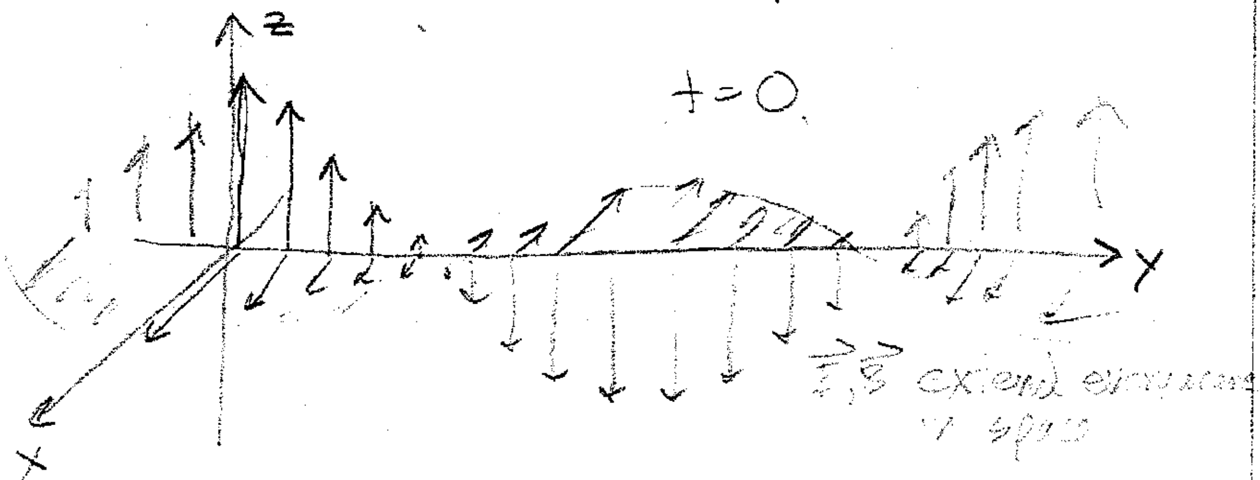
→ And let there be light

$$\rho = \vec{J} = 0 \quad \text{(free space)}$$

$$\text{curl } \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\text{curl } \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

<u>Ansatz</u>: $\vec{E} = \hat{z} E_0 \cos(y - vt)$




$$\vec{B} = \hat{x} B_0 \cos(y - vt)$$

<u>Try now</u>

$$\text{curl } \vec{E} : \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_0 \cos(y-vt) \end{vmatrix} = \hat{x} \cdot \frac{\partial}{\partial y}\left(E_0 \cos(y-vt)\right) - \hat{y}\, \cancel{\frac{\partial}{\partial x}\left(E_0 \cos(y-vt)\right)} \rightarrow 0$$

$$= -\hat{x} E_0 \sin(y - vt)$$

$$\text{curl } \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_0 \cos(y-vt) & 0 & 0 \end{vmatrix} = \begin{array}{l} \hat{x} \cdot 0 \\ -\hat{y} \cdot \left(-\frac{\partial}{\partial z} B_0 \cos(y-vt)\right) \\ +\hat{z}\left(-\frac{\partial}{\partial y} B_0 \cos(y-vt)\right) \end{array} = +\hat{z} B_0 \sin(y-vt)$$

$$-\frac{1}{c}\frac{\partial}{\partial t}\vec{B} = -\frac{1}{c}\frac{\partial}{\partial t}\hat{x}B_0\cos(y-vt)$$

$$= -\frac{v}{c}\hat{x}B_0\sin(y-vt)$$

$$\text{curl}\,\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$-\hat{x}E_0\sin(y-vt) = -\frac{v}{c}\hat{x}B_0\sin(y-vt)$$

$$E_0 = \frac{v}{c}B_0$$

$$\text{curl}\,\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t}$$

$$\frac{\partial\vec{E}}{\partial t} = \frac{\partial}{\partial t}\left(\hat{z}E_0\cos(y-vt)\right) = +v\hat{z}E_0\sin(y-vt)$$

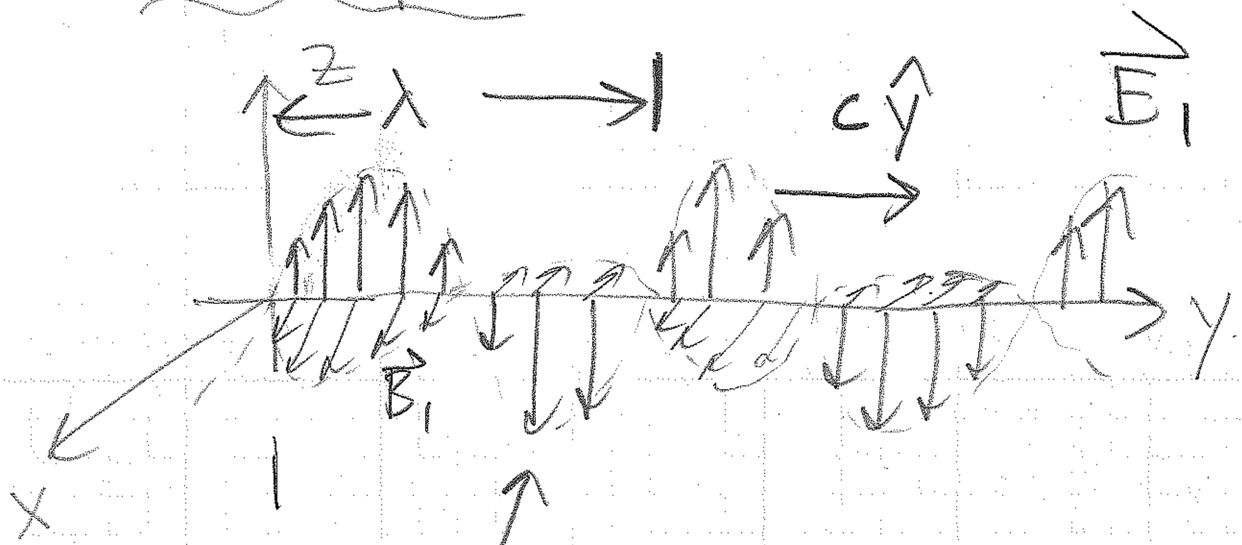
$$-\hat{z}B_0\sin(y-vt) = +\frac{v}{c}\hat{z}E_0\sin(y-vt)$$

$$B_0 = \frac{v}{c}E_0$$

$$\cancel{E_0} = \frac{v}{c}\cdot\frac{v}{c}\cancel{E_0}$$

$$\left(\frac{v}{c}\right)^2 = 1 \qquad \boxed{v = \pm c}$$

More general electromagnetic wave in free space




$$\vec{E}_1 = \hat{z} E_0 \sin\left[\frac{2\pi}{\lambda}(y - ct)\right]$$

deduce $\vec{B}_1$ because $\vec{E}_1 \times \vec{B}_1$ must point in $\hat{y}$ direction

$$\vec{B}_1 = \hat{x} E_0 \sin\left[\frac{2\pi}{\lambda}(y - ct)\right]$$

1 statvolt/cm = 1 gauss!

Standing Wave: • like guitar string

• useful to match boundary condition: $\vec{E} = 0$ at conductor

Superpose wave moving in opposite direction . . .

$$\vec{E}_2 = \hat{z}\, E_0 \sin\left[\frac{2\pi}{\lambda}(y + ct)\right]$$

$$\vec{E}_1 + \vec{E}_2 = \hat{z}\, E_0 \left[\sin \frac{2\pi}{\lambda}(y - ct) + \sin \frac{2\pi}{\lambda}(y + ct)\right]$$

$$= \hat{z}\, E_0 \Big[\sin \frac{2\pi}{\lambda} y \cos\left(\frac{2\pi}{\lambda}(-ct)\right) + \cos \frac{2\pi}{\lambda} y \sin\left(\frac{2\pi}{\lambda}(-ct)\right)$$

$$+ \underbrace{\sin \frac{2\pi}{\lambda} y \cos\left(\frac{2\pi}{\lambda}(ct)\right)}_{\text{add}} + \underline{\cos \frac{2\pi}{\lambda} y \sin\left(\frac{2\pi}{\lambda} ct\right)}\Big]$$

cancel

$$\boxed{\vec{E}_1 + \vec{E}_2 = 2\hat{z}\, E_0 \sin\left(\frac{2\pi}{\lambda} y\right) \cos\left(\frac{2\pi}{\lambda} ct\right)}$$

stable in space

"co"-sinusoidal in time.

Corresponding $\vec{B}$ has a surprise . . .

$$\vec{B}_2 = \hat{x}\, E_0 \sin\left[\frac{2\pi}{\lambda}(y + ct)\right]$$

so $\vec{E}_2 \times \vec{B}_2$ is in $-\hat{y}$ direction

→ And let there be light

$$\rho = \vec{J} = 0 \quad \text{(free space)}$$

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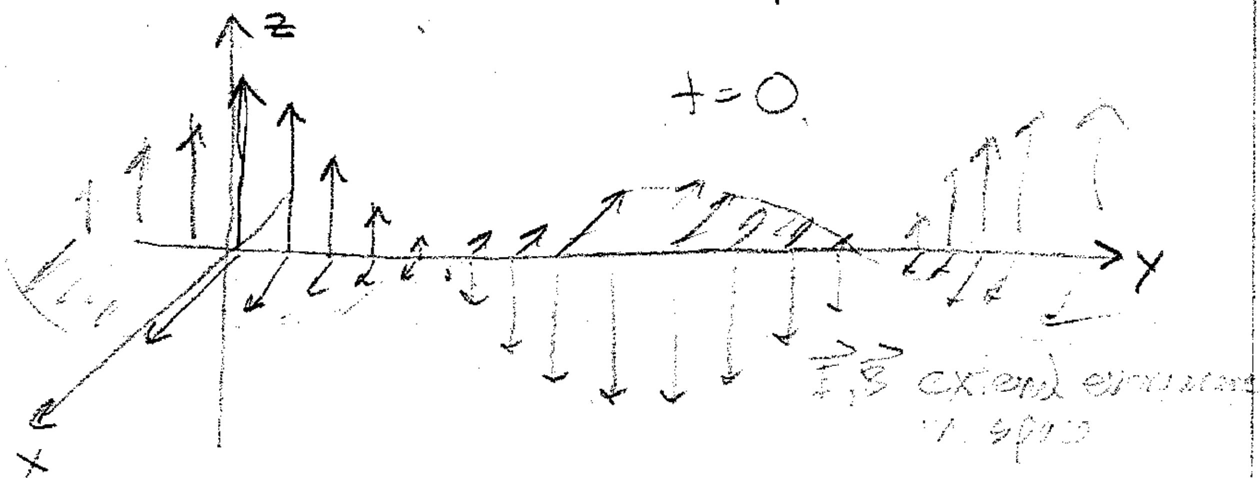


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Try now

$$\text{curl}\,\vec{E}: \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_0\cos(y-vt) \end{vmatrix} = \hat{x}\cdot\frac{\partial}{\partial y}\left(E_0\cos(y-vt)\right) - \hat{y}\,\frac{\partial}{\partial x}\left(E_0\cos(y-vt)\right) \xrightarrow{} 0$$

$$= -\hat{x} E_0 \sin(y - vt)$$

$$\text{curl}\,\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_0\cos(y-vt) & 0 & 0 \end{vmatrix} = \begin{array}{l} \hat{x}\cdot 0 \\ -\hat{y}\cdot\left(-\frac{\partial}{\partial z} B_0\cos(y-vt)\right) \\ +\hat{z}\left(-\frac{\partial}{\partial y} B_0\cos(y-vt)\right) \end{array} = +\hat{z} B_0 \sin(y-vt)$$

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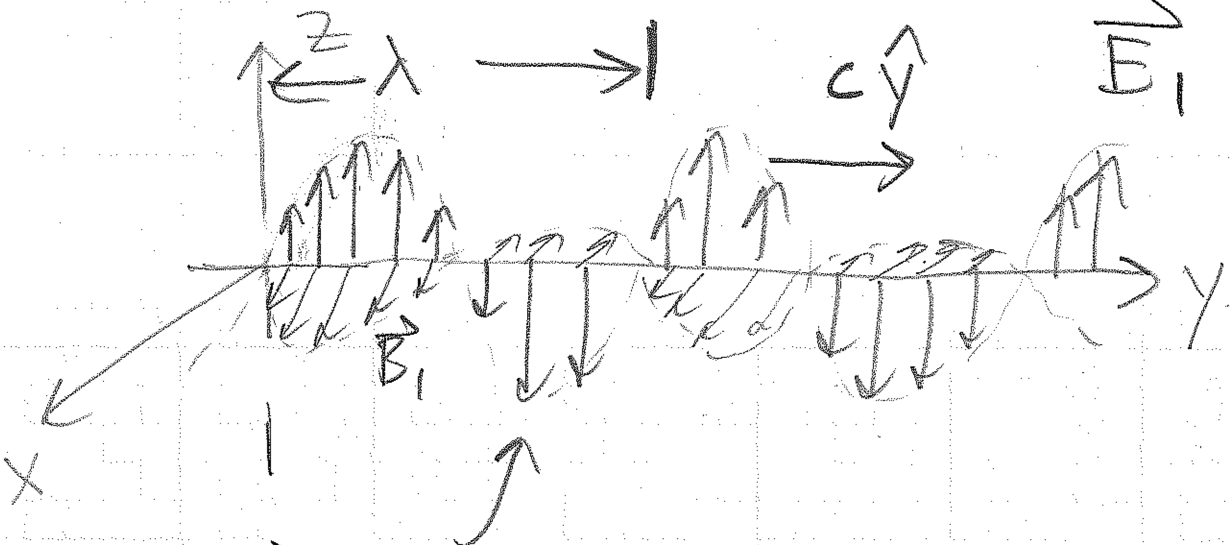
More general electromagnetic wave in <u>free space</u>




$$\vec{E}_1 = \hat{z} E_0 \sin\left[\frac{2\pi}{\lambda}(y - ct)\right]$$

deduce $\vec{B}_1$ because $\vec{E}_1 \times \vec{B}_1$ must point in $\hat{y}$ direction

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1 statvolt/cm = 1 gauss!

<u>Standing Wave</u> : • like guitar string

• useful to match boundary condition: $\vec{E} = 0$ at conductor b

<u>Superpose</u> wave moving in opposite direction . . .

$$\vec{E}_2 = \hat{z} E_0 \sin\left[\frac{2\pi}{\lambda}(y + ct)\right]$$

$$\vec{E}_1 + \vec{E}_2 = \hat{z} E_0 \left[\sin \frac{2\pi}{\lambda}(y - ct) + \sin \frac{2\pi}{\lambda}(y + ct)\right]$$

$$= \hat{z} E_0 \Big[\sin \frac{2\pi}{\lambda} y \cos\left(\frac{2\pi}{\lambda}(-ct)\right) + \cos \frac{2\pi}{\lambda} y \sin\left(\frac{2\pi}{\lambda}(-ct)\right)$$

$$+ \underbrace{\sin \frac{2\pi}{\lambda} y \cos\left(\frac{2\pi}{\lambda}(ct)\right)}_{\text{add}} + \underbrace{\cos \frac{2\pi}{\lambda} y \sin\left(\frac{2\pi}{\lambda} ct\right)}_{\text{cancel}}\Big]$$

$$\boxed{\vec{E}_1 + \vec{E}_2 = 2\hat{z} E_0 \sin\left(\frac{2\pi}{\lambda} y\right) \cos\left(\frac{2\pi}{\lambda} ct\right)}$$

stable in space

"co"-sinusoidal in time

---

Corresponding $\vec{B}$ has a surprise . . .

$$\vec{B}_2 = \hat{x} E_0 \sin\left[\frac{2\pi}{\lambda}(y + ct)\right]$$

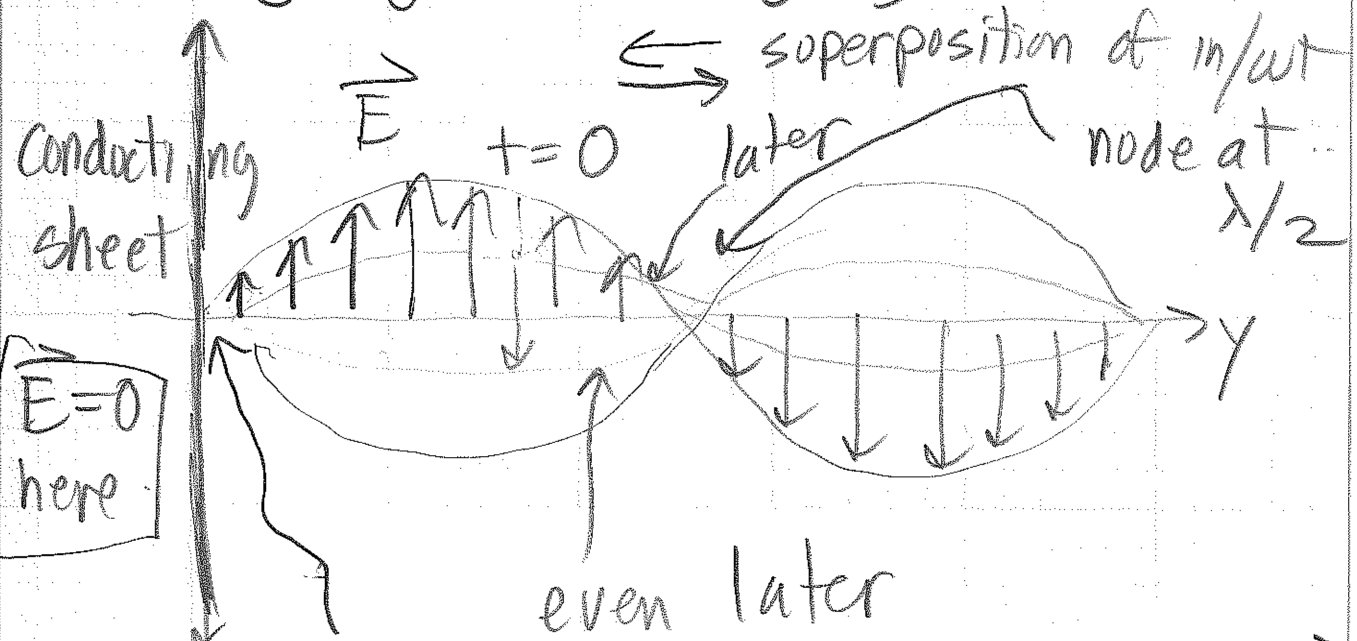
so $\vec{E}_2 \times \vec{B}_2$ is in $-\hat{y}$ direction so

$$\vec{B}_1 + \vec{B}_2 = -2\hat{x}E_0\cos\left(\frac{2\pi}{\lambda}y\right)\sin\left(\frac{2\pi}{\lambda}ct\right)$$

out of phase by $90^\circ$ w/r to $\vec{E}$

also!

Figure 9.9 depicts very well...
Useful to describe wave incident on perfect conductor ... to maintain $\vec{E} = 0$ at conductor need the superposition of ingoing and outgoing waves...



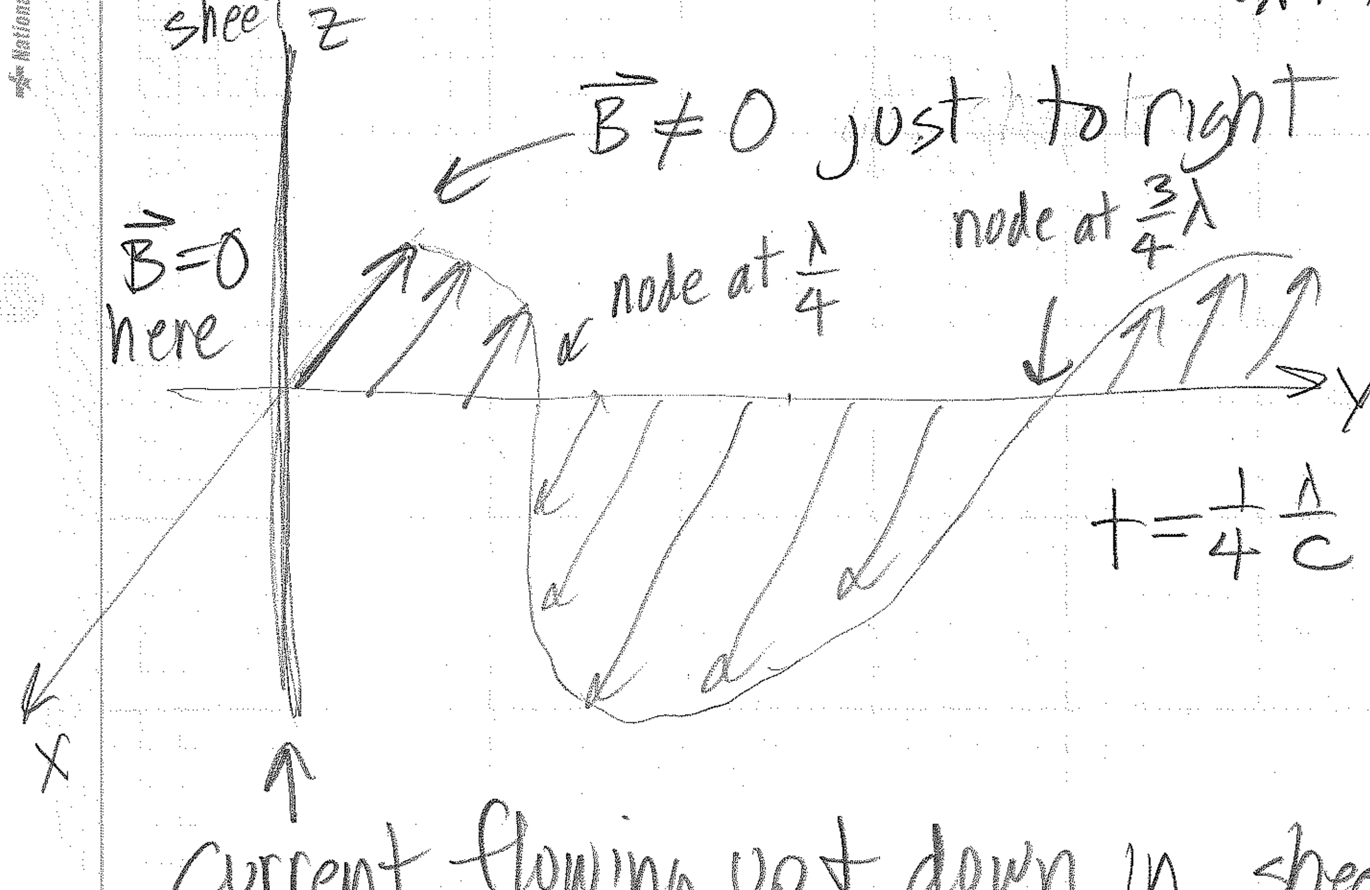

however, current flows... making $\vec{B}$!

How $\vec{B}$ looks... a little new...

First.. maximum not at $t=0$, but when $\sin\left(\frac{2\pi}{\lambda}ct\right)=1 \Rightarrow \frac{2\pi}{\lambda}ct=\pi/2$

later $t=\frac{1}{4}\frac{\lambda}{c}$ .. $\vec{E}(t)=0$!

Second.. in space.. $\propto -2\hat{x}E_0\cos\left(\frac{2\pi}{\lambda}y\right)$




Current flowing up + down in sheet!

at $t=\frac{1}{4}\frac{\lambda}{c}$ $\vec{E}=0$

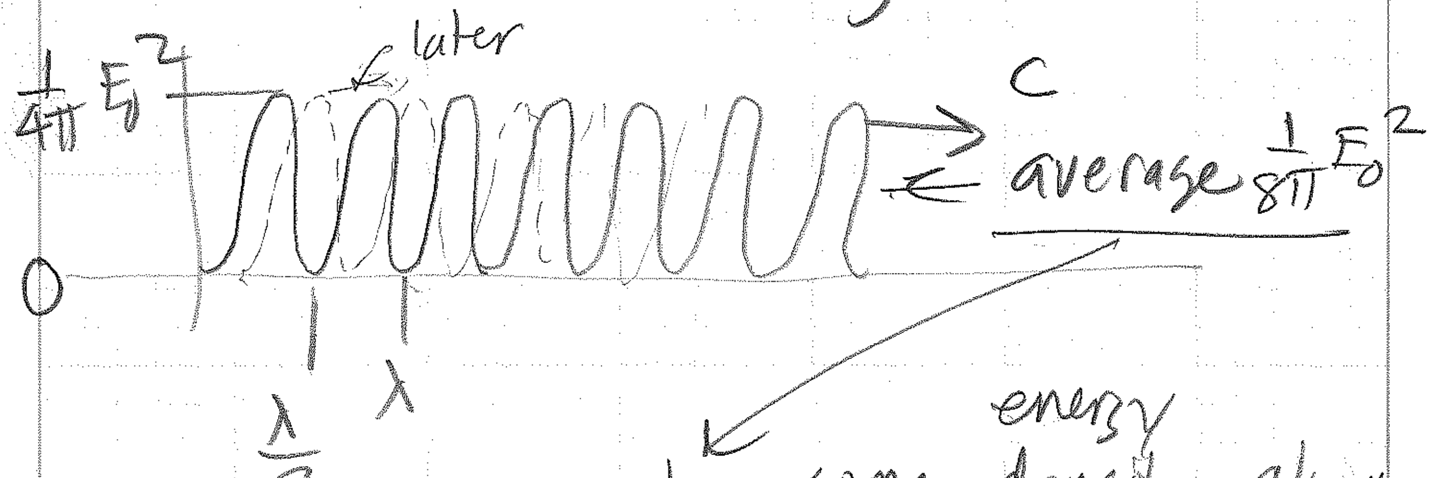
# Power + Energy

$$\vec{E}_1 = \hat{z} E_0 \sin\left[\frac{2\pi}{\lambda}(y - ct)\right]$$

$$\vec{B}_1 = \hat{x} E_0 \sin\left[\frac{2\pi}{\lambda}(y - ct)\right]$$

Energy density is $\frac{1}{8\pi}(E^2 + B^2)$

$$= \underbrace{\frac{1}{4\pi} E_0^2 \sin^2\left[\frac{2\pi}{\lambda}(y - ct)\right]}_{\text{still moving!}}$$




note, some energy density always there

$$\frac{\text{Energy}}{\text{volume}} \times \text{speed} = \frac{\text{Energy}}{\text{area time}} \Rightarrow \text{flux}$$

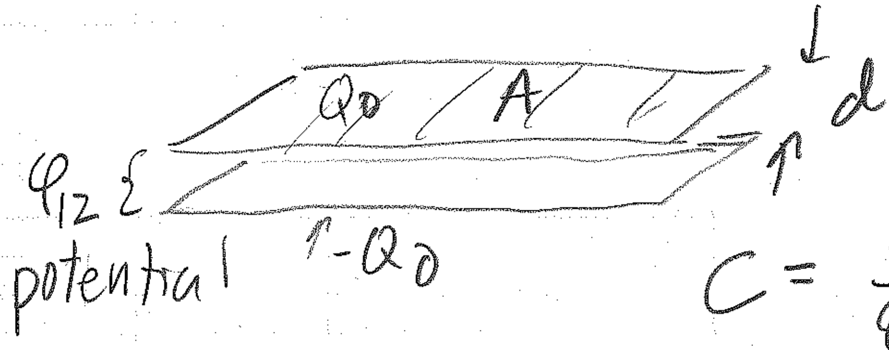
$$S = \frac{1}{8\pi} E_0^2 = \frac{1}{4\pi} \overline{E^2} \leftarrow \text{mean squared}$$

This is one where the SI units are important ...

$$S = \frac{\overline{E^2}}{\sqrt{\mu_0/\varepsilon_0}} = \frac{\overline{E^2}\left(\frac{\text{volts}}{\text{meter}}\right)^2}{377 \text{ ohms}}$$

comes out in $\frac{\text{watts}}{m^2}$

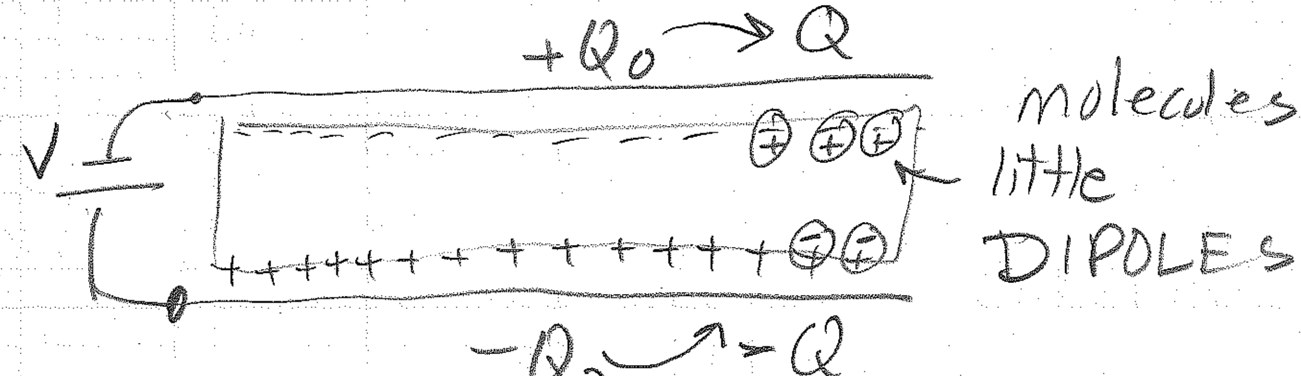
# Dielectrics




$C = \dfrac{Q_0}{\varphi_{12}}$ $\quad Q = CV$

$V$

Imagine holding $\varphi_{12} = V$ constant, and putting an insulator.. a "dielectric" in the gap...




2 ways: $\quad Q = \varepsilon Q_0 \quad$ $\varepsilon$: "DIELECTRIC" constant

can be $\gg 1$ !, (para-electric)

Generally.. not $< 1$

physical meaning?