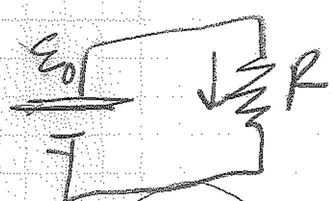
Chapter 8

DC : $\mathcal{E}_0$

R

$$I = \frac{V}{R}$$



$$I = \frac{V}{R}$$

$$IV = \frac{V^2}{R} = I^2 R$$

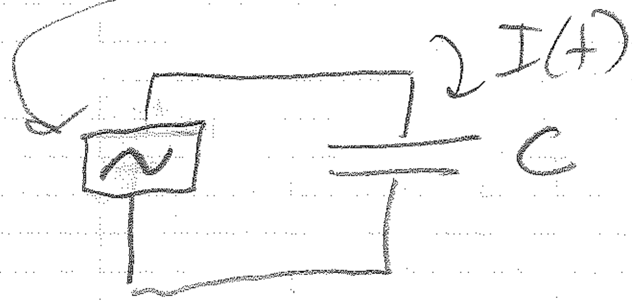
power always dissipated (DC)

60 Hz

$\omega = 377$ rad/s

AC : $\mathcal{E}_0 \cos(\omega t)$

$= \mathrm{Re}(\mathcal{E}_0 e^{i\omega t})$



more subtle relationship.

$$Q = CV$$

$$\int I\,dt = \frac{I_0}{i\omega} \cancel{e^{i\omega t}} = C\mathcal{E}_0 \cancel{e^{i\omega t}}$$

$\mathrm{Re}(I_0 e^{i\omega t})$

same as driver

$$I_0 = \underbrace{i\omega C}_{\text{like "1/R"}} \mathcal{E}_0$$

"Admittance"

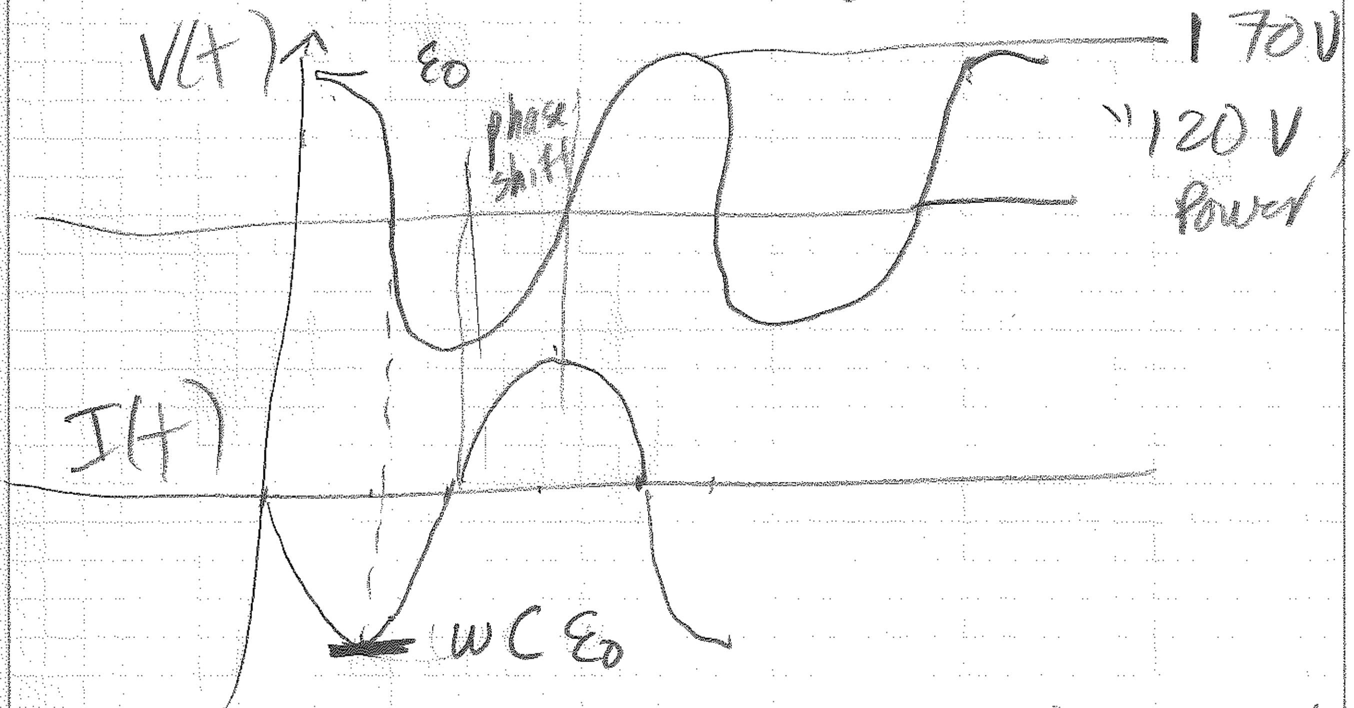
$$\mathcal{E}_0 = \frac{I_0}{i\omega C} = \underbrace{\frac{-i}{\omega C}}_{\text{"Impedance"}} I_0$$

What does the i mean in I_0?

means : $I(t) = \mathrm{Re}\left(i\omega C \mathcal{E}_0 e^{i\omega t}\right)$

$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$ $= \mathrm{Re}\left(i\omega C \mathcal{E}_0 \cos(\omega t) - \omega C \mathcal{E}_0 \sin(\omega t)\right)$

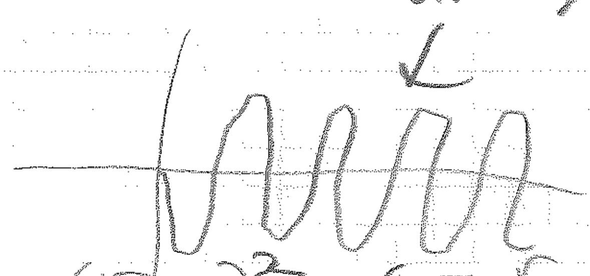
$I(t) = -\omega C \mathcal{E}_0 \sin(\omega t)$



Note $\underbrace{I(t)V(t)}$ $= -\omega C\mathcal{E}_0 \sin(\omega t) \cdot \mathcal{E}_0 \cos(\omega t)$

instant. power dissipated $= \dfrac{-\omega C\mathcal{E}_0^2}{2} \sin(2\omega t)$

averages to 0!

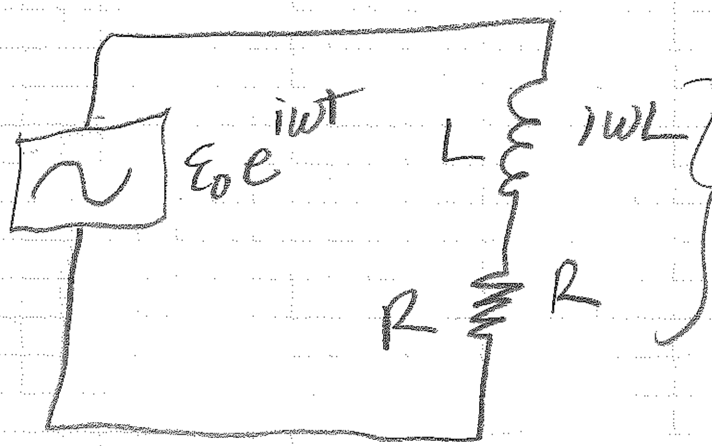


$\langle V^2 \rangle = \frac{1}{2}\mathcal{E}_0 = (120V)^2$, $\mathcal{E}_0 = \sqrt{2} \times 120 = 170V$

Inductor... too

Symbol	Admittance	Impedance
R	$1/R$	R
C	$i\omega C$	$-\frac{i}{\omega C}$
L	$-\frac{i}{\omega L}$	$i\omega L$

Fun...



$Z = R + i\omega L$

$$I_0 = \frac{\mathcal{E}_0}{R + i\omega L}$$

lossy

not Lossy

$$= \frac{\mathcal{E}_0 \cdot (R - i\omega L)}{R^2 + \omega^2 L^2}$$

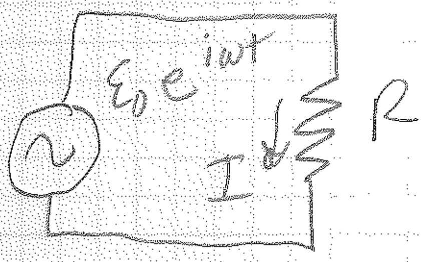
$$\langle IV \rangle = \frac{\mathcal{E}_0^2 R}{R^2 + \omega^2 L^2} \times \frac{1}{2} = \frac{V_{rms}^2}{R\left(1 + \omega^2 \frac{L^2}{R^2}\right)} \; !$$

Single Resistive Element

AC Voltage

$$I = \frac{1}{R} V = \frac{1}{R} \mathcal{E}_0 e^{-i\omega t}$$

real



Key: zoom in on the Power dissipation

$$\underbrace{I(t)\,V(t)}_{\text{uh-oh, not LINEAR!}} = \frac{1}{R}\mathcal{E}_0 \cos(\omega t)\, \mathcal{E}_0 \cos(\omega t)$$

$$= \underbrace{\frac{\mathcal{E}_0^2}{R} \cos^2(\omega t)}_{\text{now time average!}}$$

go back to REAL PARTs

$$\overline{P} = \frac{\mathcal{E}_0^2}{R} \times \frac{1}{2}$$

square root $\equiv V_{rms}$

$$V_{rms} = \frac{1}{\sqrt{2}} \mathcal{E}_0 = 120V$$

↑ U.S.

$$\mathcal{E}_0 = \sqrt{2} \cdot 120V$$
$$= 169.7V$$

"AC"

$$I_{rms} = \frac{V_{rms}}{R}$$

$$I_{rms} V_{rms} = \frac{V_{rms}^2}{R} = \overline{P}$$

THIS IS TRUE IN GENERAL ONLY FOR RESISTIVE ELEMENTS

<u>Generally</u> when $I = I_0 e^{i\omega t + \varphi}$

$$\overline{P} = I_{rms} V_{rms} \cos\varphi$$

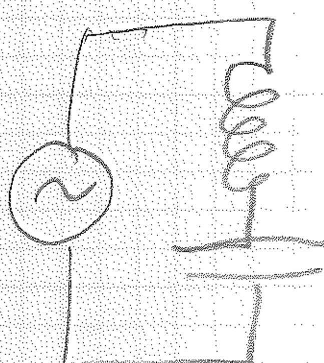
$\boxed{\text{Purely}}$ imaginary impedances give $\varphi = \pm \pi/2$

$+\pi/2$: Capacitor

$-\pi/2$: Inductor

and $\overline{P} = 0$.

<u>How about</u>

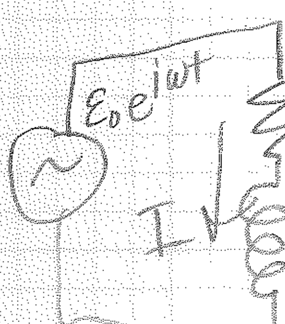


$$\left.\begin{array}{l} Z = i\omega L \\ c = -\dfrac{i}{\omega C} \end{array}\right\} \quad Z_{tot} = i\omega L - \frac{i}{\omega C}$$

= purely imaginary.

NO Power Dissipated.

$\boxed{8.2}$ (almost).



$$\left.\begin{array}{l} R = 2000\,\Omega \\ L = 10^{-6}\,H \end{array}\right\} \quad Z = R + i\omega L$$

$$I = \frac{\mathcal{E}_0 e^{i\omega t}}{R + i\omega L}$$

get out of denom

$$= \frac{\mathcal{E}_0 (R - i\omega L) e^{i\omega t}}{R^2 + \omega^2 L^2}$$

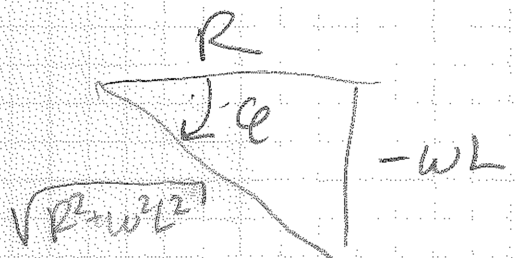
<u>go polar</u>: $R - i\omega L = \sqrt{R^2+\omega^2L^2}\, e^{i\tan^{-1}\left(\frac{-\omega L}{R}\right)}$

$$I = \frac{\varepsilon_0}{\sqrt{R^2+\omega^2L^2}}\, e^{i\omega t + i\tan^{-1}\left(\frac{-\omega L}{R}\right)}$$

(take real part)

$$I_{rms} = \frac{1}{\sqrt{2}} \frac{\varepsilon_0}{\sqrt{R^2+\omega^2L^2}} \qquad V_{rms} = \frac{1}{\sqrt{2}}\varepsilon_0$$

$$\cos\left(\tan^{-1}\left(\frac{-\omega L}{R}\right)\right)$$



$$\cos\varphi = \frac{R}{\sqrt{R^2+\omega^2L^2}}$$

$$\overline{P} = I_{rms} V_{rms} \cos\varphi$$

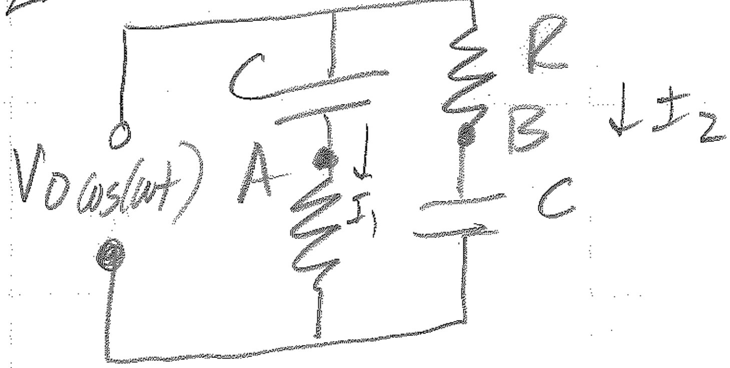
$$= \frac{1}{2} \frac{\varepsilon_0^2}{\sqrt{R^2+\omega^2L^2}} \frac{R}{\sqrt{R^2+\omega^2L^2}}$$

$R \to 0$, what

NO POWER DISSIPATED

<u>THE WEIRDNESS</u> ... $(V_{rms})_R + (V_{rms})_L \neq V_{rms}$

8.12



$$V_{AB} = V_B - V_A \qquad \text{show } |V_{AB}|^2 = V_0^2$$

for any ω

$$Z_1 = \frac{-i}{\omega C} + R \qquad Z_2 = R - \frac{i}{\omega C} = Z$$

$$I_1 = \frac{V_0 e^{i\omega t}}{\frac{-i}{\omega C} + R} \quad \rightleftarrows \text{ same!} \quad I_2 = \frac{V_0 e^{i\omega t}}{R - \frac{i}{\omega C}}$$

$$V_A = I_1 R \qquad V_B = I_2\left(\frac{-i}{\omega C}\right)$$

$$= \frac{R V_0 e^{i\omega t}}{\frac{-i}{\omega C} + R} \qquad V_B = \frac{\frac{-i}{\omega C} V_0 e^{i\omega t}}{R - \frac{i}{\omega C}}$$

$$V_B - V_A = \frac{\left(\frac{-i}{\omega C} - R\right)}{\left(R - \frac{i}{\omega C}\right)} V_0 e^{i\omega t}$$

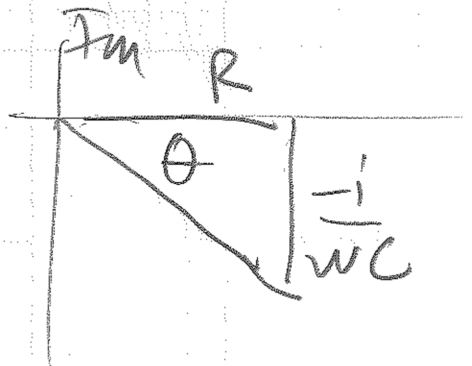
$$= \frac{-\left(R + \frac{i}{\omega C}\right)}{\left(R - \frac{i}{\omega C}\right)} V_0 e^{i\omega t}$$

$$= -\frac{Z^*}{Z} V_0 e^{i\omega t}$$

"a pure phase" $\Rightarrow \left|\frac{Z^*}{Z}\right| = \frac{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$

$$= \underline{1}$$

$$-\frac{Z^*}{Z} = \frac{-e^{-i\theta}}{e^{i\theta}} = -e^{-2i\theta}$$



$$\theta = \tan^{-1}\left(\frac{-1}{\omega C R}\right)$$

Really, $Re(V_B - V_A)$

$$= Re\left[-V_0 e^{i(\omega t - 2\theta)}\right]$$

$$Re(V_B - V_A) = -V_0 \cos\left(\omega t - 2\tan^{-1}\left(\frac{1}{\omega C R}\right)\right)$$

$$2\tan^{-1}\left(\frac{1}{wCR}\right) = \frac{\pi}{2} = 90^\circ$$

$$\tan^{-1}\left(\frac{1}{wCR}\right) = \frac{\pi}{4} = 45^\circ$$

$$wCR = 1$$

$$w = \frac{1}{RC}$$

Electromagnetic Equations

$$\mathrm{div}\,\vec{E} = 4\pi\rho \qquad \mathrm{div}\,\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\rho = \frac{\text{local charge}}{\text{volume}}$$

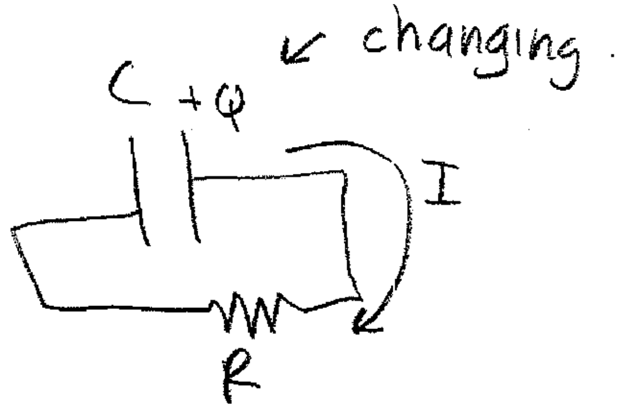
$$\underbrace{\mathrm{div}\,\vec{J} = -\frac{\partial \rho}{\partial t}}_{\text{always true.}} \qquad \vec{J} = \frac{\text{charge}}{\text{area time}} \times (\widehat{\text{direction}})$$

when $\vec{J}$ is constant in time $\left(\frac{\partial \vec{J}}{\partial t} = 0\right)$

$$\mathrm{curl}\,\vec{B} = \frac{4\pi}{c}\vec{J} \qquad \mathrm{curl}\,\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix}$$

Contradiction

suppose $\frac{\partial \rho}{\partial t} \neq 0$



then $\mathrm{div}\,\vec{J} = -\frac{\partial \rho}{\partial t} \neq 0$

but... $\mathrm{div}\,(\underbrace{\mathrm{curl}\,\vec{B}}_{\text{no sources}}) = 0 = \frac{4\pi}{c}\,\mathrm{div}\,\vec{J}$

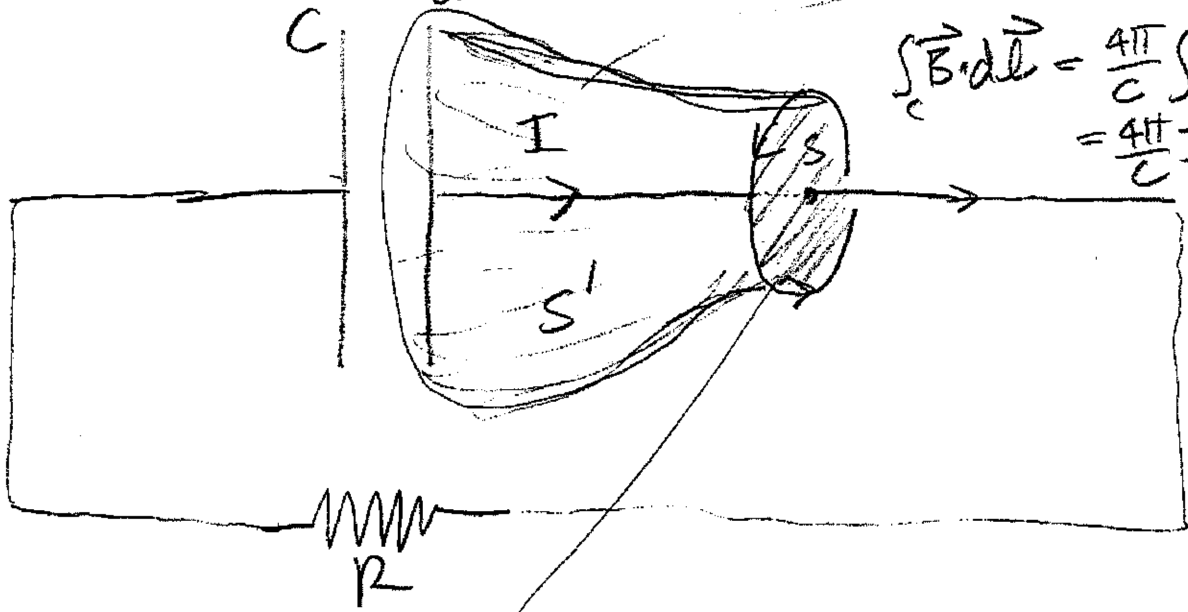
That's the contradiction.

Integral Statement

$Q \to Q = Q_0 e^{-t/RC} \to I = \frac{Q_0}{RC} e^{-t/RC}$

$\int_c \vec{B}\cdot d\vec{l} = \frac{4\pi}{c}\int_s \vec{J}\cdot d\vec{A}$

$= \frac{4\pi}{c} I$



but that is not the only surface one could use!

$$\int_{S'} \vec{J}\cdot dA = 0 \quad !$$

so, $\quad \text{curl}\,\vec{B} = \frac{4\pi}{c}\vec{J} + \left(\begin{matrix}\text{new}\\ \text{term}\end{matrix}\right)$

$\uparrow$ some $\frac{\partial(\;)}{\partial t}$

Look at $\quad \text{curl}\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$

recall symmetry of $\vec{E}, \vec{B}$ in "Lorentz" transformation

$$\vec{E}_{\parallel}' = \vec{E}_{\parallel} \qquad \vec{E}_{\perp}' = \gamma(\vec{E}_{\perp} + \vec{\beta} \times \vec{B}_{\perp})$$
$$\vec{B}_{\parallel}' = \vec{B}_{\parallel} \qquad \vec{B}_{\perp}' = \gamma(\vec{B}_{\perp} - \vec{\beta} \times \vec{E}_{\perp})$$

– sign.

$$\text{curl}\,\vec{B} = \frac{4\pi}{c}\vec{J} + \underbrace{\frac{1}{c}\frac{\partial E}{\partial t}}$$

changing $\vec{E}$ field, integrated across surface, gives a "circular" $\vec{B}$ (like E.M.F.)

"LENZ" ⇒ other way.

$$\text{curl}\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\text{curl}\,\vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

$$\text{div}\,\vec{E} = 4\pi\rho$$

$$\text{div}\,\vec{B} = 0$$

Maxwell's Equations (CGS)

MKS/SI:

$$\text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{curl}\,\vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

$$\text{div}\,\vec{E} = \varepsilon_0 \rho$$

$$\text{div}\,\vec{B} = 0 \qquad \mu_0 \varepsilon_0 = \frac{1}{c^2}$$