac{\tilde{L}(c+u) + \tilde{L}(c-u)}{(c-u)(c-u)} = \frac{2\tilde{L}c}{c^2-u^2}$$

$$\Delta\tilde{t} = \frac{2\tilde{L}c}{c^2} \cdot \frac{1}{1-\frac{u^2}{c^2}} = \frac{2\tilde{L}}{c}\frac{1}{1-\left(\frac{u}{c}\right)^2}$$

$$\frac{2\tilde{L}}{c}\frac{1}{1-\left(\frac{u}{c}\right)^2} = \frac{2L}{c}\frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$ ← from time dilitation

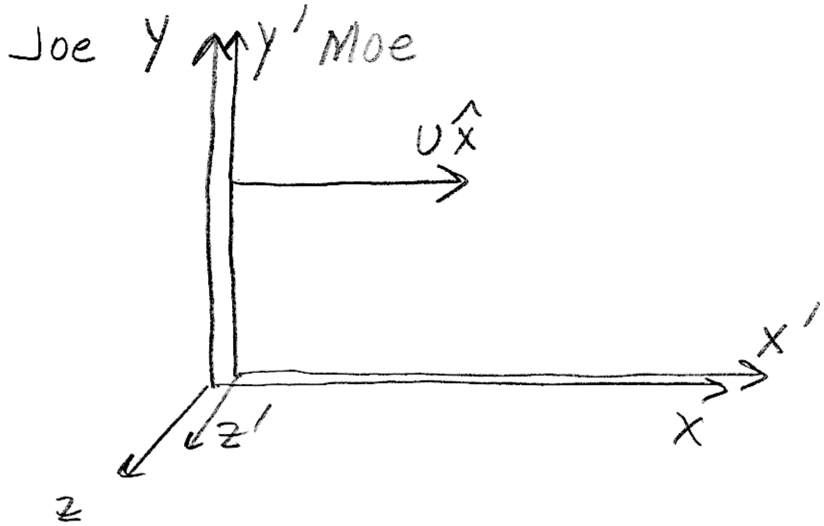
How is this resolved???

$\boxed{\Rightarrow \text{LENGTH CONTRACTION!}}$

The length of an object $\parallel$ to motion that is moving appears <u>shortened</u> by a factor of $\sqrt{1-\left(\frac{u}{c}\right)^2}$

$$\tilde{L} = L \cdot \sqrt{1-\left(\frac{u}{c}\right)^2}$$

Let's make this more systematic...
Imagine 2 coordinate systems,
"Joe" and "Moe"



at $t = t' = 0$ coordinate systems align.

at later times, $y = y'$, $z = z'$

origin of Moe's at $x = ut$ in Joe's.

What about more generally?

Suppose we know the 4 coordinates of an event in Joe's frame:

$$x, y, z, t \leftarrow \#4!$$

How do we calculate x', y', z', t'??

① Suppose $y' = y$, $z' = z$ (simplicity)
why? $\perp$ to direction of motion.

(2) Accomodate length contraction...
Something x' from Moe's origin appears to be only $x' \cdot \sqrt{1-(\frac{v}{c})^2}$ from Moe's origin to Joe...

$$x = vt + x' \cdot \sqrt{1-\left(\frac{v}{c}\right)^2}$$

for Joe

location of Moe's origin

But.. why is Joe looking at Moe so special... must also be true that...

$$x' = -vt' + x \cdot \sqrt{1-\left(\frac{v}{c}\right)^2}$$

for Mo

This should disturb you, length contraction goes both ways!! Solving.

$$x' = \frac{x - vt}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

but these can be cross substituted to arrive at similar equations for

$$x' = \frac{\frac{x'+vt'}{\sqrt{1-(\frac{v}{c})^2}} - vt}{\sqrt{1-(\frac{v}{c})^2}} \qquad x = \frac{\frac{x-vt}{\sqrt{1-(\frac{v}{c})^2}} + vt'}{\sqrt{1-(\frac{v}{c})^2}}$$

$$x' = \frac{x'+vt'}{1-(\frac{v}{c})^2} - \frac{vt}{\sqrt{1-(\frac{v}{c})^2}} \qquad x = \frac{x-vt}{1-(\frac{v}{c})^2} + \frac{vt'}{\sqrt{1-(\frac{v}{c})^2}}$$

$$x' \left(\frac{1-(\frac{v}{c})^2}{1-(\frac{v}{c})^2} - \frac{1}{1-(\frac{v}{c})^2} \right) - \frac{vt'}{1-(\frac{v}{c})^2} = -\frac{vt}{\sqrt{1-(\frac{v}{c})^2}}$$

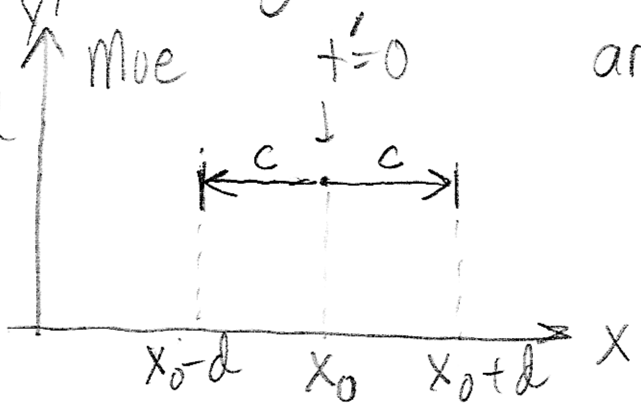
$$t = \frac{t' + (\frac{v}{c})(\frac{x'}{c})}{\sqrt{1-(\frac{v}{c})^2}} \Rightarrow t' = \frac{t - (\frac{v}{c})(\frac{x}{c})}{\sqrt{1-(\frac{v}{c})^2}}$$

SURPRISE: not just time dilitation (factor of $1/\sqrt{1-(\frac{v}{c})^2}$), but time in another frame depends on space too! ($\frac{x'}{c}$ factor)

Physical Origin of This:

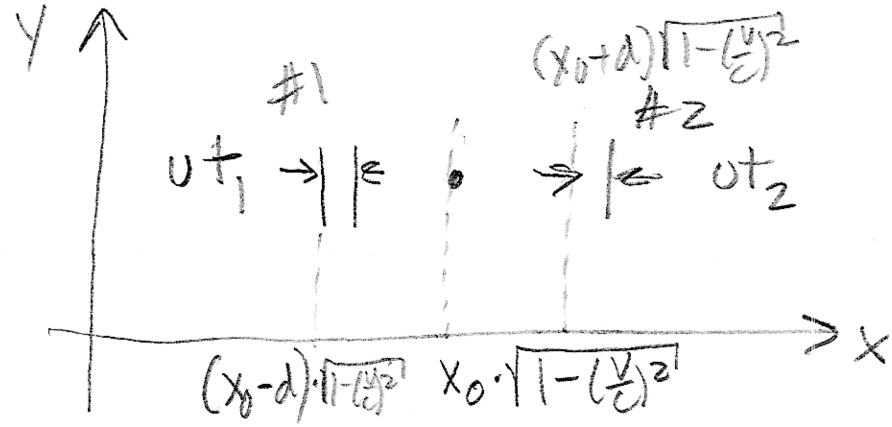
Imagine two light beams in Moe's frame:

both emitted from x_0, one toward $+x$, one toward $-x$, so



arrive "simultaneously" at $t' = \frac{d}{c}$

In Joe's Frame ...



appears that #1 arrives first...

$$vt_1 + ct_1 = d\cdot\sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$t_1 = \frac{d\sqrt{1-\left(\frac{v}{c}\right)^2}}{c+v}$$

from picture.

$$t_1 = \frac{d}{c}\,\frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1+\frac{v}{c}} = \frac{d}{c}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

from equations

$$t_1 = t_{reception} - t_{emission}$$

$$= \frac{\frac{d}{c} + \frac{v}{c}\left(\frac{x_0-d}{c}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - \frac{0 + \frac{v}{c}\left(\frac{x_0}{c}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$= \frac{d}{c}\,\frac{\left(1-\frac{v}{c}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{d}{c}\,\frac{\left(1-\frac{v}{c}\right)}{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}} = \frac{d}{c}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

$$\boxed{t_1 = \frac{d}{c}\cdot\frac{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}{1+\frac{v}{c}} = \frac{d}{c}\,\frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1+\frac{v}{c}} \quad \text{same}}$$

$$ct_2 - ut_2 = d\cdot\sqrt{1-\left(\tfrac{u}{c}\right)^2}$$

$$t_2 = \frac{d\sqrt{1-\left(\frac{u}{c}\right)^2}}{c-u} = \frac{d}{c}\cdot\frac{\sqrt{1-\left(\frac{u}{c}\right)^2}}{1-\frac{u}{c}}$$

$$= \frac{d}{c}\,\frac{\sqrt{\left(1+\frac{u}{c}\right)\left(1-\frac{u}{c}\right)}}{1-\frac{u}{c}}$$

$$t_2 = \frac{d}{c}\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}$$

$$t_2 = t_{\text{reception}} - t_{\text{emission}}$$

$$= \frac{\frac{d}{c}+\frac{u}{c}\left(\frac{x_0+d}{c}\right)}{\sqrt{1-\left(\frac{u}{c}\right)^2}} - \frac{0+\frac{u}{c}\left(\frac{x_0}{c}\right)}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$

$$= \frac{d}{c}\,\frac{\left(1+\frac{u}{c}\right)}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = \frac{d}{c}\,\frac{\left(1+\frac{u}{c}\right)}{\sqrt{\left(1+\frac{u}{c}\right)\left(1-\frac{u}{c}\right)}}$$

$$\boxed{t_2 = \frac{d}{c}\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}}$$

$t_1 \neq t_2$, events not simultaneous,

<u>and</u> $\quad t_1 \simeq \frac{d}{c}\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} \simeq \frac{d}{c}\left(1-\frac{2u}{c}\right)^{1/2} \simeq \frac{d}{c}\left(1-\frac{u}{c}\right)$

$$t_2 \simeq \frac{d}{c}\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}} \simeq \frac{d}{c}\left(1+\frac{2u}{c}\right)^{1/2} \simeq \frac{d}{c}\left(1+\frac{u}{c}\right); \quad t_2 - t_1 \simeq 2\frac{du}{c^2}$$

Finally, the Lorentz Transformation

x', y', z' moving $v\hat{x}$ w/r to x, y, z

$$x' = \frac{x - vt}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - \left(\frac{v}{c}\right)\left(\frac{x}{c}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad t = \frac{t' + \left(\frac{v}{c}\right)\left(\frac{x'}{c}\right)}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$