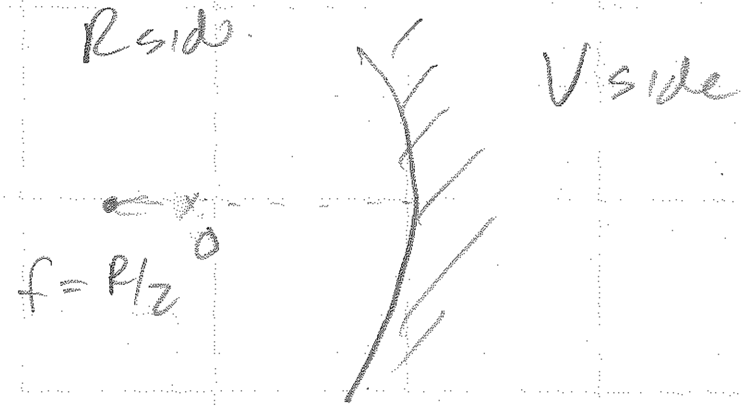
$f > 0$

when $i, o > 0$ $\quad \frac{1}{o} + \frac{1}{i} = \frac{1}{f}$ $\quad o, i$ always ___ than f

o: always > 0 for physical object, but $o < 0$ ...

when is $i < 0$

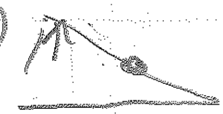
$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} < 0$ when $\boxed{o < f}$



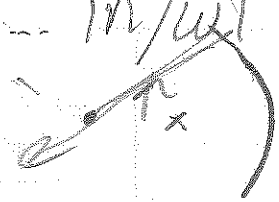
Visual Technique

4 rays: 1) Parallel In

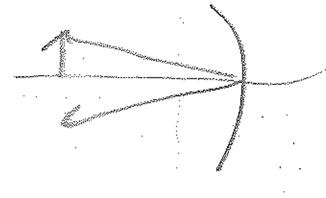
2) Through focus in, Parallel Out



3) Center of curvature in/out



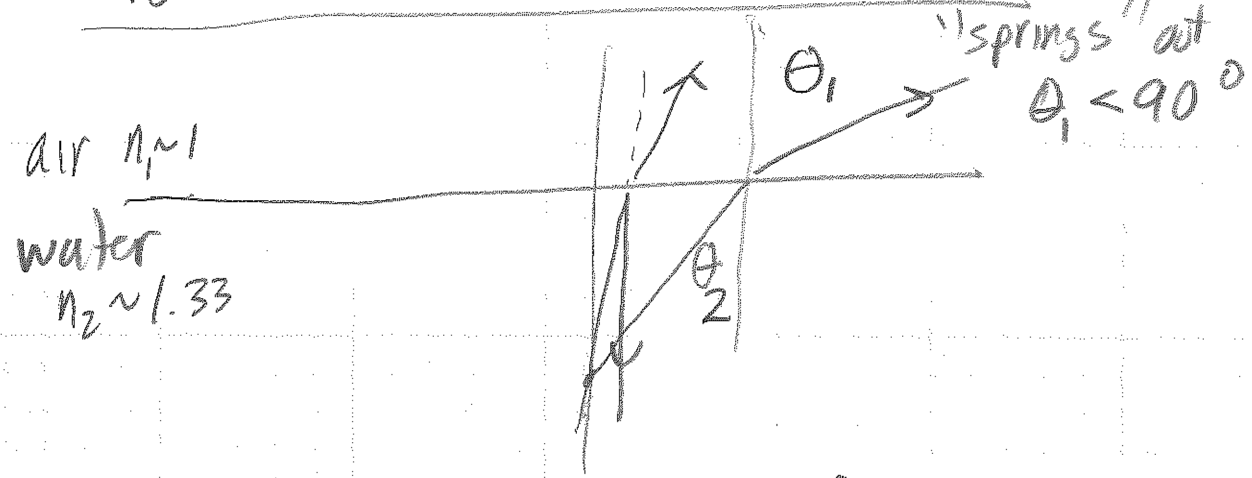
4) Mirror vertex



all should agree

Demo: multiple images (# bounces)

Total Internal Reflection



$$n_2 \sin\theta_2 = n_1 \sin\theta_1 \leftarrow \text{max}, \ \theta_1 = 90^\circ$$

$$n_2 \sin\theta_{2c} = n_1$$

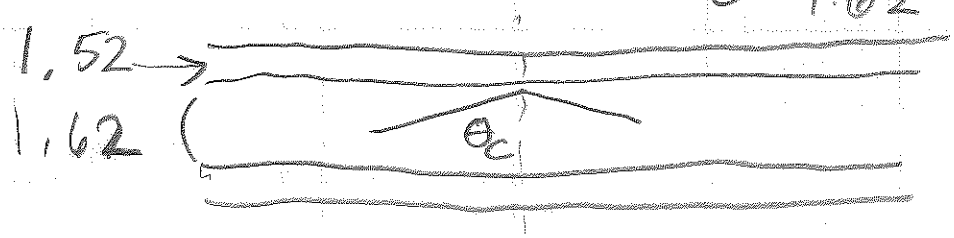
$$\sin\theta_{2c} = \frac{n_1}{n_2} < 1$$

$$\theta_{2c} = \sin^{-1}\left(\frac{n_1}{n_2}\right) = \sin^{-1}\left(\frac{1}{1.33}\right)$$

$$= 48.8^\circ \text{ (from Vertical)!}$$

100% reflection! Better than mirror! Basis of fiber optics

$$\sin\theta_c = \frac{1.52}{1.62} \qquad \theta_c = 69.8^\circ$$



$$T_{BA} = \frac{\sqrt{h_1^2 + x^2}}{(c/n_1)} + \frac{\sqrt{h_2^2 + (d-x)^2}}{(c/n_2)}$$

$$\frac{dT}{dx} = \frac{n_1}{c}\frac{x}{\sqrt{h_1^2+x^2}} - \frac{n_2}{c}\frac{(d-x)}{\sqrt{h_2^2+(d-x)^2}} = 0$$

$$n_1 \cdot \sin\theta_1 = n_2 \sin\theta_2$$

This is an illustration of "Fermat's Principle" that...

$$\int_{A,\text{ path}}^{B} n(x)\,dx$$ is an extremum for actual path

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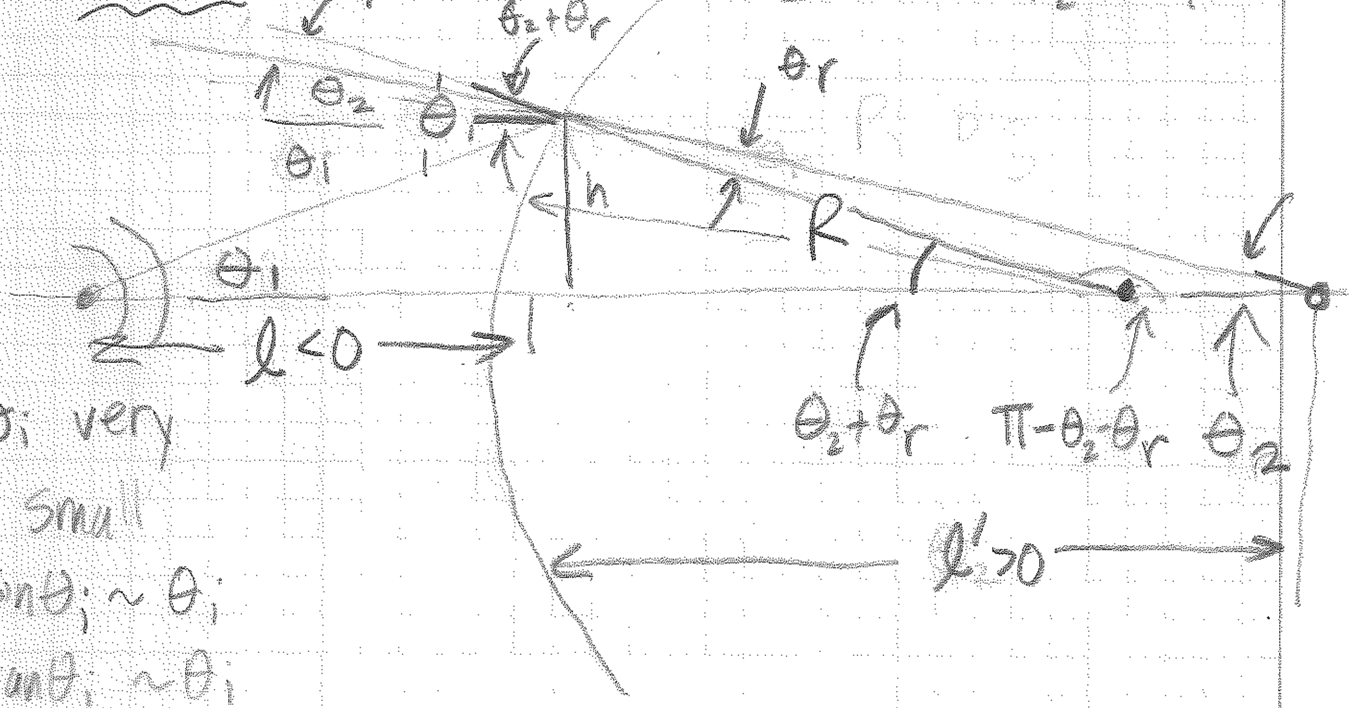
Lenses



θ_i very small

$\sin\theta_i \sim \theta_i$

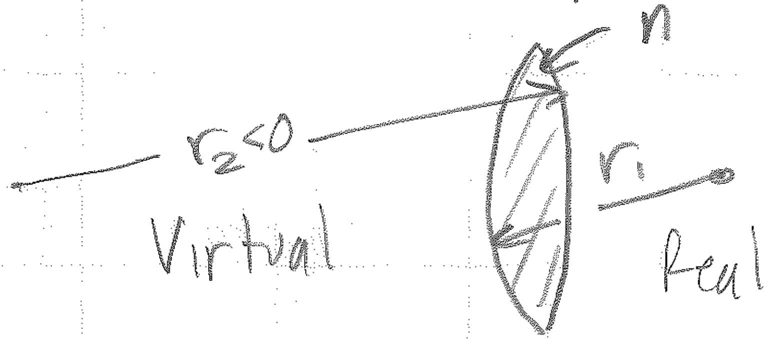
$\tan\theta_i \sim \theta_i$

$\cos\theta_i \simeq 1$

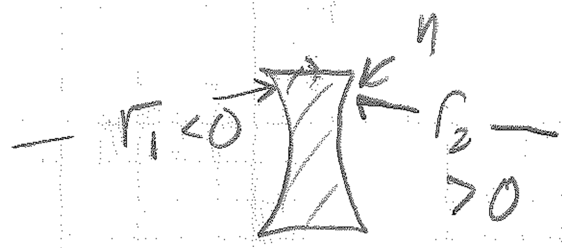
$$\theta_1' = \theta_1 + \theta_2 + \theta_r$$

$$\theta_1 \simeq -\frac{h}{l} \qquad \theta_2 \simeq \frac{h}{l'}$$

Lens Maker's Equation (thin lens)

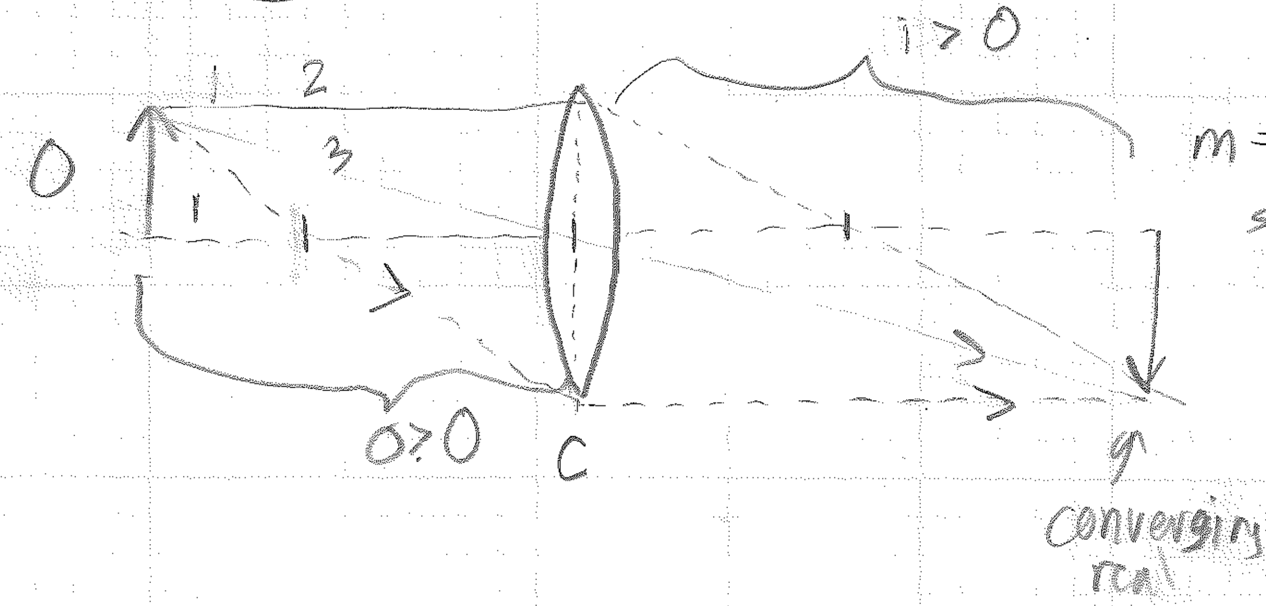


$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) > 0$$



$\frac{1}{f} < 0$ no matter what.

Thin lens: thickness irrelevant.



$m = -\frac{i}{o}$ still

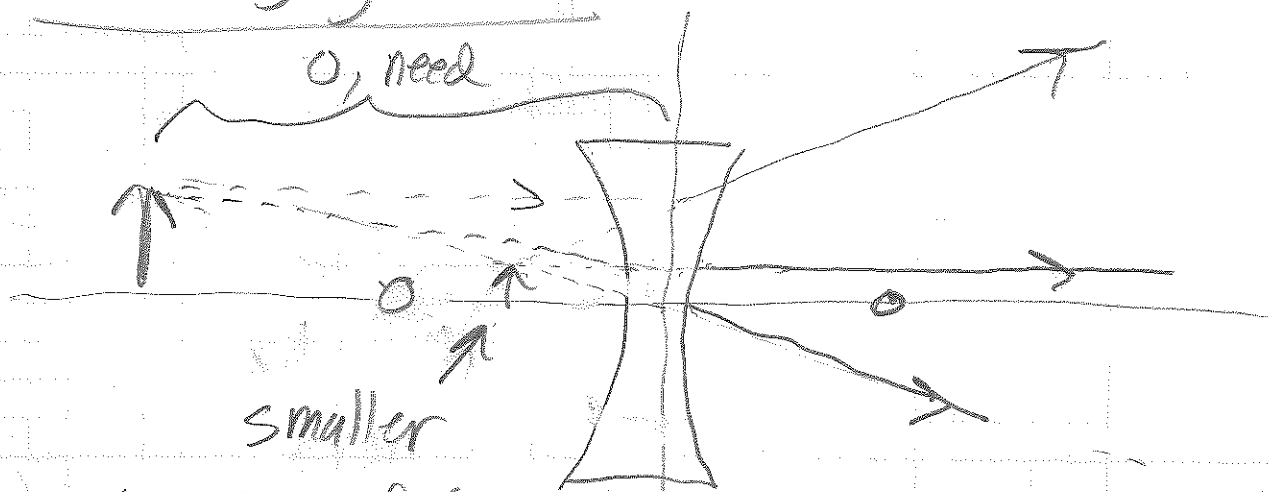
3 principal rays: 1) First focus, out ∥

2) in ∥, through second.

3) Undeflected through center.

IGNORE: headed toward other.

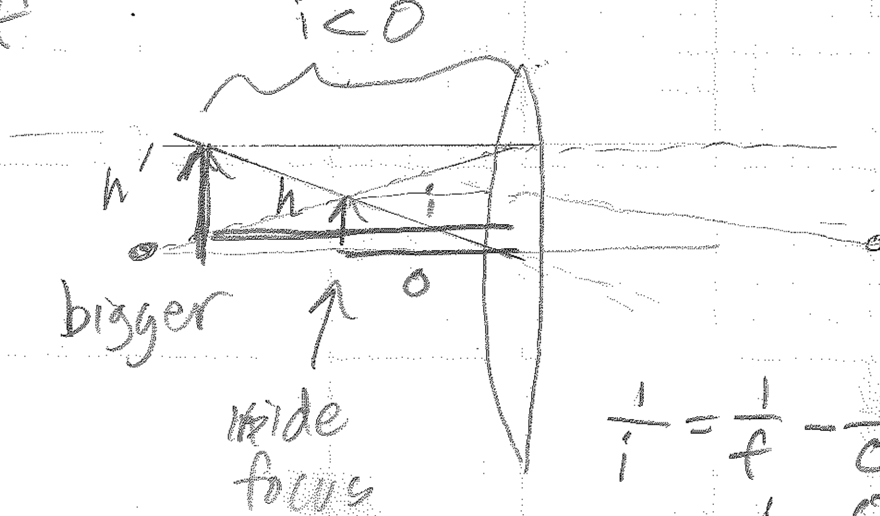
Diverging Lens



(no sign flip for real objects)

Fun with converging lenses

$o < f$



$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} < 0$$

$o < f$

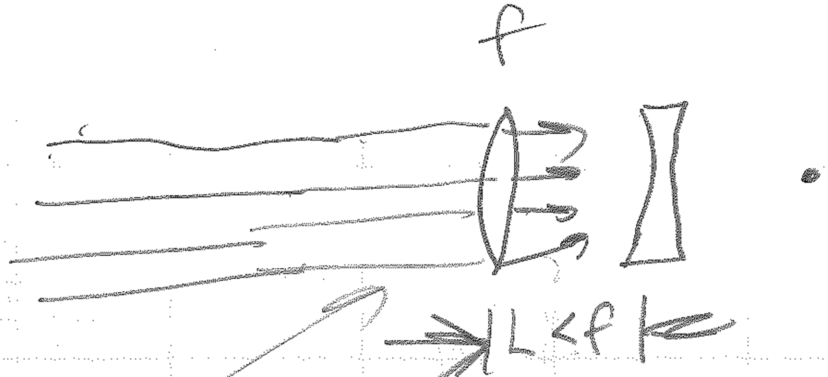
$$\frac{1}{i} = \frac{o-f}{fo}$$

$$\frac{h'}{h} = \frac{-i}{o} = \frac{f}{f-o}$$

"magnifying glass"

Best: $o \simeq f$

Lens System



$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \qquad i = f$$

$o = \infty \quad i = f \qquad o' = -(f-L) \qquad$ KEY

virtual object, converging

$$\frac{1}{o'} + \frac{1}{i} = -\frac{1}{f}$$

$$-\frac{1}{f-L} + \frac{1}{i} = -\frac{1}{f}$$

$$\frac{1}{i} = -\frac{1}{f} + \frac{1}{f-L} = \frac{-(f-L)+f}{f(f-L)}$$

NET CONVERGENCE $\qquad i = \frac{f}{L}(f-L) > 0!$

when $i, o > 0$ $\quad \frac{1}{o} + \frac{1}{i} = \frac{1}{f}$ $\quad f > 0$ $\quad o, i$ always ___ than f

o : always > 0 for physical object, but $o < 0$

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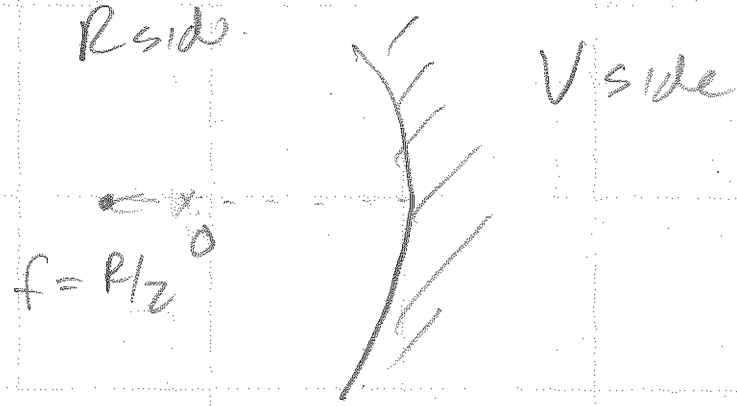

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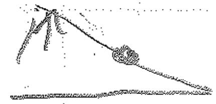
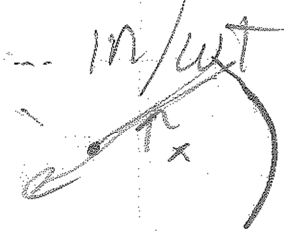
2) Through focus in, Parallel Out 
3) Center of curvature in/out 
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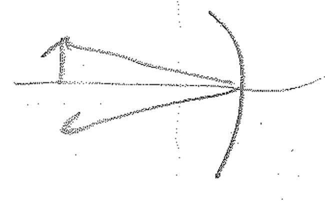

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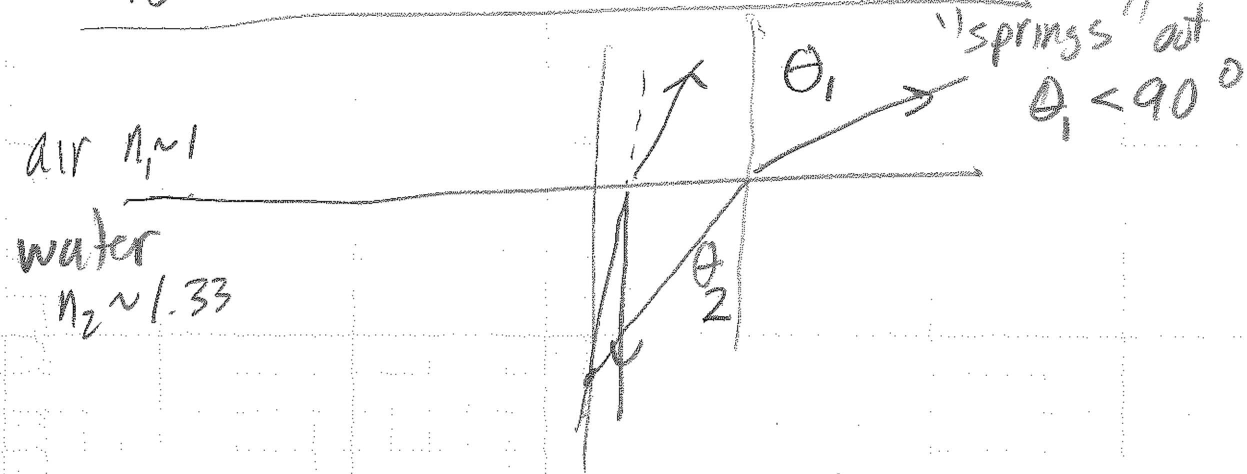

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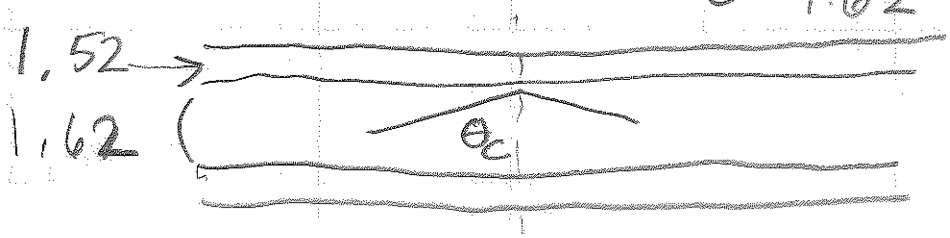

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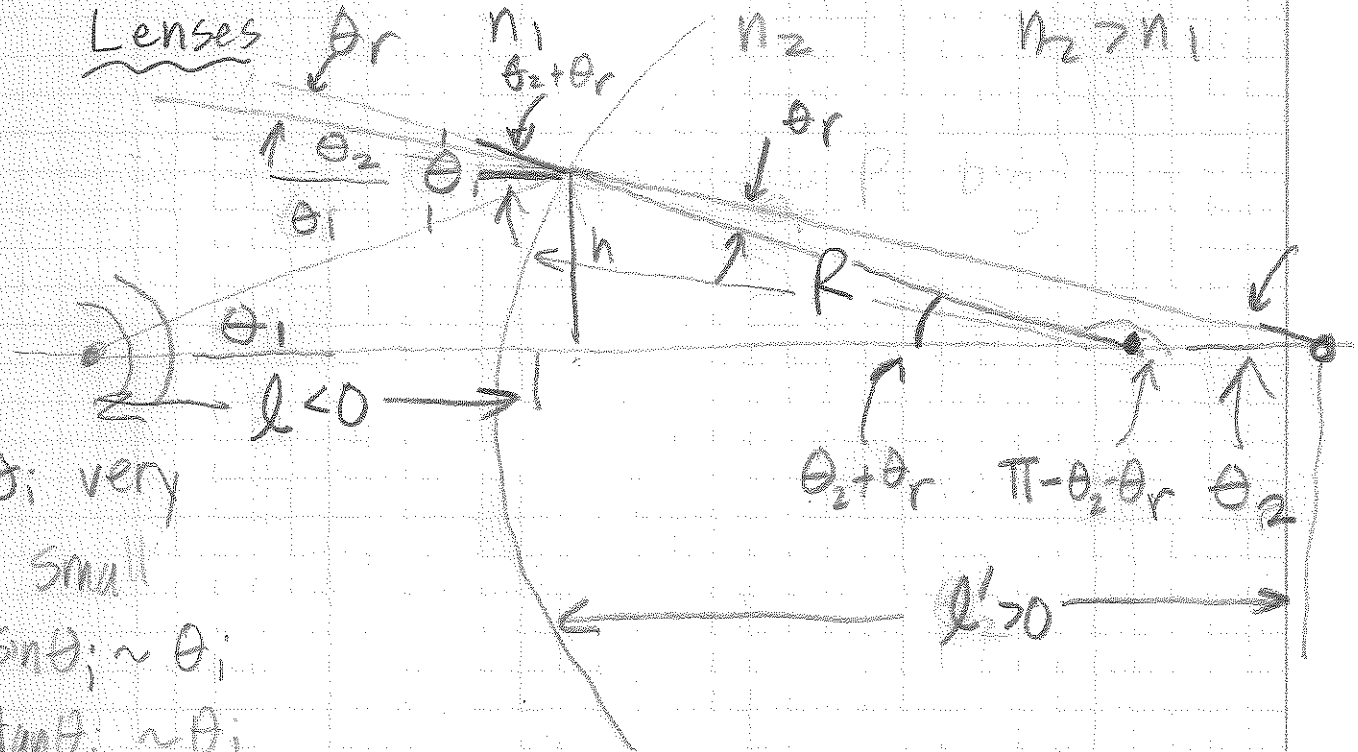
Lenses



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$$\theta_i = \theta_1 + \theta_2 + \theta_r$$

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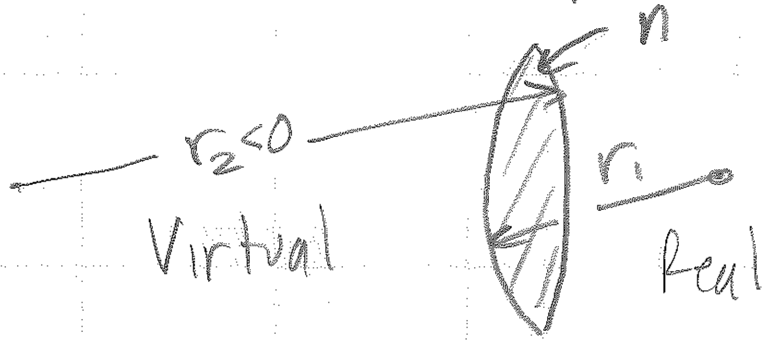
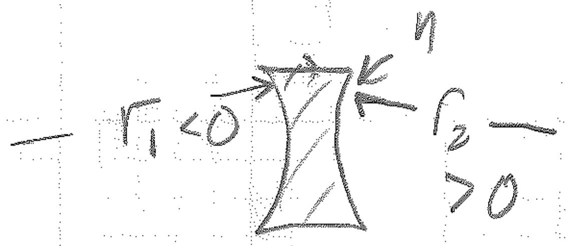
Lens Maker's Equation (thin lens)



$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

> 0



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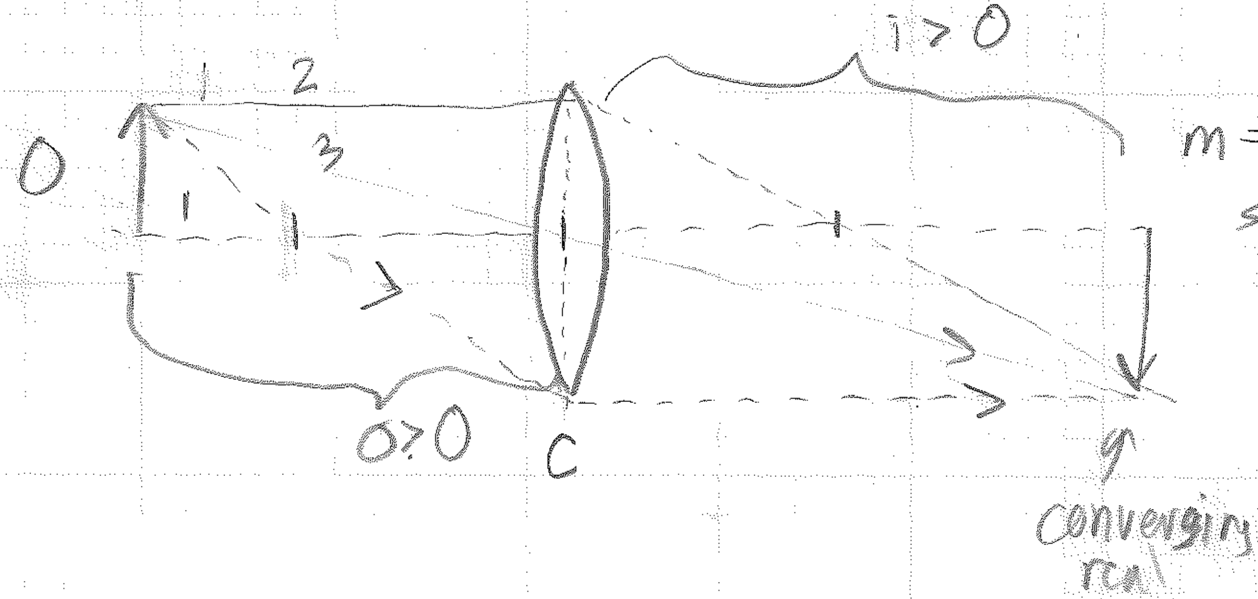
Thin lens: thickness irrelevant.



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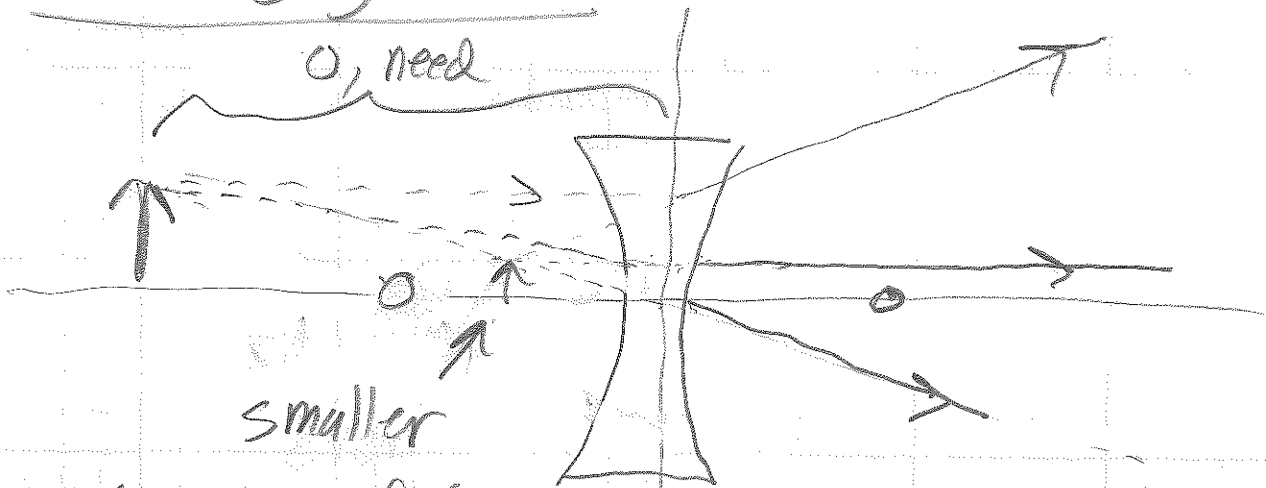
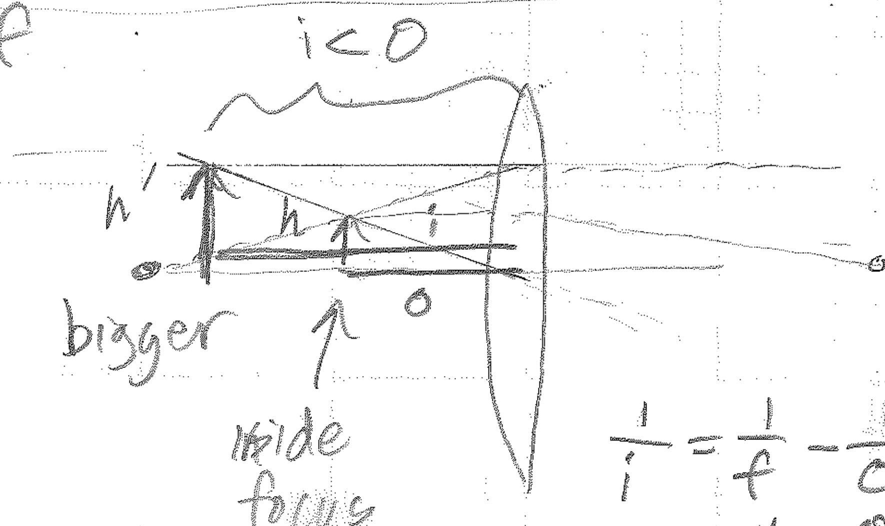
Diverging Lens



(no sign flip for real objects)

Fun with converging lenses

$o < f$



$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} < 0$$

$$\downarrow \quad o < f$$

$$\frac{1}{i} = \frac{o-f}{fo}$$

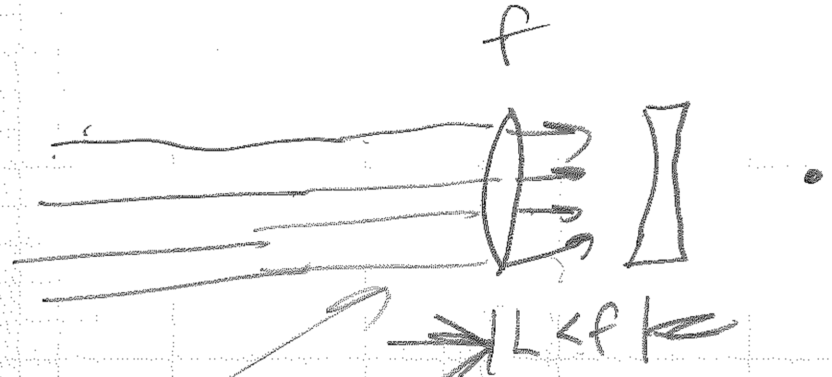
$$\frac{h'}{h} = \frac{-i}{o}$$

$$= \frac{f}{f-o}$$

"magnifying glass"

Best: $o \simeq f$

Lens System



$\frac{1}{i}+\frac{1}{o}=\frac{1}{f}$ $\quad$ $| \, i=f \, |$

$o=\infty \quad i=f \qquad o' = -(f-L)$ $\quad$ KEY

virtual object, converging

$$\frac{1}{o'}+\frac{1}{i}=-\frac{1}{f}$$

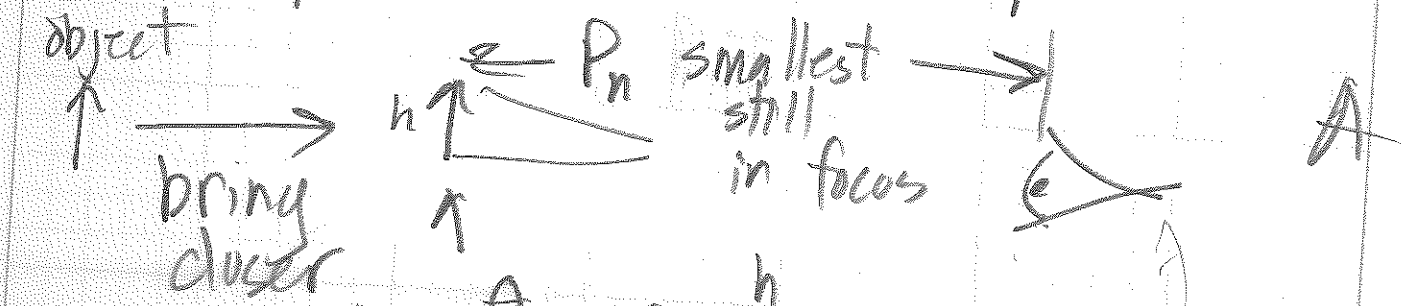
$$-\frac{1}{f-L}+\frac{1}{i}=-\frac{1}{f}$$

$$\frac{1}{i}=-\frac{1}{f}+\frac{1}{f-L}=\frac{-(f-L)+f}{f(f-L)}$$

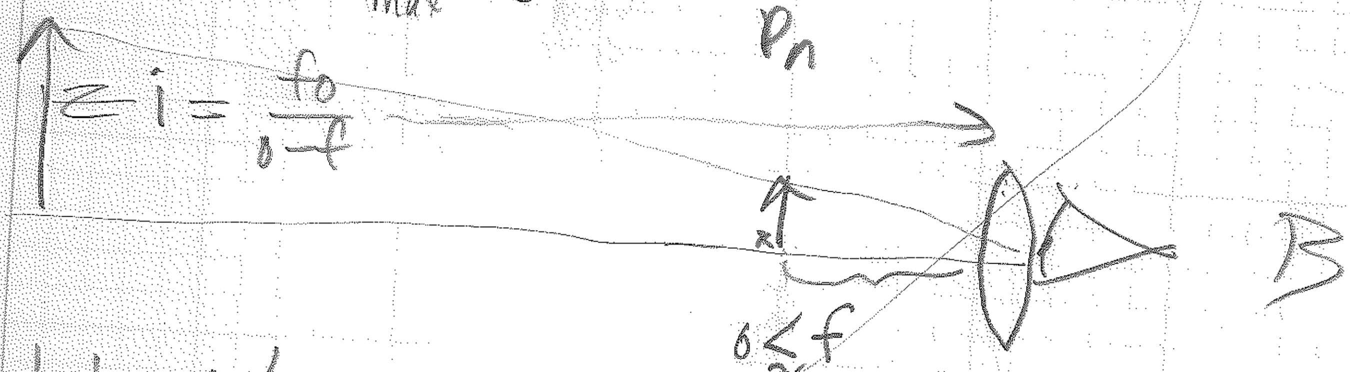
NET CONVERGENCE $\qquad i=\frac{f}{L}(f-L) > 0!$

Magnifying Glass

Really helps your near point

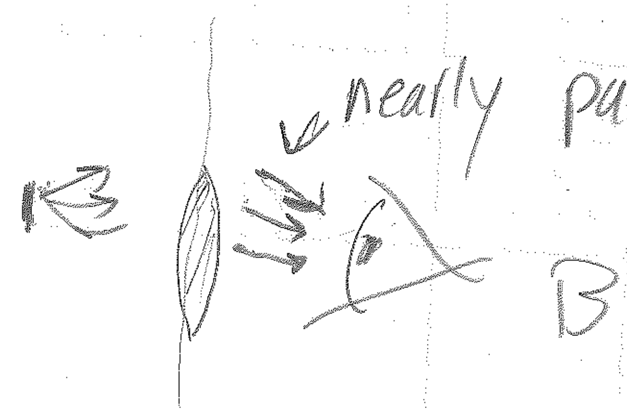


$\theta_{max} \sim \frac{h}{P_n}$



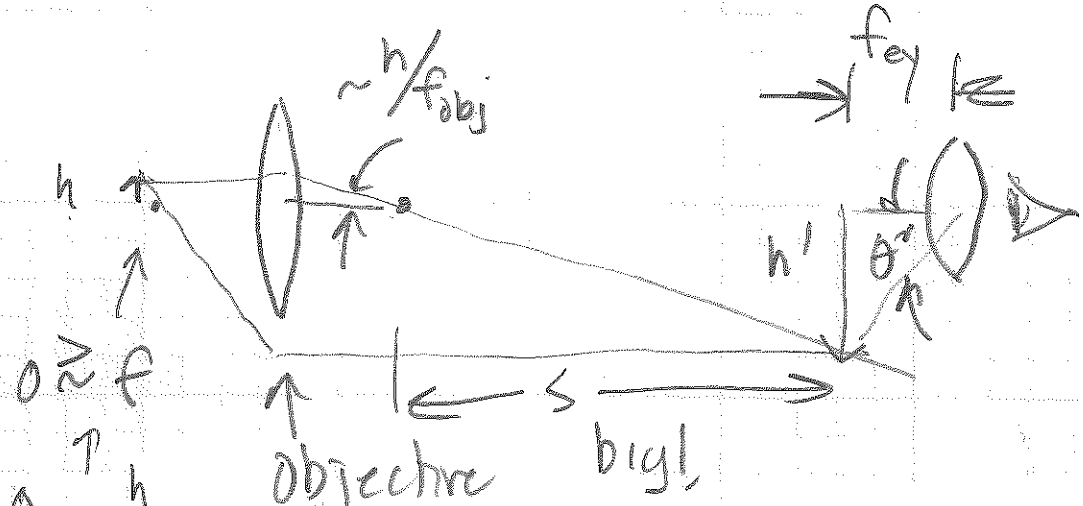
but $\theta' = \frac{h'}{i} = \frac{-i}{io} h \simeq \frac{h}{f}$ (just undeflected ray)

$\frac{\theta'}{\theta_{max}} = \frac{P_n}{f}$ $f < P_n$



Microscope

Add a stage to "blow up" little image...



$$\theta \sim \frac{h}{P_n}$$

$$h \sim P_n \theta$$

$$-\frac{h'}{s} = \frac{h}{f_{obj}}$$

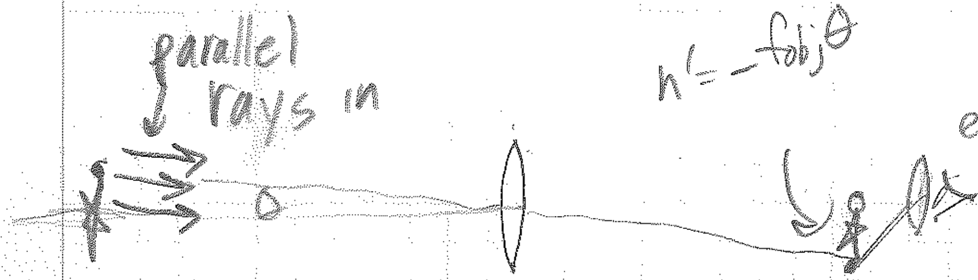
$$h' = -\frac{h}{f_{obj}} s$$

$$\theta' = \frac{h'}{f_{eye}} = -\frac{h s}{f_{obj} f_{eye}} \sim -\frac{P \theta s}{f_{obj} f_{ey}}$$

$$\boxed{M = \frac{\theta'}{\theta} = -\frac{P_n s}{f_{obj} f_{ex}}}$$

Telescope

Like microscope, but stuff of interest ∞ far away



$$\theta' = \frac{h'}{f_{eye}} = \frac{-f_{obj}}{f_{eye}}\,\theta$$

$$\boxed{\frac{\theta'}{\theta} = \frac{-f_{obj}}{f_{ey}}}$$

Key concept: off axis parallel rays converge in focal plane