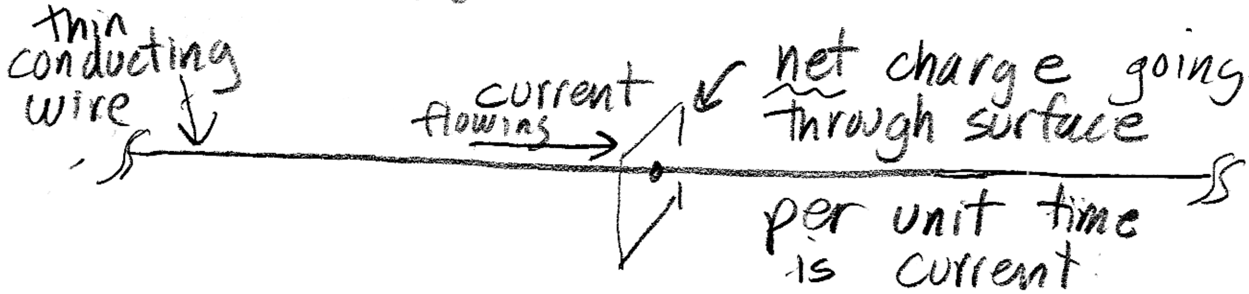
Electric Current

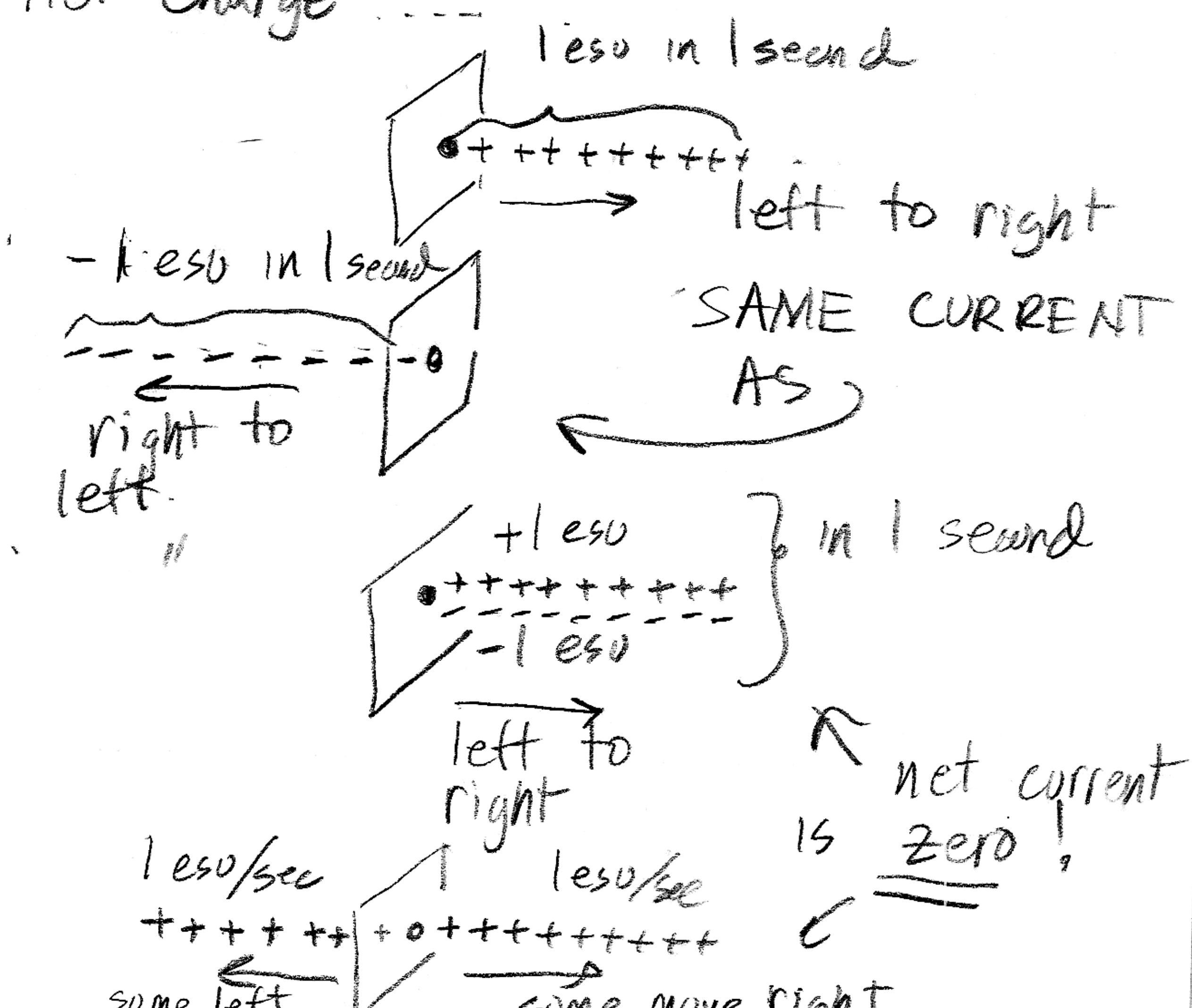
is $\frac{\text{charge}}{\text{second}}$ → flow of charge



CGS: units of current are $\frac{\text{esu}}{\text{sec}}$

MKS/SI: $\frac{\text{coulombs}}{\text{second}} \equiv$ Ampere

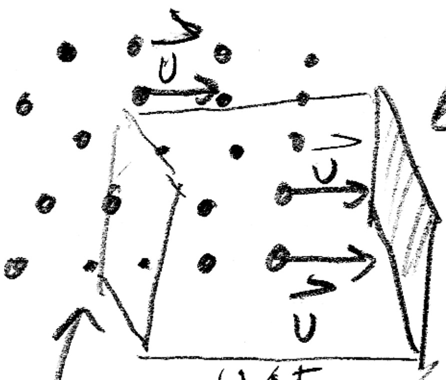
"Net Charge" ...



Conclude: what matters, somehow, is **product** of $q\vec{v}$

Whoops check units: $esu \times \frac{cm}{s}$

"Back Up" to more fundamental picture.



area a, $\perp$ to direction.

herd of charges

$n = \frac{\#}{\text{volume}}$

each has charge q

$\rho = nq$

ρ: $\frac{\text{charge}}{\text{volume}}$

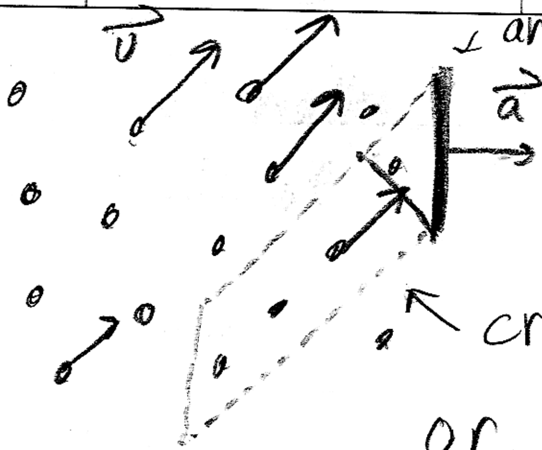
suppose each charge has identical velocity $\vec{v}$

Area $\vec{a}$, $\perp$ to direction.

$$I = \frac{(\text{Charge through area})}{\text{time}} = \frac{(nq \cdot a \cdot v\Delta t)}{\Delta t}$$

$$\boxed{I = nqva}$$

now consider angle between $\vec{a}$ + $\vec{v}$



area, edge on · $\vec{a}$ vector, ⊥ to surface

cross section smaller, or, equivalently, volume smaller...

$$\boxed{I = nq\vec{v}\cdot\vec{a}}$$

Usually, a variety of charges (q_i) with a variety of densities (n_i) and a variety of velocities, $\vec{v}_i$. (no variation across area).

then $$I = \underbrace{\left(\sum_i n_i q_i \vec{v}_i\right)}_{\text{why not name this something?}} \cdot \vec{a}$$

<u>Current Density</u> $$\vec{J} = \sum_i n_i q_i \vec{v}_i$$

$$I = \vec{J}\cdot\vec{a} \quad \Leftarrow \text{ when } \vec{J} \text{ is constant across a}$$

when $q_i = q =$ same (one species!)

$$\vec{J} = q\sum_i n_i \vec{v}_i = N_q q \left(\frac{\sum_i n_i \vec{v}_i}{N_q}\right)$$

$$N_q = \frac{\#\ q\text{'s, all velocities}}{\text{volume}}$$

average velocity $\overline{\vec{v}} \equiv \dfrac{\sum_i n_i \vec{v}_i}{N_q}$

can be zero or small

even when each $\vec{v}_i$ is large, since, variety of <u>directions</u>

$$\vec{J} = N_q q \overline{\vec{v}}$$

most common case: $q = -e$ (electrons)

$$\vec{J} = -N_e e \overline{\vec{v}_e}$$

<u>Current Conservation</u>

more generally,

$$I = \int_S \vec{J} \cdot d\vec{a}$$

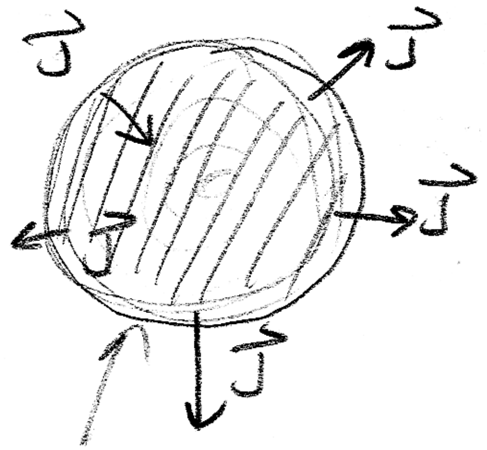
← review 1.9

For a <u>closed</u> surface S

$$\oint_S \vec{J} \cdot d\vec{a} = I = -\frac{dQ}{dt} = -\frac{d}{dt}\int_V \rho \, dV$$

surface around volume

+ current out means <u>loss</u> of charge



net flux out is:

$$\int_S \vec{J}\cdot d\vec{a}$$

partial, V fixed, time only one matters

$$\int_S \vec{J}\cdot d\vec{a} = -\int_V \frac{\partial \rho}{\partial t}\, dV$$

Q inside is $\int_V \rho\, dV$

$$\int_S \vec{J}\cdot d\vec{a} = \int_V \operatorname{div}\vec{J}\, dV = \int_V \left(-\frac{\partial \rho}{\partial t}\right) dV$$

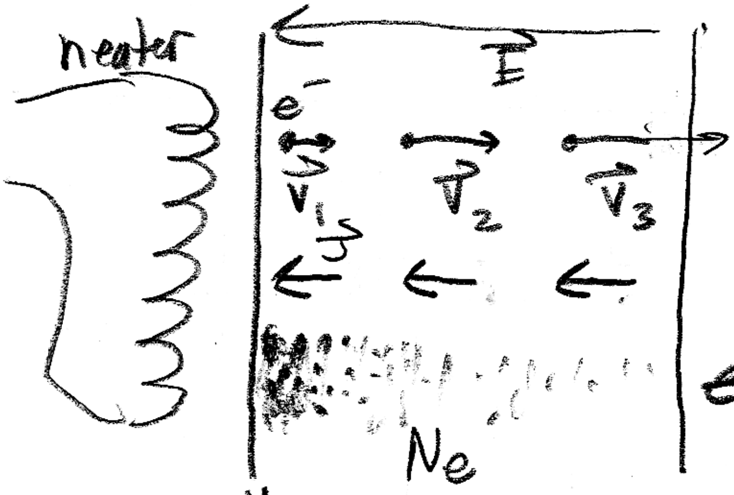
$$\operatorname{div}\vec{J} = -\frac{d\rho}{dt}$$

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = -\frac{\partial \rho}{\partial t}$$

"Steady state" when $\frac{\partial \rho}{\partial t} = 0$

"Current Conservation"

Vacuum Diode in steady state!



(acceleration)

$\frac{\partial J_x}{\partial x} = 0$ (steady)

← density of electrons varies!

− voltage electrode

+ voltage (anode) electrode