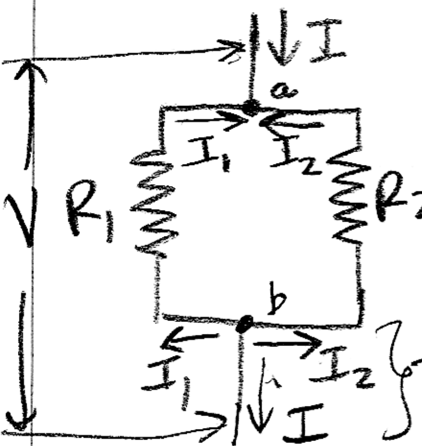
Physics 24 | UCSB | N



at a: $I_1 + I_2 + I = 0$

$I_1 + I_2 = -I$ (#1,

at b: flip

$-I_1 - I_2 - I = 0$

all $-$signs, since leaving.

#2: Start at (a), work around.

$$\underbrace{-I_1 R_1}_{\text{drop from a to b}} \underbrace{+ I_2 R_2}_{\text{drop from b to a}} = 0$$

$$I_2 = \left(\frac{R_1}{R_2}\right) I_1 \qquad \left(\begin{matrix} R_2 \gg R_1 \\ I_2 \ll I_1 \end{matrix}\right)$$

$$I_1 + \left(\frac{R_1}{R_2}\right) I_1 = -I$$

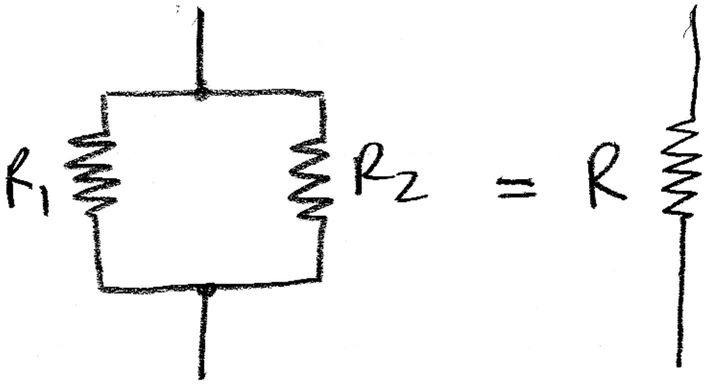
$$I_1 = \frac{-I}{1 + \frac{R_1}{R_2}} = -\frac{R_2}{R_1 + R_2} I$$

$$I_2 = \left(\frac{R_1}{R_2}\right) I_1 = -\frac{R_1}{R_1 + R_2} I$$

#3 $-I_1 R_1 - V = 0$

$$V = -I_1 R_1 = \frac{R_1 R_2}{R_1 + R_2} I$$

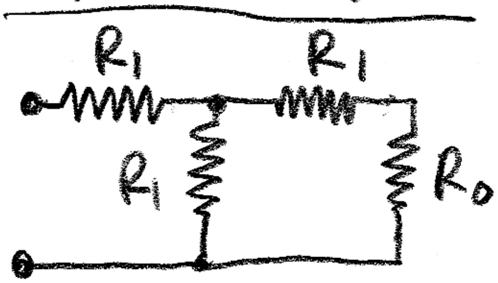
$$\boxed{V = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} I = R I}$$



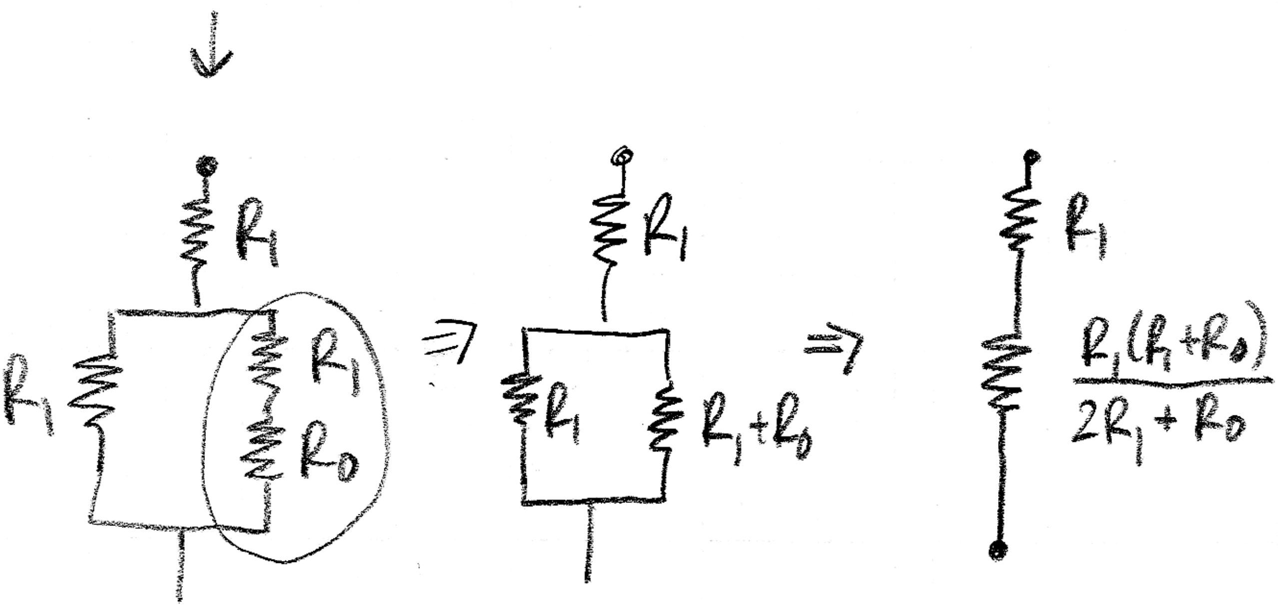
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Learn to "condense" resistors to make overall equivalent resistor

Problem 4.16 :



Pick R_1 so that equivalent resistance equals R_0



want $\quad R_0 = R_1 + \dfrac{R_1(R_1+R_0)}{2R_1+R_0}$

$$2\cancel{R_1}R_0 + R_0^2 = 2R_1^2 + \cancel{R_1R_0} + R_1^2 + \cancel{R_1R_0}$$

$$R_0^2 = 3R_1^2 \qquad \boxed{R_1 = R_0/\sqrt{3}}$$

<u>Check</u>: $$\frac{R_0}{\sqrt{3}} + \frac{\frac{R_0}{\sqrt{3}}\left(\frac{R_0}{\sqrt{3}} + R_0\right)}{2\frac{R_0}{\sqrt{3}} + R_0} = R_0\left[\frac{1}{\sqrt{3}} + \frac{1+\sqrt{3}}{2\sqrt{3}+3}\right]$$

$$= R_0\left[\frac{2\sqrt{3}+3 + \sqrt{3}(1+\sqrt{3})}{\sqrt{3}(2\sqrt{3}+3)}\right]$$

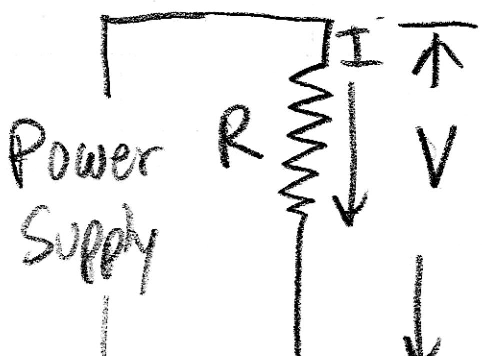
$$= R_0\left[\frac{\sqrt{3}\left[2+\sqrt{3}+1+\sqrt{3}\right]}{\sqrt{3}(2\sqrt{3}+3)}\right]$$

$$= R_0\left[\frac{\sqrt{3}\cdot\left[2\sqrt{3}+3\right]}{\sqrt{3}\left[2\sqrt{3}+3\right]}\right] = R_0$$

Energy Dissipation

When a <u>constant voltage</u> (provided, say, by a power supply) moves charge Q from higher voltage to lower voltage, the power supply does work $Q \cdot V$

$$W = Q \cdot V$$



$$\frac{dW}{dt} = \frac{dQ}{dt} \cdot V + Q\cancel{\frac{dV}{dt}}^{\,0}$$

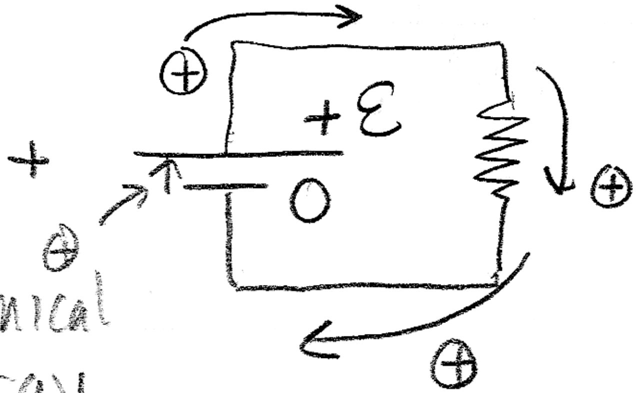
$$P = \text{Power} = \frac{dW}{dt} = IV$$

$V = IR$

note: square

$$\boxed{P = I^2 R = \frac{V^2}{R}}$$

<u>Batteries</u>: use chemical energy to push charge "back up hill"



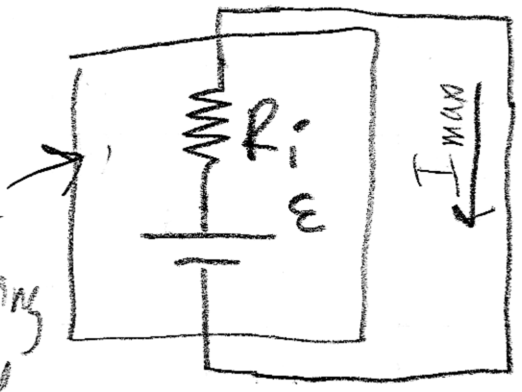
voltage = ε

positive charge "falling down hill"

Chemical energy pushes charges back up hill! (in a Van de Graaf, mechanical energy does this!)

<u>Batteries actually have resistance</u>

They cannot provide ∞ current!



Box Representing Battery

Short Circuit!

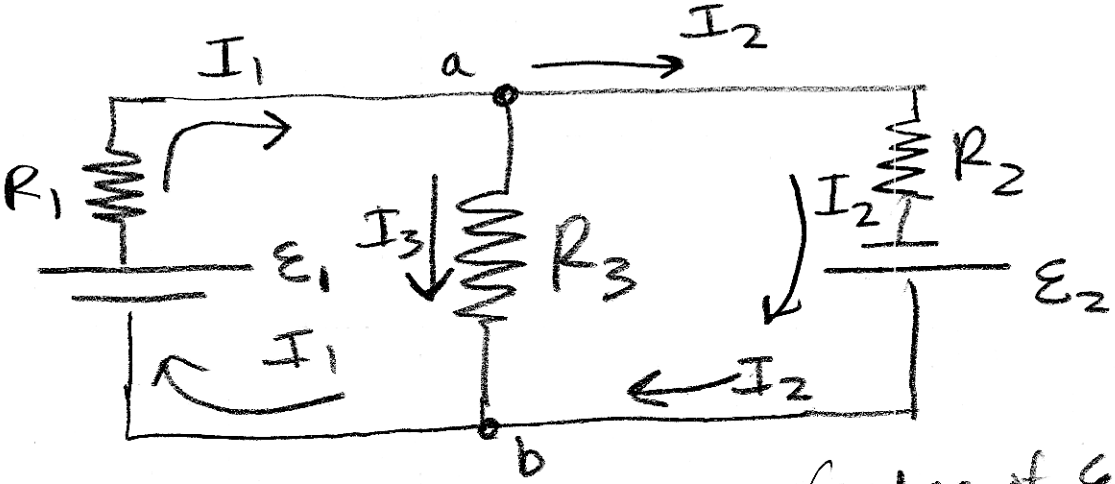
$$I_{max} = \frac{\varepsilon}{R_i}$$

internal to battery.

Not all problems take this effect into account.

Networks:

Consider two opposing batteries:



Suppose $R_1, R_2, R_3, \varepsilon_1, \varepsilon_2$ known

(order of ε_2, R_2 not important.

Find I_1, I_2, I_3 → will be 3 equations in 3 unknowns

At a: current conservation:

$$I_1 - I_2 - I_3 = 0$$

Loop with ε_1, R_3:

drops $-\varepsilon_1 + I_1 R_1 + I_3 R_3 = 0$

ups $\varepsilon_1 - I_1 R_1 - I_3 R_3 = 0$

Loop with ε_2, R_3:

ups: $\varepsilon_2 + R_3 I_3 - I_2 R_2 = 0$

Straightforward to solve with mathematica:

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In[1]:= Solve[{I1 - I2 - I3 = 0, E1 - I1 * R1 - I3 * R3 = 0, E2 + R3 * I3 - I2 * R2 = 0}, {I1, I2, I3}]
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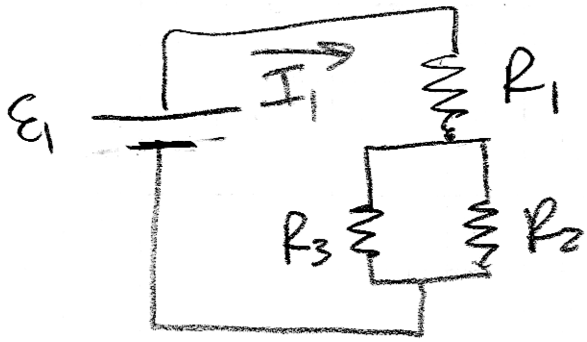
$$\text{Out[1]}= \left\{\left\{I1 \to -\frac{-E1\,R2 - E1\,R3 - E2\,R3}{R1\,R2 + R1\,R3 + R2\,R3},\ I2 \to -\frac{-E2\,R1 - E1\,R3 - E2\,R3}{R1\,R2 + R1\,R3 + R2\,R3},\ I3 \to -\frac{E2\,R1 - E1\,R2}{R1\,R2 + R1\,R3 + R2\,R3}\right\}\right\}$$

Checks: $\mathcal{E}_2 \to 0$



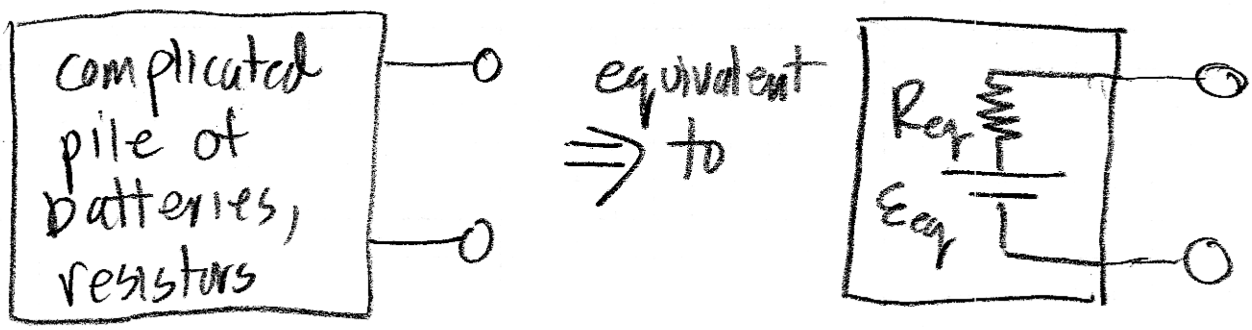
$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

$$I_1 = \frac{\mathcal{E}_1}{R_{eq}} = \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \checkmark$$

$$I_3 = \frac{R_2}{R_2 + R_3} I_1 \checkmark \qquad I_2 = \frac{R_3}{R_2 + R_3} \cdot I_1 \checkmark$$

Worthwhile to do checks.

Thévinin Equivalent



How to determine R_{eq}, $\mathcal{E}_{eq}$?