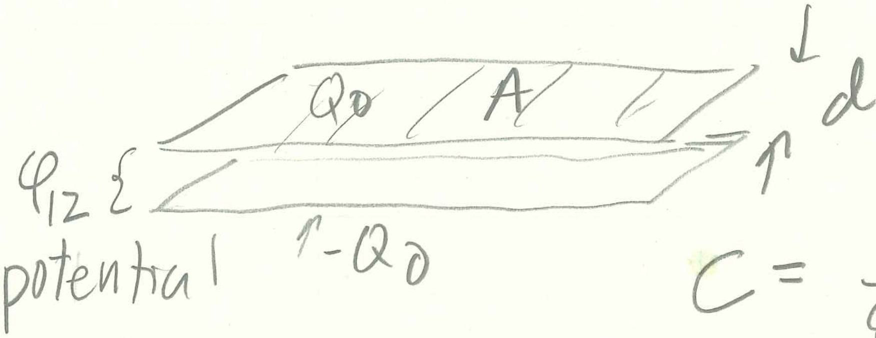
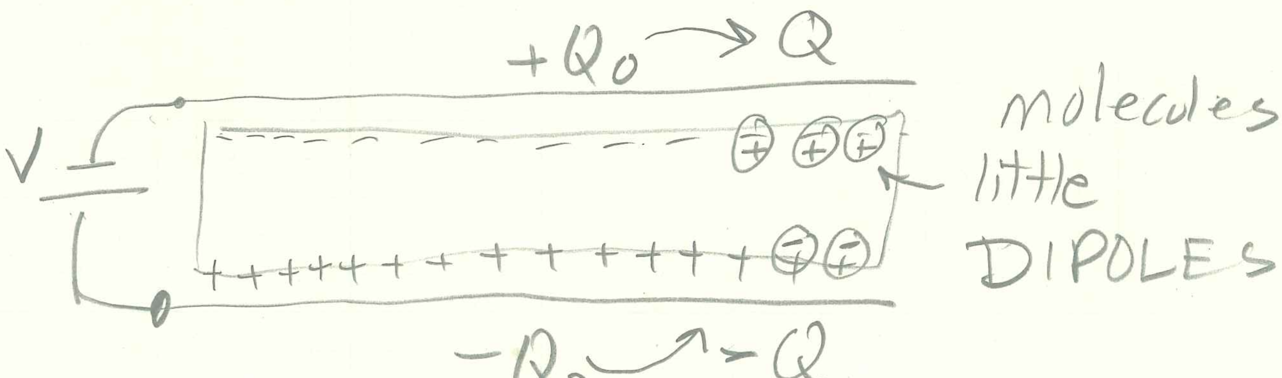
Dielectrics



$$C = \frac{Q_0}{\underbrace{\varphi_{12}}_{V}} \qquad Q = CV$$

Imagine holding $\varphi_{12} = V$ constant, and putting an insulator... a "dielectric" in the gap...



2 ways: $\quad Q = \varepsilon Q_0 \qquad \varepsilon$: "DIELECTRIC" constant.

can be $\gg 1$!, (para-electric)

Generally... not < 1

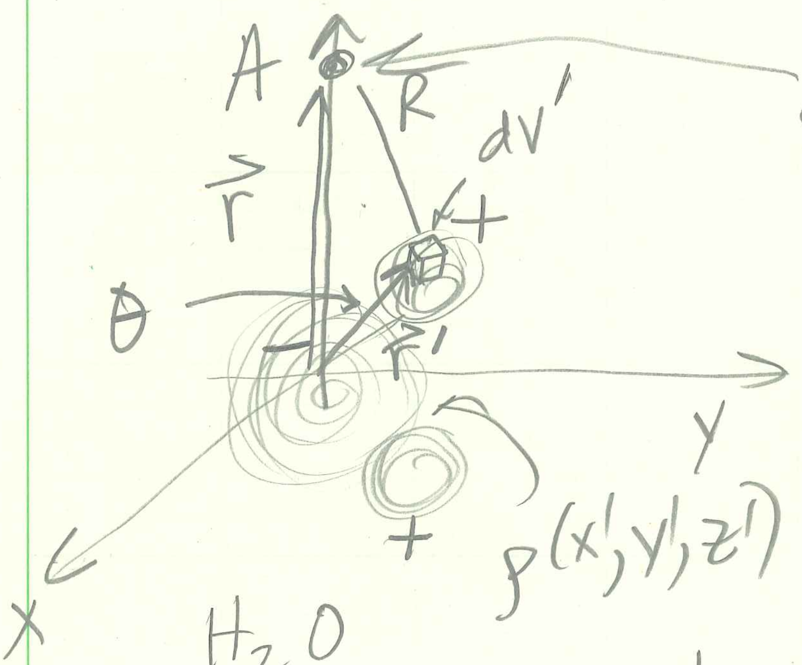
physical meaning?

Moments of a Charge Distribution

$\rho(x', y', z') \rightarrow$ charge density

or... bunch of point charges

$z \Leftarrow$ chosen direction



$$\varphi_A = \int \frac{\rho(x', y', z')\, dV'}{R}$$

a hard integral

systematic approximation

$$R = (r^2 + r'^2 - 2rr'\cos\theta)^{1/2}$$

$$= r\left(1 + \left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cos\theta\right)^{1/2}$$

as $r \rightarrow \infty$ ... $(1 + \text{small})^{1/2}$

$$\frac{1}{R} = \frac{1}{r} \frac{1}{(1+\text{small})^{1/2}} = \frac{1}{r} \frac{1}{(1+\delta)^{1/2}}$$

$$\delta = \left(\frac{r'}{r}\right)^2 - 2\frac{r'}{r}\cos\theta$$

$$\frac{1}{(1+\delta)^{1/2}} \simeq 1 - \frac{1}{2}\delta + \frac{3}{8}\delta^2$$

$$\simeq 1 - \frac{1}{2}\left[\left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cos\theta\right] + \frac{3}{8}\left[\left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cos\theta\right]^2$$

$$\simeq 1 + \frac{r'}{r}\cos\theta + \left(\frac{r'}{r}\right)^2\left(-\frac{1}{2} + \underset{\substack{\uparrow \\ 3/2}}{\frac{12}{8}}\cos^2\theta\right) + \cdots$$

$$\varphi_A \simeq \int \frac{\rho(x', y', z')}{r} \times \left(1 + \frac{r'}{r}\cos\theta + \left(\frac{r'}{r}\right)^2\left(\frac{3\cos^2\theta - 1}{2}\right) + \right) dv'$$

r factors out!

$$= \frac{K_0}{r} + \frac{K_1}{r^2} + \frac{K_2}{r^3} + \cdots$$

$K_0 = \int \rho(x', y', z')\, dv' = Q$! (what if $= 0$!)

$K_1 = \int r' \cos\theta \, \rho \, dv'$ ⇐ related to dipole moment

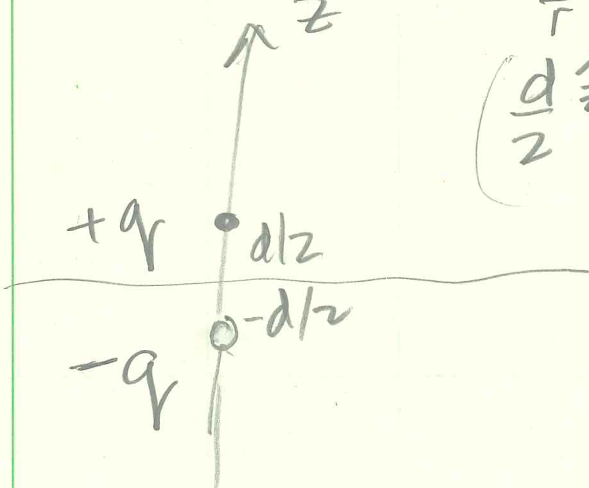
$K_2 = \int r'^2 \left(\frac{3\cos^2\theta - 1}{2}\right) \rho \, dv'$ ⇐ Quadrupole

$K_1 = \int r' \cos\theta \rho \, dV'$ is the z component of...

$\vec{p} = \int \vec{r}\,' \rho \, dV'$ ← called the dipole moment.

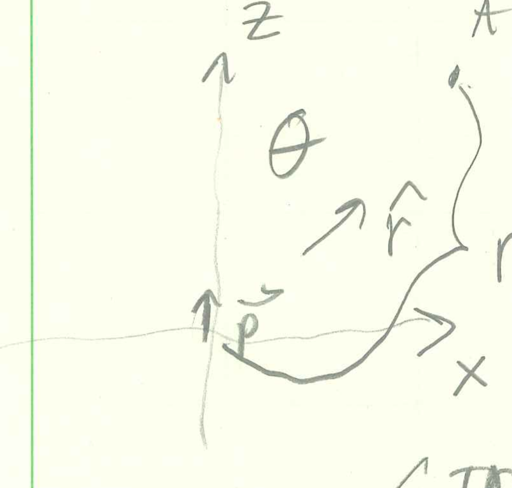
↓

point charge version



$\left(\overbrace{\frac{d}{2}\hat{z}}^{\vec{r}\,'}\right)\cdot(+q) + \left(\frac{d}{2}(-\hat{z})\right)(-q)$

$\vec{p} = \underbrace{q\,d}_{p}\,\hat{z}$ (Location?)



$r = \sqrt{x^2+z^2}$

$\varphi \sim \frac{\hat{r}\cdot\vec{p}}{r^2} = \frac{p\cos\theta}{r^2}$

Electric field from this?

↓

STAY in z-x plane

$$\vec{E} = -\vec{\nabla}\varphi$$ cartesians

$$= \left(-\hat{x}\frac{\partial}{\partial x} - \hat{y}\frac{\partial}{\partial y} - \hat{z}\frac{\partial}{\partial z}\right)\left(\frac{z}{r^3}\right)\cdot p$$

$$\frac{\partial}{\partial x}\frac{1}{r^3} = -\frac{3}{r^4}\frac{\partial r}{\partial z}$$

$r = \sqrt{x^2+z^2}$

$$\frac{\partial r}{\partial x} = \frac{1}{2}\frac{2x}{\sqrt{x^2+z^2}} = \frac{x}{r}$$

$\frac{\partial}{\partial y}\frac{1}{r^2} = 0$ in x-z plane.

$$\frac{\partial}{\partial z}\frac{1}{r^3} = 3\frac{1}{r^3}\frac{z}{\sqrt{x^2+z^2}}$$

biggest? $\theta = 0$? $= \pi/4$? $= \pi/2$?

$$E_x = \frac{3p\,xz}{r^5} = \frac{3p\,\overbrace{\sin\theta\cos\theta}}{r^3}$$

$E_z =$ (2 terms)

$$= \frac{3pz^2}{r^5} - \frac{p}{r^3}$$

$$E_z = \frac{p(3\cos^2\theta - 1)}{r^3}$$

$\theta = 0$

$\theta = \pi/4$.

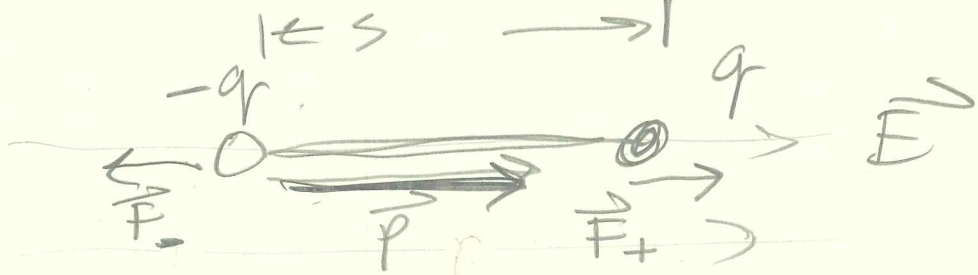
$\theta = \pi/2$.

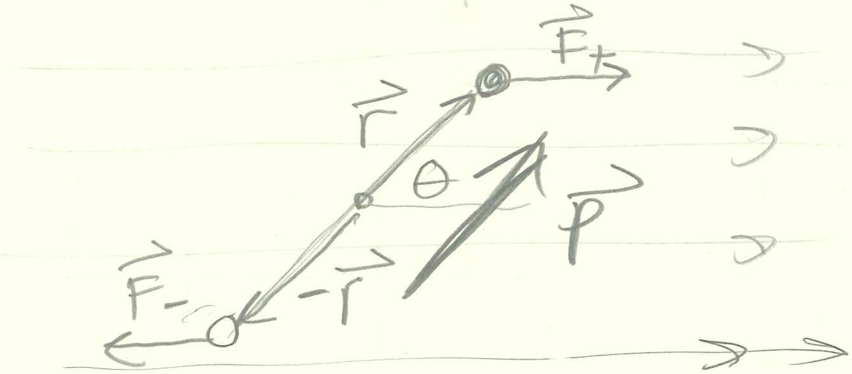
Force & Torque

Uniform $\vec{E}$: no net $\vec{F}$ on $\vec{p}$

yes net torque

$p = sq$





$$\vec{\tau} = \vec{r} \times \vec{F}_+ + (-\vec{r}) \times \vec{F}_-$$

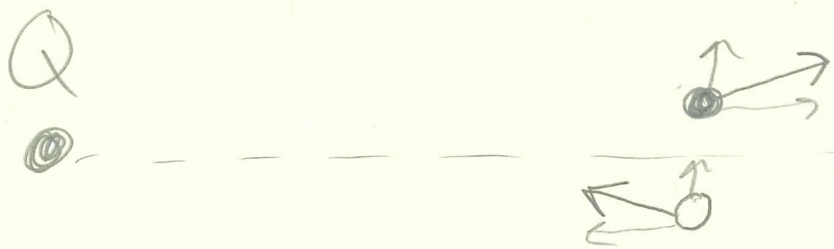
$$= \frac{s}{2} Eq \sin\theta - - \frac{s}{2} Eq \sin\theta$$

$$= sqE \sin\theta = pE \sin\theta$$ tries to line up $\vec{p}$ with $\vec{E}$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$\vec{F}$ non-zero: $\vec{p}$ in non uniform $\vec{E}$

bigger repulsive than attractive



↑ net upward

<u>in general</u>:

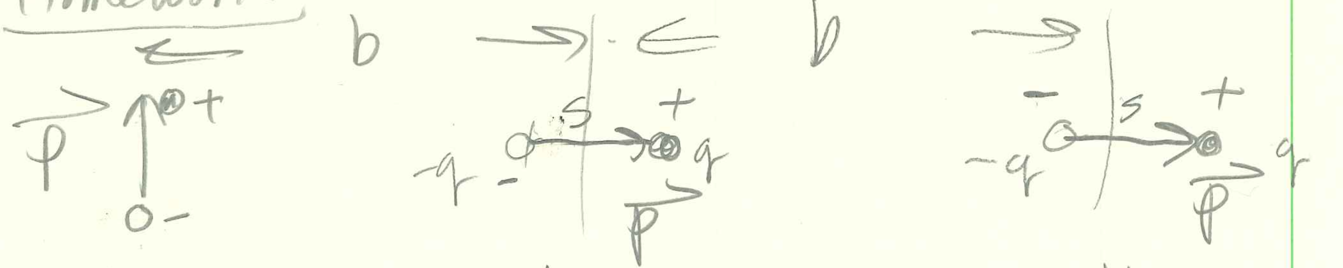
$$F_x = \vec{p} \cdot \text{grad } E_x$$

$$F_y = \vec{p} \cdot \text{grad } E_y$$

$$F_z = \vec{p} \cdot \text{grad } E_z$$

$$\text{grad } E_y = \hat{x}\frac{\partial E_y}{\partial x} + \hat{y}\frac{\partial E_y}{\partial y} + \hat{z}\frac{\partial E_y}{\partial z}$$

<u>Homework</u>:



#1

↑ force on that

#2 all from right

$$F = -\frac{q^2}{b^2} - \frac{q^2}{b^2} + \underbrace{\frac{q^2}{(b+s)^2} + \frac{q^2}{(b-s)^2}}_{\text{expand!}}$$

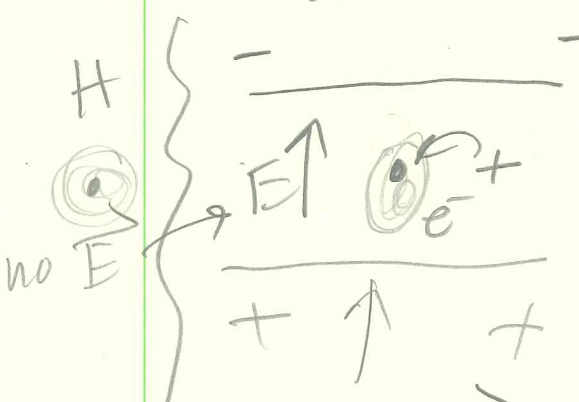
$$\frac{1}{(1+\delta)^2} = 1 - 2\delta + 3\delta^2$$

$$F = -\frac{q^2}{b^2} - \frac{q^2}{b^2} + \frac{q^2}{b^2}\left(1 - 2\frac{s}{b} + 3\left(\frac{s}{b}\right)^2\right) + \frac{q^2}{b^2}\left(1 + 2\frac{s}{b} + 3\left(\frac{s}{b}\right)^2\right)$$

$$= \frac{6\,q^2 s^2}{b^4} = \frac{6\,p^2}{b^4}$$

Induced, permanent molecular dipoles

Induced



$\vec{p} = \alpha \vec{E}$

$\alpha \sim 10^{-24}\ cm^2$

(induction)

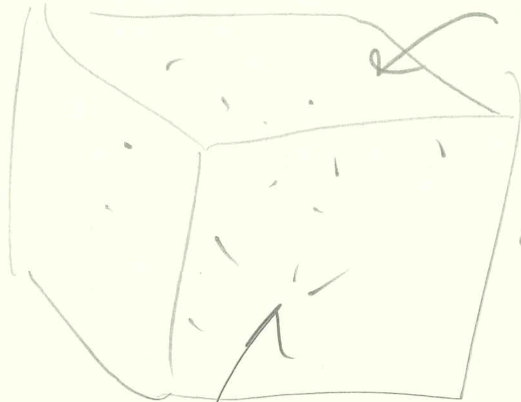
Permanent

$\downarrow p = 1.84 \cdot 10^{-18}$ esu/cm

$\vec{p}$

(alignment)

Polarizability in matter...



$N \to \# / cm^3$

dV cm^3

$$\vec{P} = \vec{p} N$$

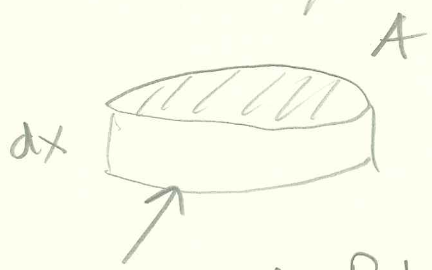
$$\frac{\text{dipoles}}{cm^3} \to \frac{esu \cdot cm}{cm^3}$$

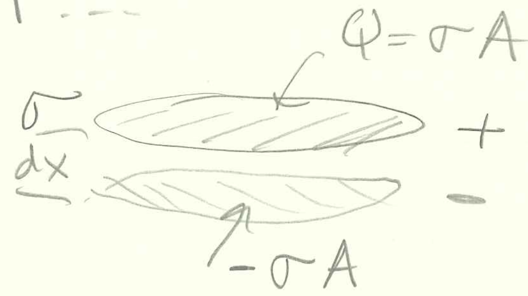
each molecule has $\vec{p}$

units... $\frac{esu}{cm^2}$

Assume they all line up (

Two ways to view P...





$p = P \cdot dV = P\,dx\,A$

$= (PA)\,dx$

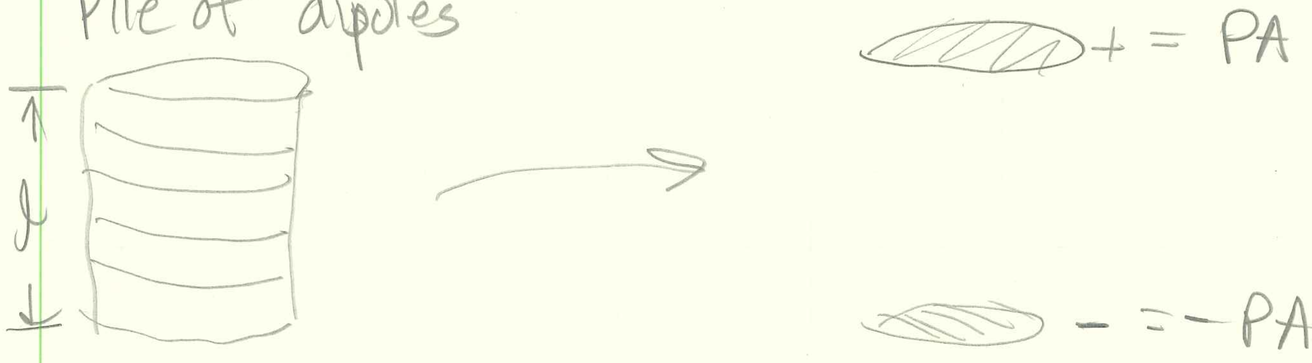
real physics,

$\frac{\#\text{ dipoles}}{\text{unit volume}} = P$ like σ

$p = (\sigma A) \cdot dx$

Q

Pile of dipoles



$p = P \cdot A \cdot l$

Connection with ε

εQ_0 $\quad Q_0 = C_0 V \quad C_0 = \frac{A}{4\pi s}$

$-\varepsilon Q_0$

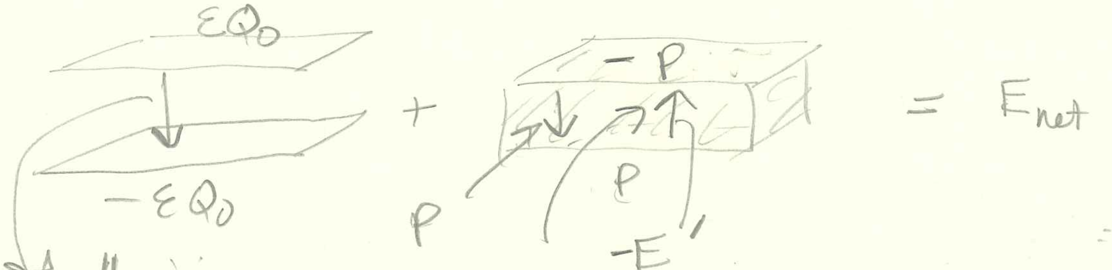
$E_{net} = \frac{V}{s}$

ε

$E_{net} = E'' - E'$

from εQ_0 on plates

dipoles contribution



$E'' = \varepsilon E_{net}$ $\qquad$ $= -4\pi P$

downward is positive

$$\varepsilon E_{net} - 4\pi P = E_{net}$$

$$4\pi P = (\varepsilon - 1) E_{net}$$

$$\frac{P}{E_{net}} = \frac{\varepsilon - 1}{4\pi}$$

but $\quad E_{net} = \dfrac{E''}{\varepsilon} = \dfrac{E_{external}}{\varepsilon}$

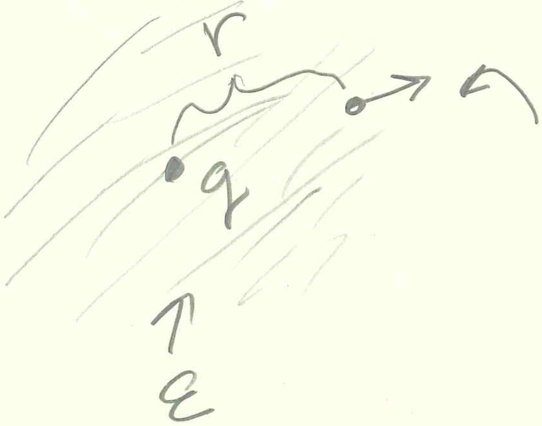
$$\frac{P}{\frac{E_{external}}{\varepsilon}} = \frac{\varepsilon - 1}{4\pi}$$

so $\quad \dfrac{P}{E_{external}} = \dfrac{1}{4\pi}\left(1 - \dfrac{1}{\varepsilon}\right)$

or $\quad 4\pi P = \left(1 - \dfrac{1}{\varepsilon}\right) E_{external}$

- when $\varepsilon = 1$, no induced/aligned $\dfrac{\text{dipole}}{\text{volume}}$.
- when $\varepsilon \to \infty$, P induced to just cancel External field.

$$E_{net} = \frac{E_{external}}{\varepsilon}$$



$$E_{net} = \frac{q}{\varepsilon r^2}$$

$\varepsilon \to \infty$, "shielded"

when accunted for in Maxwell, ... $c \to \frac{c}{\sqrt{\varepsilon}}$

$\sqrt{\varepsilon}$ known as .. index of refraction