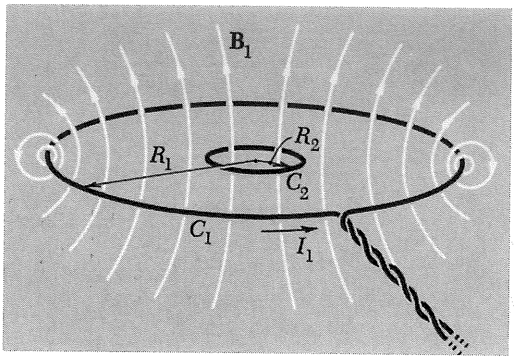


FIGURE 7.20

Current I_1 in ring C_1 causes a field $\mathbf{B}_1$ which is approximately uniform over the region of the small ring C_2.

#1s :

$$\mathcal{E}_{max} = -\frac{1}{3 \cdot 10^{10} \frac{cm}{\cancel{s}}} \cdot \overset{\text{gauss} \cdot cm^2 \; \text{cm}}{\overset{\downarrow}{15{,}700}} \cdot 377 \frac{1}{\cancel{s}}$$

$= 0.000193$ statvolts

$\times 300 = 0.0592$ volts

$= 59.2$ millivolts

want volts?

$$= \frac{3 \cdot 10^2}{3 \cdot 10^{10}} \cdot \frac{d\Phi}{dt} = 10^{-8} \frac{d\Phi}{dt}$$

Gauss, cm^2

Mutual Inductance

$V = \dfrac{Q}{C}$ Capacitance.

Self Capacitance.

Q

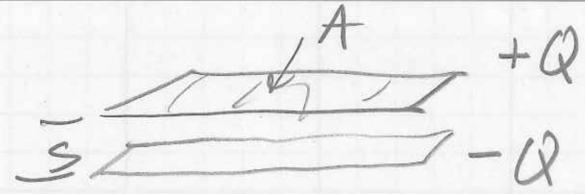
$E_r = \dfrac{Q}{r^2}$ $r > a$

$V = \dfrac{Q}{a}$ $C = a$

$(V = 0$ at $\infty)$

Mutual:

net $Q = 0$



$$C = \frac{A}{4\pi s}$$

Capacitance $\Longleftrightarrow$ geometry of situation.

Inductance: start Mutual.

Figure 7.19

$\mathcal{E}_{21} \rightarrow$ "voltage induced in C_2 due to C_1"

How? Φ_{21}, note $\propto I_1$

$\uparrow$

depends on Geometry!

$$\frac{\Phi_{21}}{I_1} = M_{21}\, c$$

$\uparrow$ mutual inductance $\uparrow$ speed of light.

$$\mathcal{E}_{21} = -\frac{1}{c}\frac{d\Phi_{21}}{dt}$$

$$= -\frac{1}{c}\frac{d}{dt}\left(M_{21} c I_1\right)$$

$$\mathcal{E}_{21} = -M_{21}\frac{dI_1}{dt}$$

Let's do one: Fig 7-20

$$B_1 \cong \frac{2\pi I_1}{cR_1}$$

$$\Phi_{21} = \pi R_2^2 \frac{2\pi I_1}{cR_1}$$

$M_{21}\, c$

$$M_{21} = \frac{2\pi^2 R_2^2}{c^2 R_1} \Bigg] \rightarrow \text{units?} \quad \frac{cm^2}{\frac{cm^2}{s^2}\, cm} = \frac{s^2}{cm}$$

<u>MKS</u>: Henry

$$1 \text{ henry} = \frac{1}{9 \cdot 10^{11}} \frac{s^2}{cm}$$

$$\mathcal{E}_{21} = -M_{21} \frac{dI_1}{dt}$$

I_1 not good enough!

<u>need</u> $\frac{dI_1}{dt}$

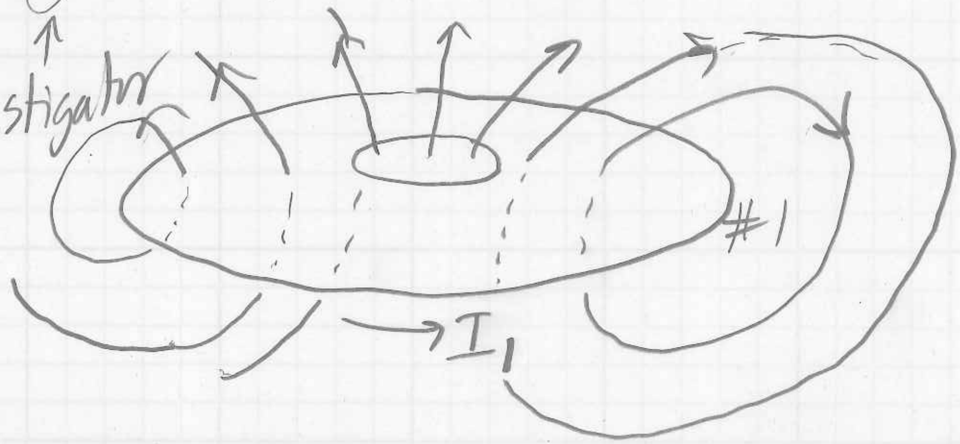
Reciprocity:

$$\mathcal{E}_{12} = -M_{12} \frac{dI_2}{dt}, \quad \underline{M_{12} = M_{21}}!!$$

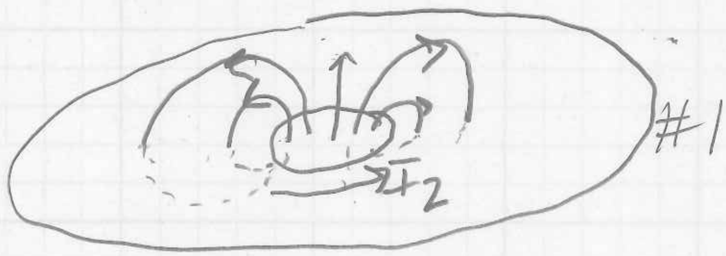
prove with $\vec{A}$

$M_{21} \propto \frac{R_2^2}{R_1}$ EASY:

↑ Instigator



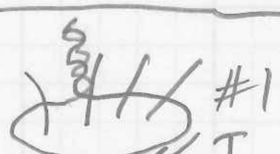
M_{12}



both + (inside) and − (outside).

why does M_{12} decrease as R_1 increases

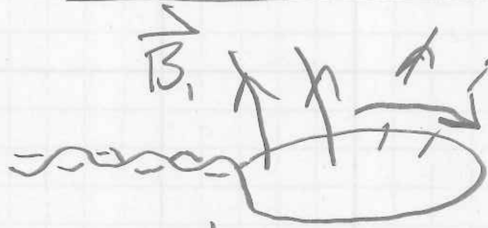
Self-Inductance



$I=0 \longrightarrow I(t)$ $\qquad \frac{dI}{dt} > 0$!

↓ makes Φ !

Self Inductance



loop

$\vec{I}_1(t)$ will create $\vec{B}_1(t)$, which will create flux $\Phi_{11}(t)$ through the loop...

when $\frac{d\Phi_{11}}{dt} \neq 0$, the loop (or more complex object) "resists" the change... a <u>voltage</u> is induced,

speed of light

$$\mathcal{E}_{11} = -\frac{1}{c}\frac{d\Phi_{11}}{dt}, \quad \Phi_{11} = L_{11}\, c\, I_1$$

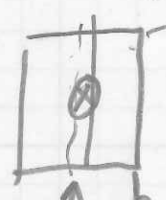
"resists"

LENZ

defines SELF INDUCTANCE

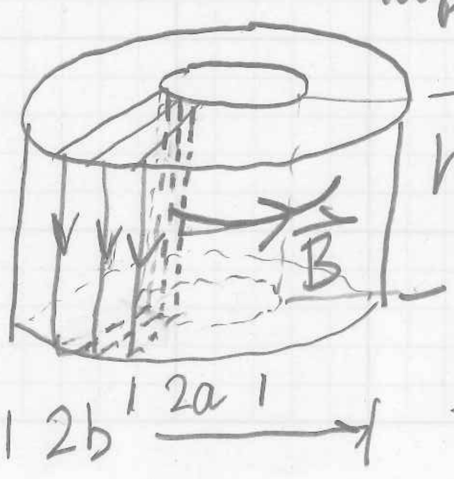
$$\mathcal{E}_{11} = -L_{11}\frac{dI_1}{dt}$$

<u>Classic</u>: one loop



$$B(r) = \frac{2NI}{cr}$$

$$\Phi_1 = h\int_a^b \frac{2NI}{cr}\,dr$$

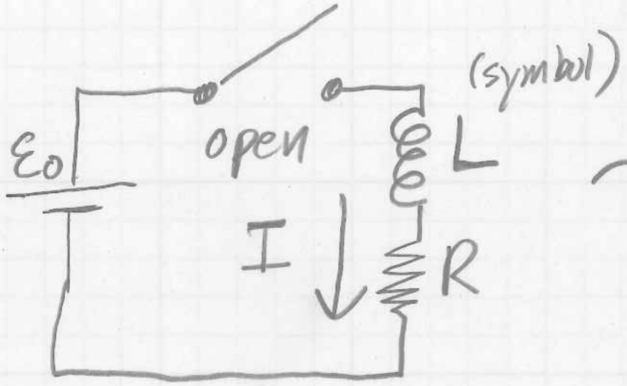


$$\Phi_1 = \frac{2NIh}{c}\ln(b/a)$$

$$\Phi_N = N\Phi_1 = \frac{2N^2Ih}{c}\ln\left(\frac{b}{a}\right), \quad \boxed{L = \frac{2N^2h}{c^2}\ln\left(\frac{b}{a}\right)}$$

Inductors in circuits usually use their self inductance, and the "oppose change"

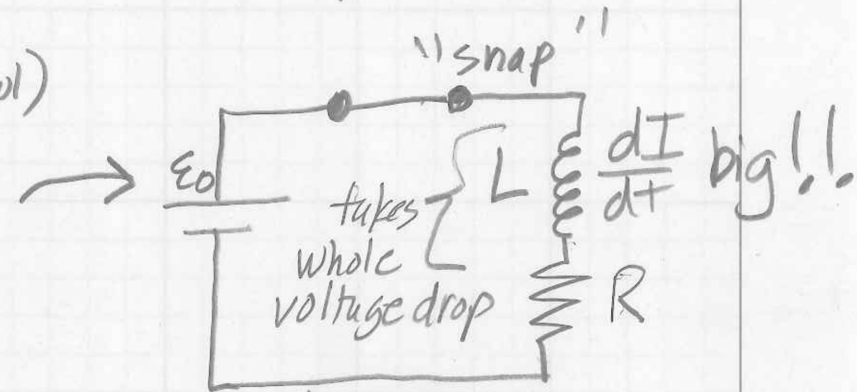
Switch S: open $-\infty < t < 0$
closes $t=0$, stays closed $t \to \infty$



$-\infty < t < 0$

NO CURRENT

$I = 0$



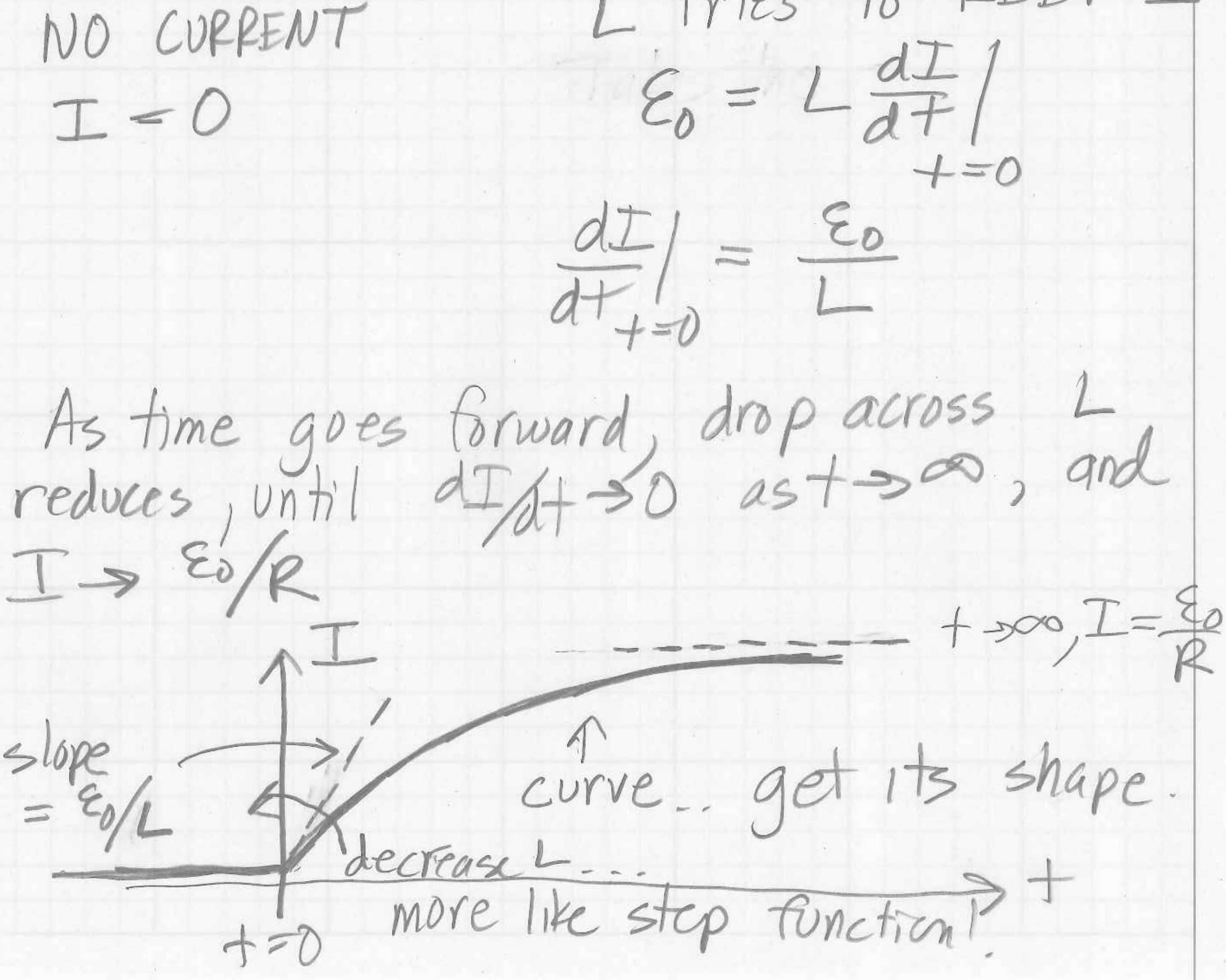
$t=0$

L tries to KEEP $I=0$

$$\varepsilon_0 = L \left.\frac{dI}{dt}\right|_{t=0}$$

$$\left.\frac{dI}{dt}\right|_{t=0} = \frac{\varepsilon_0}{L}$$

As time goes forward, drop across L reduces, until $dI/dt \to 0$ as $t \to \infty$, and $I \to \varepsilon_0/R$



general: $\mathcal{E}_0 = L\frac{dI}{dt} + RI$

"particular solution" $I_p =$ constant so $\frac{dI_p}{dt} = 0$

↑ what value? solves

$$I_p = \frac{\mathcal{E}_0}{R}$$

"homogenous solution" $0 = L\frac{dI_H}{dt} + RI_H$

$$\frac{dI_H}{I_H} = -\frac{R}{L}dt$$

$$\ln I_H = -\frac{R}{L}t + \text{constant}$$

$$I_H = (\text{constant})\, e^{-\frac{R}{L}t}$$

↑ determine with initial conditions.

General Solution = particular + homogenous

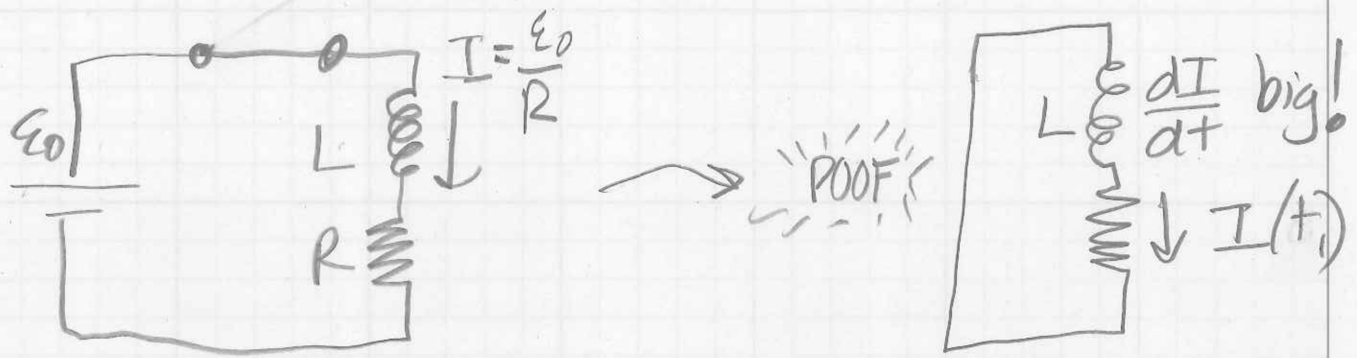
$$I(t) = \frac{\mathcal{E}_0}{R} + (\text{constant})\, e^{-\frac{R}{L}t}$$

$I(0) = 0 = \frac{\mathcal{E}_0}{R} + \text{constant}\, e^{-0}$ 1 $\text{const} = -\frac{\mathcal{E}_0}{R}$

$$\boxed{I(t) = \frac{\mathcal{E}_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

you check: $\left.\frac{dI}{dt}\right|_{t=0}$ actuall = $\frac{\mathcal{E}_0}{L}$

Now imagine a NEW, SUBSEQUENT initial condition... the $t=\infty$ situation of the previous. Imagine at a <u>new</u> $t=t_1$, shorting out voltage source + removing it



$-\infty < t < 0$

$I = \varepsilon_0/R$

$t=t_1$

$$L\frac{dI}{dt} + RI = 0$$

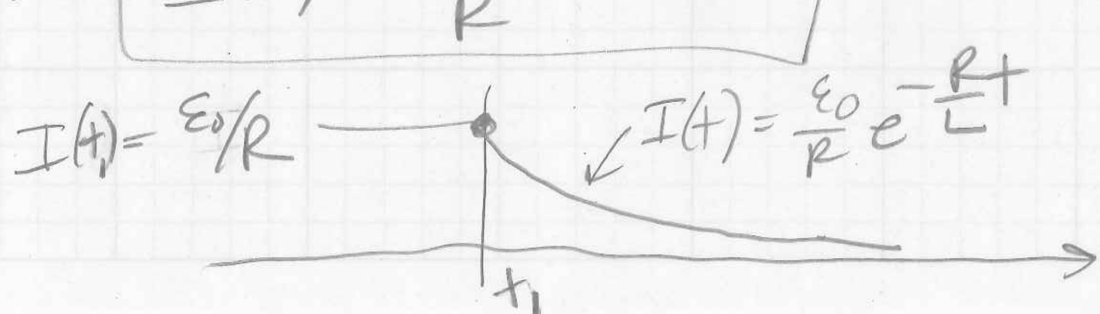
$$\left.\frac{dI}{dt}\right|_{t=t_1} = -\frac{\cancel{R}}{L} \times \frac{\varepsilon_0}{\cancel{R}} = -\frac{\varepsilon_0}{L}$$

homogenous

$$I = (\text{constant})\, e^{-\frac{R}{L}t}$$

$$I(t_1) = \frac{\varepsilon_0}{R} = (\text{constant})\, e^{-\frac{R}{L}t_1} \therefore \text{constant} = \frac{\varepsilon_0}{R} e^{\frac{R}{L}t_1}$$

so $$\boxed{I(t) = \frac{\varepsilon_0}{R} e^{-\frac{R}{L}(t-t_1)}}$$

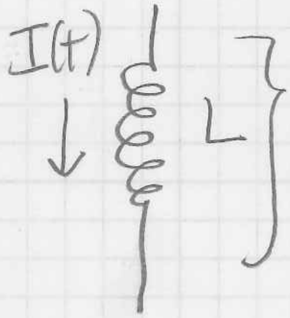


Energy:

Energy Density in $\vec{E}$ field $= \dfrac{E^2}{8\pi}$

$\Rightarrow$ in our units, in $\vec{B}$ field $= B^2/8\pi$

Portray this:



$V = -L\dfrac{dI}{dt}$

to push the current across the voltage, power

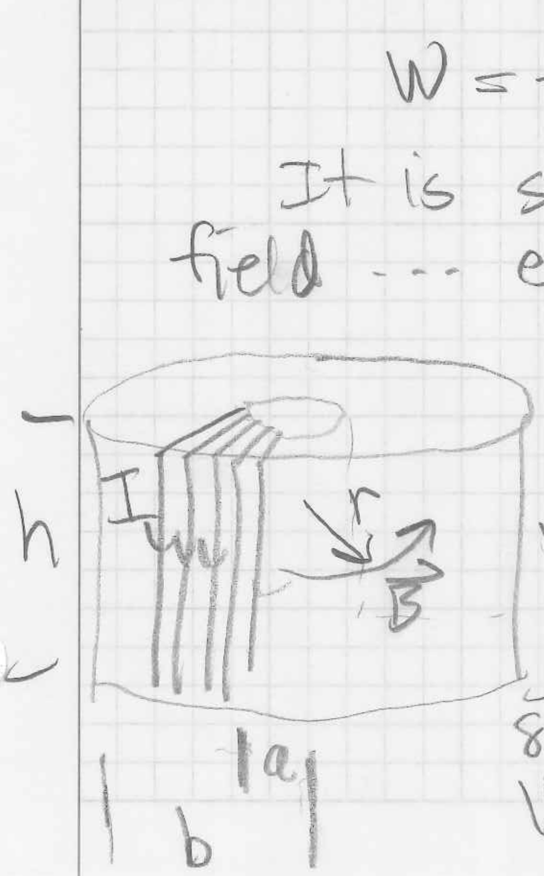
$IV = L\dfrac{dI}{dt}I$ is consumed

$$W = \int_0^t L\frac{dI}{dt} I\,dt = L\int_0^t \frac{1}{2}\frac{d}{dt}(I^2)\,dt$$

$W = \frac{1}{2}LI^2$ = energy stored

It is stored in the magnetic field ... example...

$$L = \frac{2N^2}{c^2}h\ln(b/a)$$

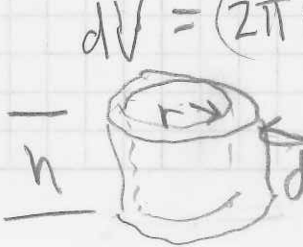


$$B = \frac{2NI}{cr}$$

"Just compute"

$$\frac{1}{8\pi}\int_{\text{Volume}} B^2\,dV$$

$dV = (2\pi r\,dr)\cdot h$ cylindrical shell



$$B^2 = \frac{4N^2I^2}{c^2r^2}$$

$$\frac{1}{8\pi}\int_{\text{Volume}} B^2\,dV = \frac{1}{8\pi}\int_a^b \frac{4N^2I^2}{c^2r^2}\,\cancel{2\pi}\,r\,dr\cdot h$$

$$= \frac{N^2I^2}{c^2}h\int_a^b \frac{dr}{r}$$

$$= \frac{N^2I^2h}{c^2}\ln(b/a)$$

$$= \frac{1}{2}\cdot\left(\frac{2N^2h}{c^2}\ln\left(\frac{b}{a}\right)\right)I^2$$

$$= \frac{1}{2}LI^2$$