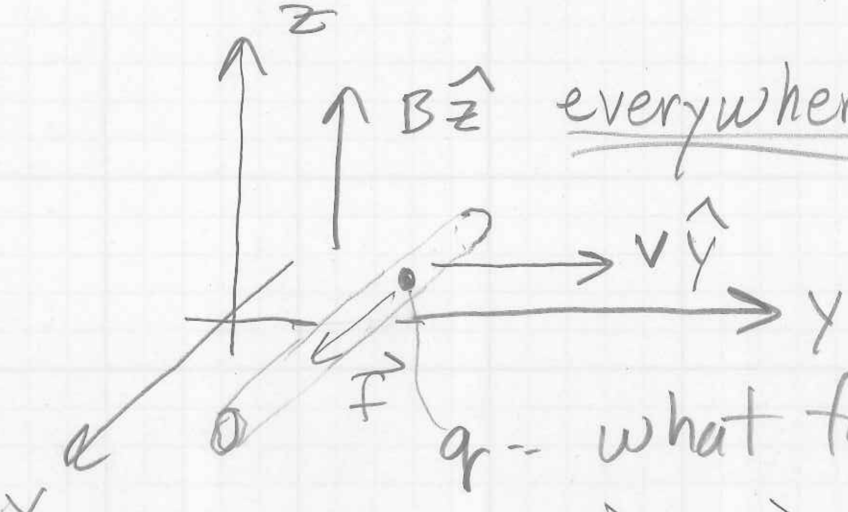
Conducting bar in uniform $\vec{B}$ field



$B\hat{z}$ everywhere $\vec{E} = 0$

$$\vec{f} = q\frac{\vec{v}}{c} \times \vec{B}$$

$$\vec{f} = \frac{qvB}{c}\hat{x}$$

$\vec{E}_i = -\frac{vB}{c}\hat{x}$

What happens now? +++ on one end (+x), ---- on other (−x)

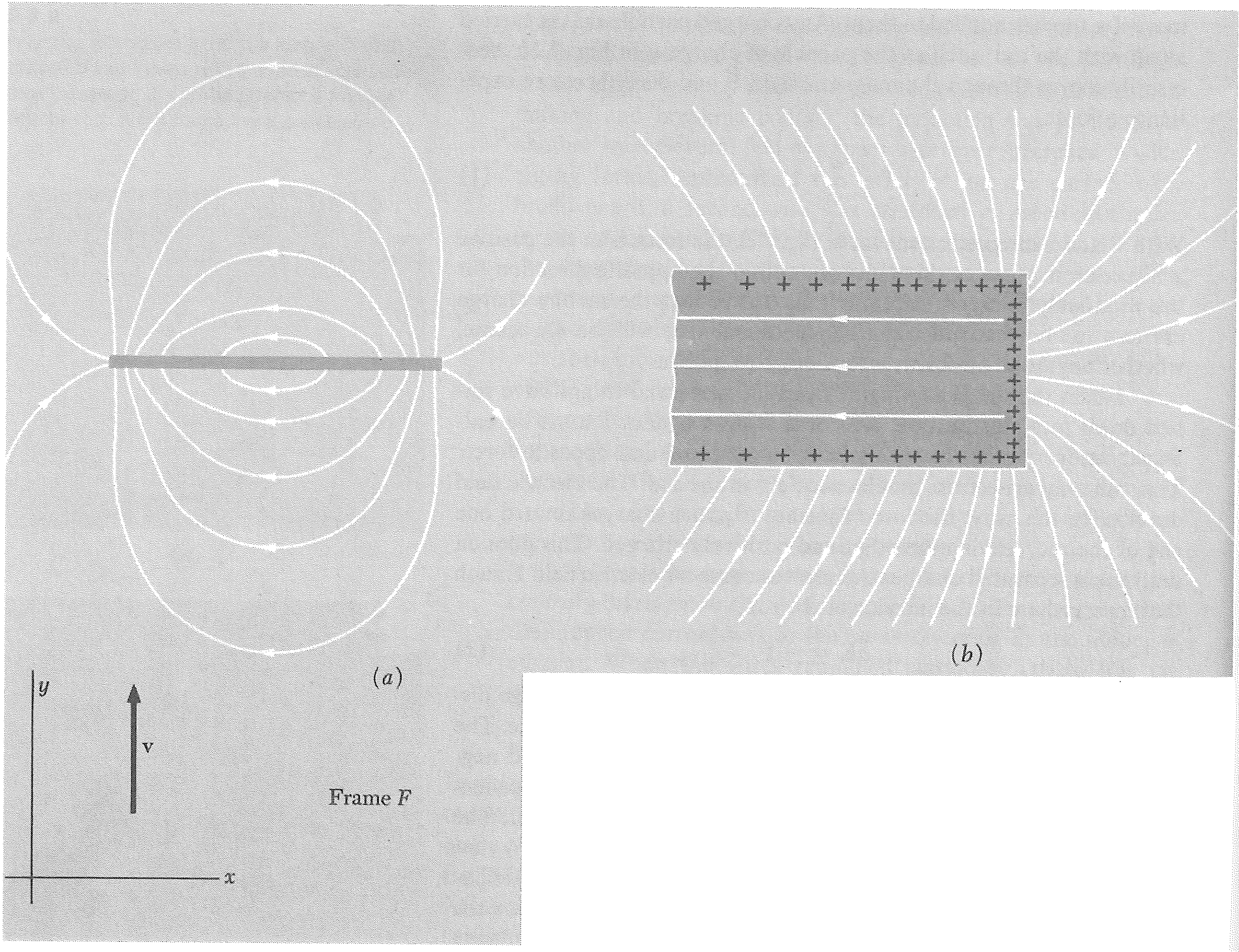
Ends of rods charge up until ... current flow stops ...

How much time? Depends on R, C of rod!

Look at Figure 7.3 ... key points ...

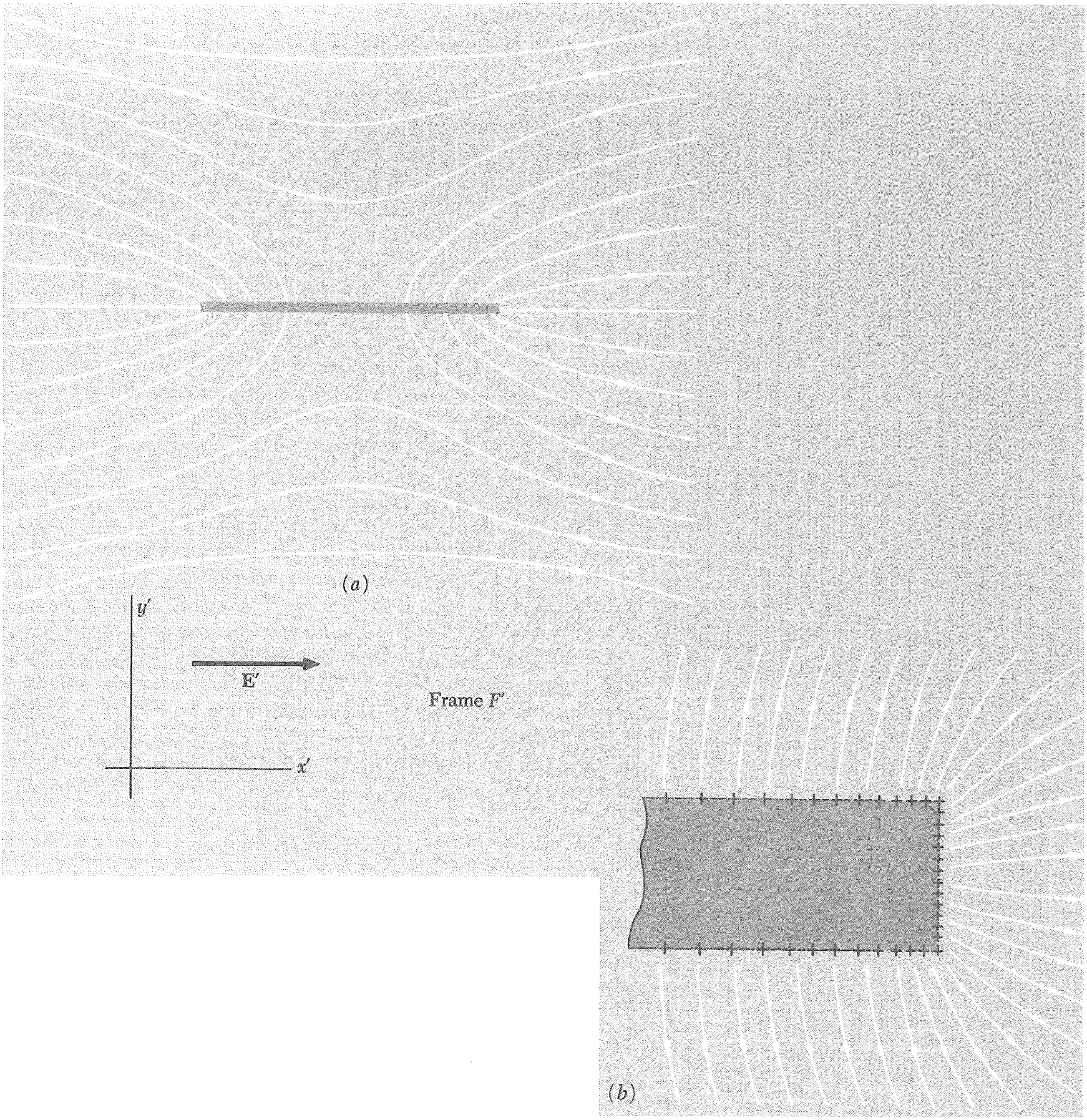
Direction • $\vec{E}$ in a conductor !!! $\vec{F} = 0$, not $\vec{E}$!

• Density of field lines



(a)

(b)



(a)

(b)

What happens in bar's rest frame?

$\vec{v}' = 0$ so no magnetic force

$\therefore q\vec{E}' = 0$

$\therefore \vec{E}' = 0$!!!

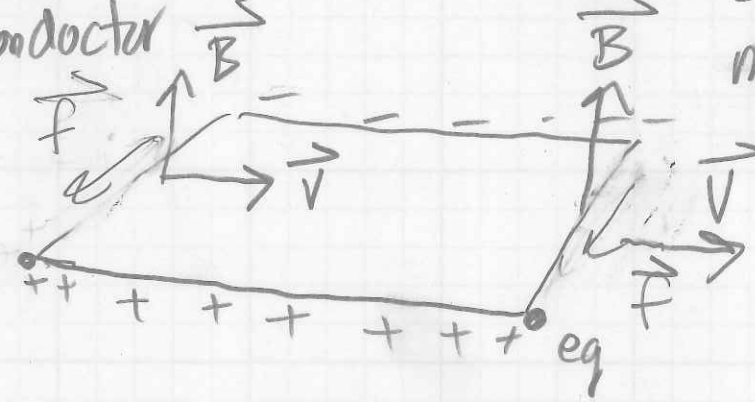
HOW? $\vec{E}'_\perp = \gamma(\vec{E}_\perp + \vec{\beta} \times \vec{B}_\perp)$

$\uparrow$

$-\frac{vB}{c}\hat{x}$ $\quad \frac{v}{c}B\hat{x}$ $= 0$!

Figure 7.4

LOOP! of conductor . at rest... $\oint_{LOOP} \vec{f} \cdot d\vec{s}$

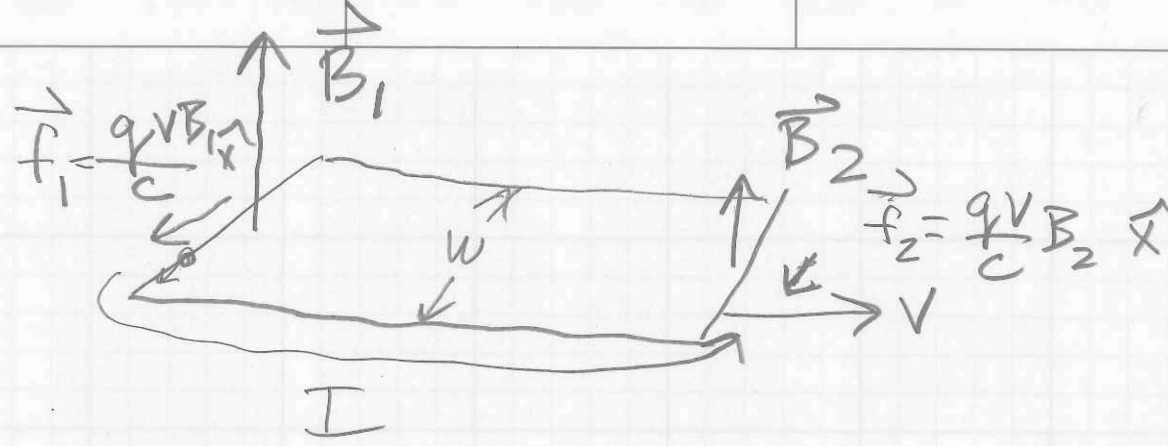


moving in ... UNIFORM $\vec{B}$

$\oint_{LOOP} \vec{f} \cdot d\vec{s} = 0$!

BUT NOW TWO $\vec{f}$'s cancel!

GET $\vec{B}$ to ... change..



$$\int_{LOOP} \vec{f}\cdot d\vec{s} = \frac{qv}{c}(B_1 - B_2)w$$

$$\mathcal{E} \equiv \frac{1}{q}\int_{LOOP} \vec{f}\cdot d\vec{s}$$ "electro motive force"

in this frame, $\int_{Loop} \vec{E}\cdot d\vec{s} = 0$

but .... $\int_{Loop} \vec{E}'\cdot ds = 0$ Loop rest frame ...

↓ 44

Makes: current flow in loop...

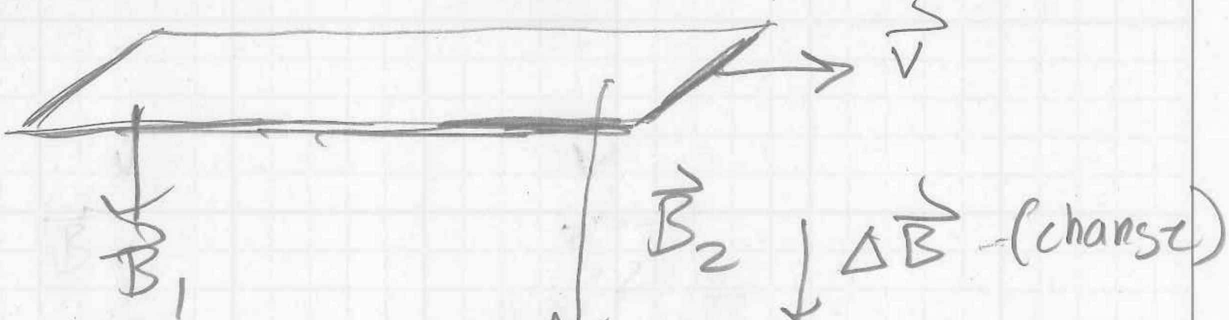
I: what is I trying to do?

$\Rightarrow$ INCREASE $\vec{B}$, as $\vec{B}$ as seen by the loop DECREASING.

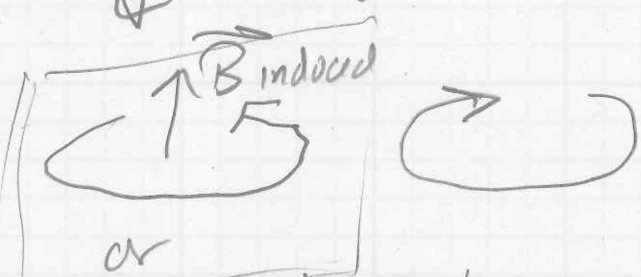
LENZ's LAW .. induced currents try to re-establish $\vec{B}$

"Ampere" Like Relationship

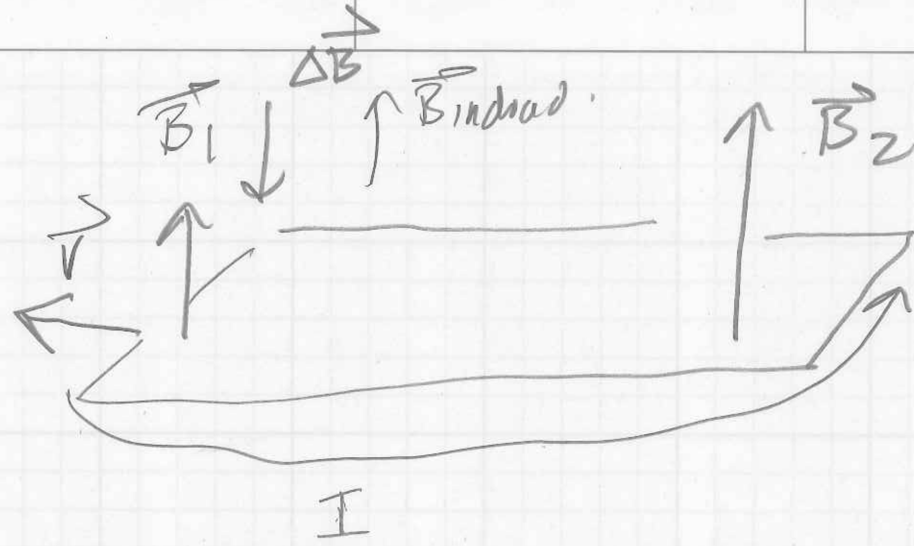
	"Penetrator"	"In the Line integral"
Ampere	Current ($\vec{J}$)	$\vec{B}$
Faraday's Law of induction	$\vec{B}$	Current ($\vec{J}$)



. $I \neq 0$ Vote:



LENZ: induced current makes a new field that opposes the change.



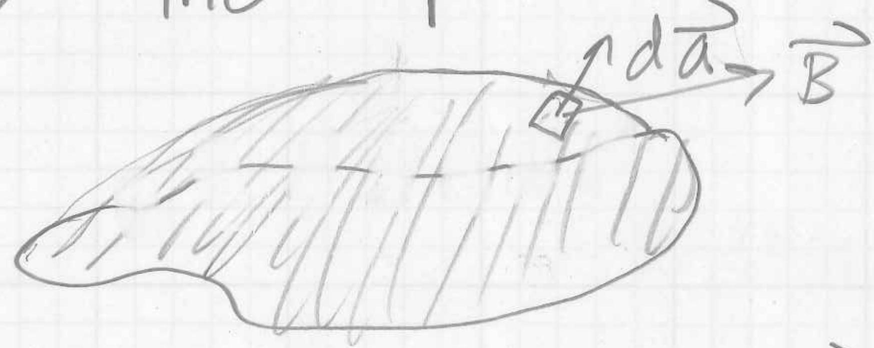
How do we quantify this? (42)

$$\mathcal{E} = \frac{vw}{c}(B_1 - B_2)$$

In general...

$$\mathcal{E} = -\frac{1}{c}\frac{d\Phi}{dt} \qquad \Phi = \text{Flux of } \vec{B}!$$

(1) ID The loop



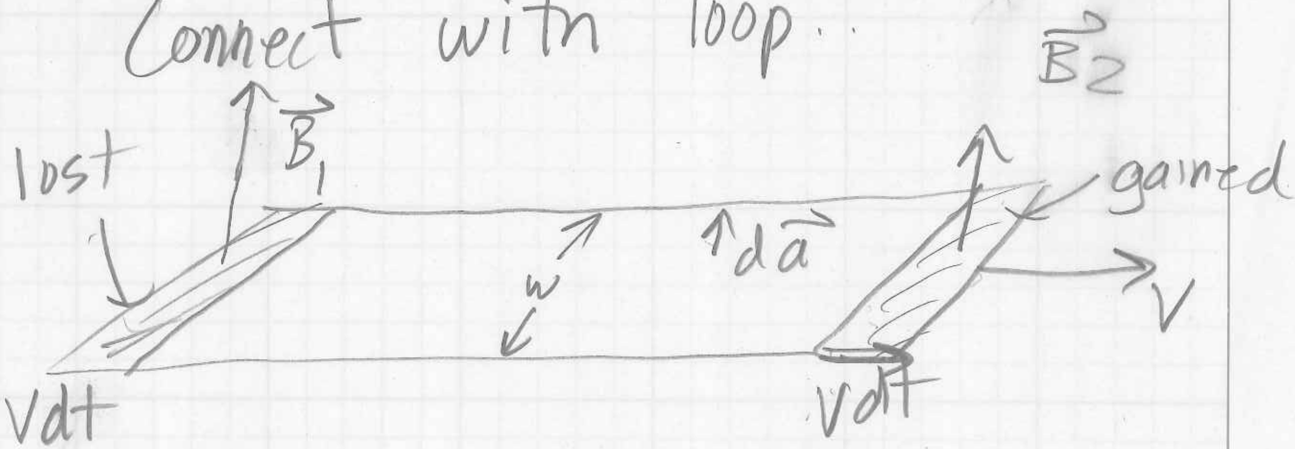
(2) surface (3) Compute $\int \vec{B} \cdot d\vec{a} = \Phi$

(4) Choice of surface... doesn't matter (Soap Bubble Videos).

(5) $\frac{d\Phi}{dt}$ matters!

(6) − sign is... "Lenz Law"

Connect with loop.

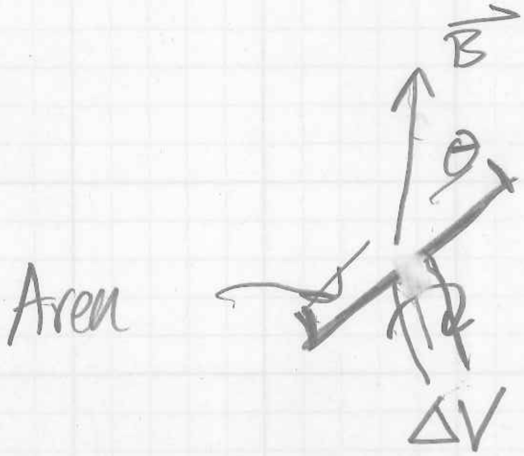


$$d\Phi = (B_2 v dt - B_1 v dt) w$$

$$\frac{1}{c}\frac{d\Phi}{dt} = \frac{v}{c} w (B_2 - B_1)$$

$$-\frac{1}{c}\frac{d\Phi}{dt} = \frac{vw}{c}(B_1 - B_2)$$

Another Classic : Fig 7.13



Φ biggest when $\theta = 90°$

$\theta = \omega t$

$$\Phi = SB \sin(\omega t + \alpha)$$

↑ $\alpha = 0$, actually.

$$\frac{d\Phi}{dt} = \omega SB \cos(\omega t + \alpha)$$

$$\Delta V = -\frac{1}{c}\frac{d\Phi}{dt} = \frac{-\omega SB}{c}\cos(\omega t + \alpha)$$

at what θ is ΔV biggest?

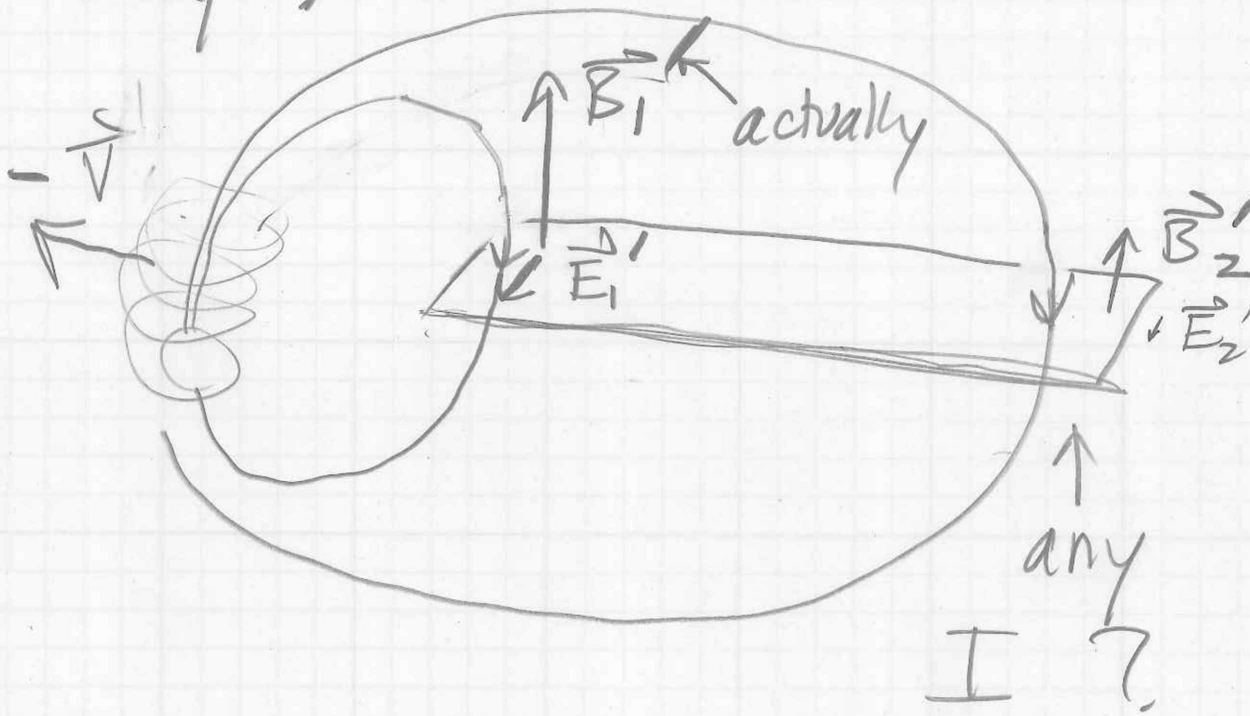
$\theta = 0$

$\theta = 90°$

$\theta = 45°$?

Which way?

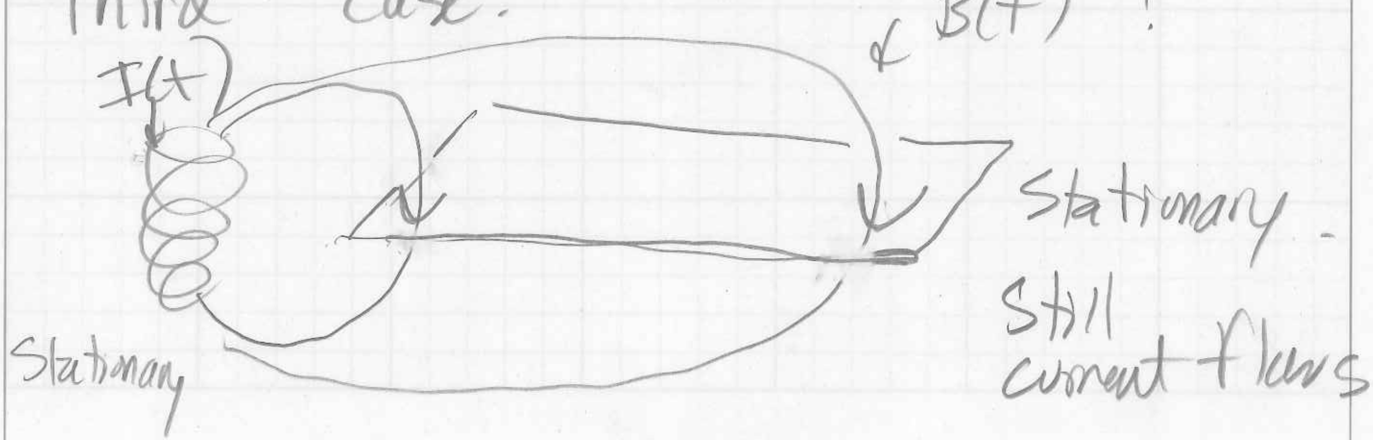
Is circuit moving the only way ?



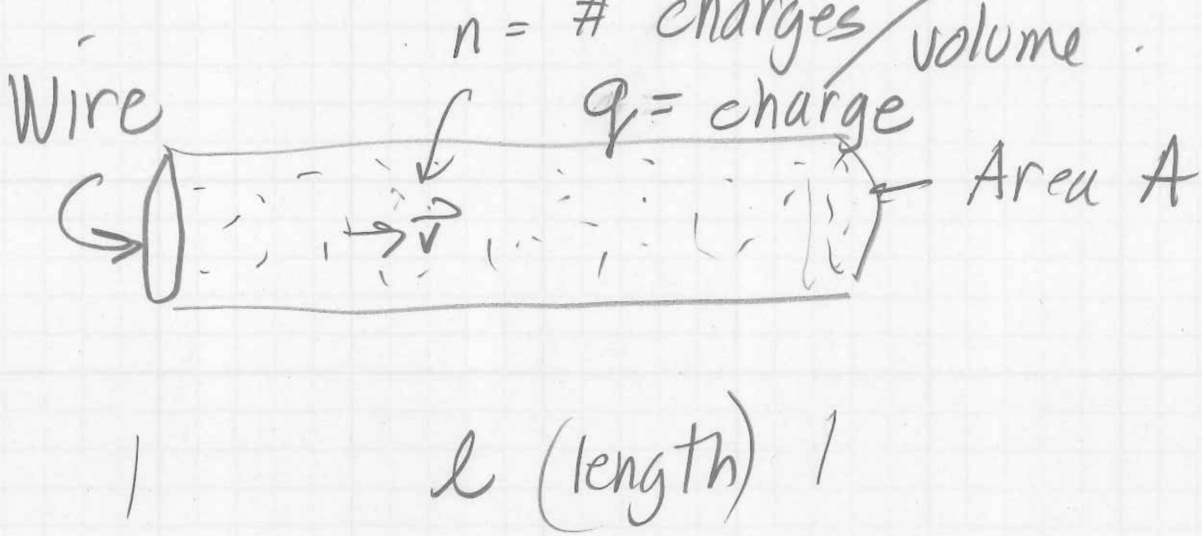
YES ... different Lorentz frame ...

in this frame $\int_C \vec{E}' \cdot d\vec{s} \neq 0$!

Third case:



Odd derivation... $Qv = I\ell$



$$Q = nq \cdot A \cdot \ell$$

$$Qv = nqA\ell v$$

$$= \underbrace{\underbrace{q(nv)}_{J} \cdot A}_{I} \cdot \ell$$

$$Qv = I\ell$$

General Law of Induction:

$$\oint_{Loop} \vec{E} \cdot d\vec{s} = \mathcal{E} = -\frac{1}{c}\frac{d}{dt}\int_S \vec{B} \cdot d\vec{a}$$

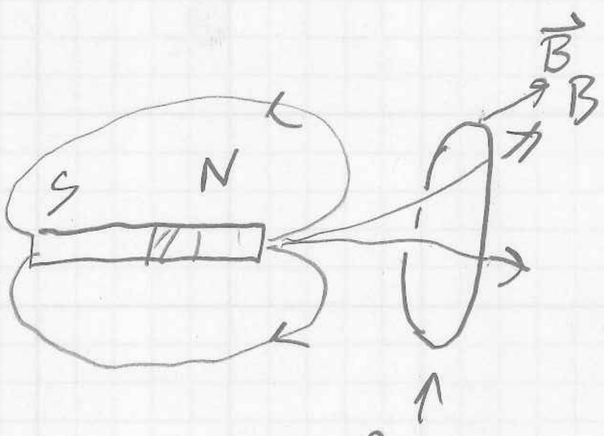
$$= -\frac{1}{c}\frac{d\Phi}{dt}$$

Stokes

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

(maxwell)

Forces from Lenz



move toward... ring should move away

↳ creates $\vec{B}_i$ that cancels...

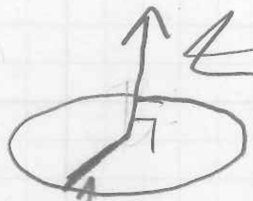
need $\frac{d\Phi}{dt}$ to get I in ring

Fig 7.16 of Purcell...

Current: $I = I_0 \sin(\overbrace{2\pi \cdot 60}^{377} \cdot t)$

2π ← f to ω

60 ← 60 cycle



$B_{max} = 50\ G$, $B = B_{max} \times \sin(377t)$

⊥ to loop.

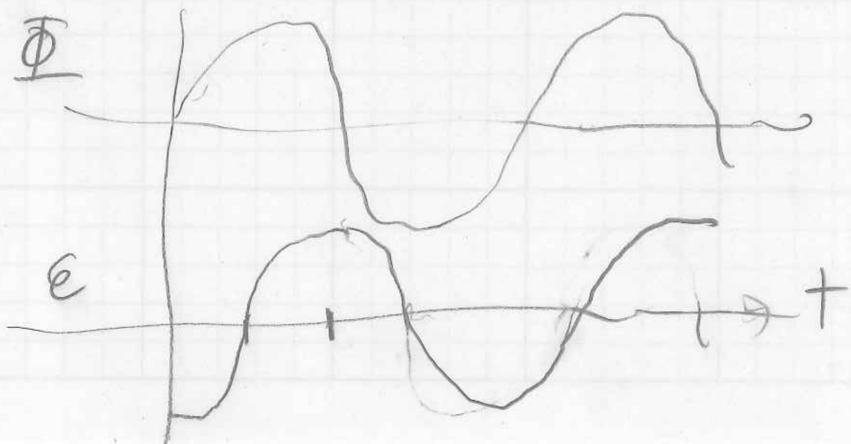
$$\Phi = \pi r^2 B$$

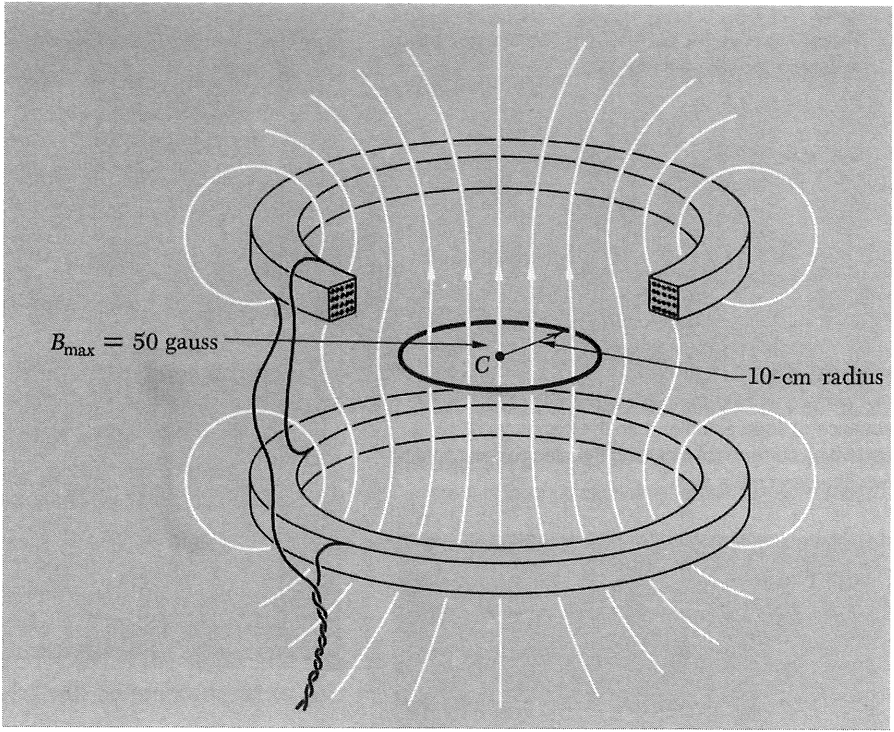
$$= \pi (10)^2\, 50 \sin(377t)$$

$$\Phi = 15{,}700 \sin(377t)$$

$$\mathcal{E} = -\frac{1}{c} \frac{d\Phi}{dt}$$

$$= -\frac{1}{c}\, 15{,}700 \cdot 377 \cos(377t)$$





$B_{max} = 50$ gauss
C
10-cm radius

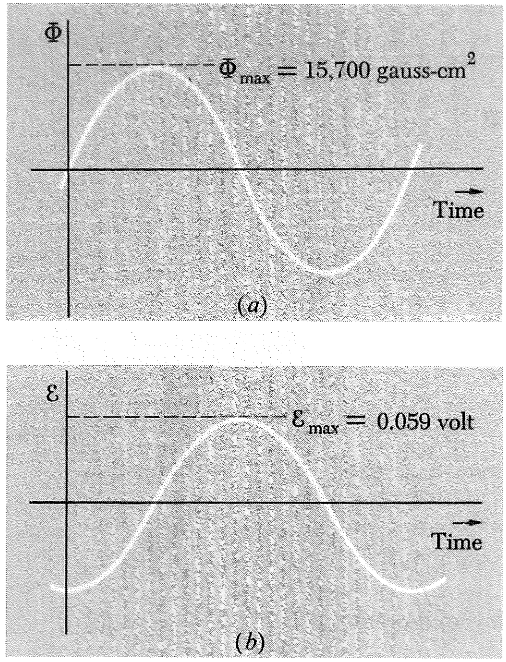


FIGURE 7.17
(*a*) The flux through the circle *C*. (*b*) The electromotive force associated with the path *C*.

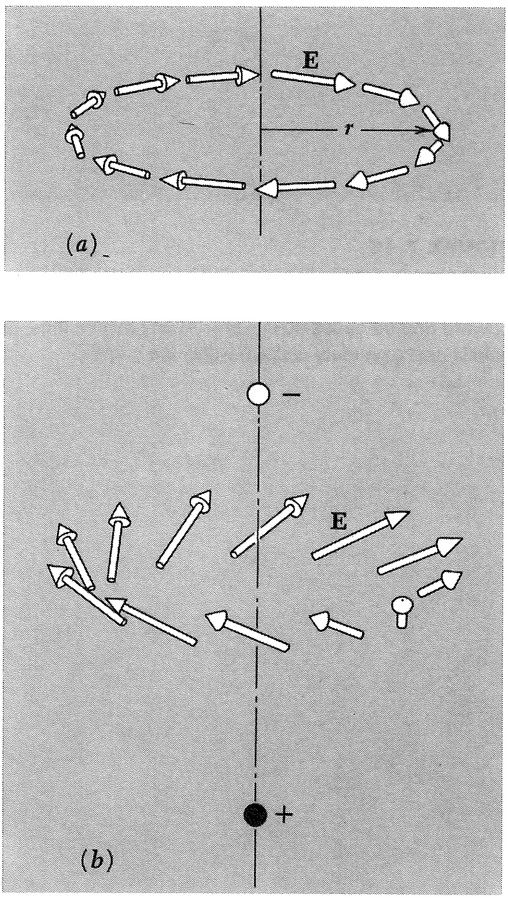
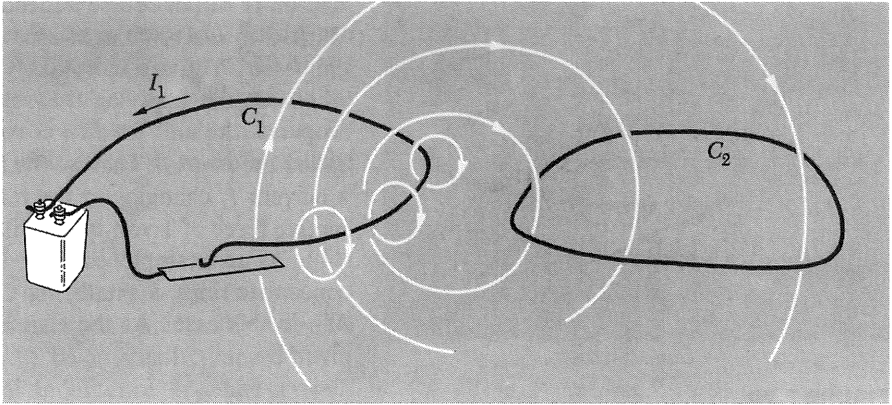


FIGURE 7.18
The electric field on the circular path *C*. (*a*) In the absence of sources other than the symmetrical, oscillating current. (*b*) Including the electrostatic field of two charges on the axis.

E from a knowledge of curl **E** alone. However, our path C is here a circle around the center of a symmetrical system. *If there are no other* electric fields around, we may assume that, on the circle C, **E** lies in that plane and has a constant magnitude. Then it is a trivial matter to predict its magnitude, since $\int_C \mathbf{E} \cdot d\mathbf{s} = 2\pi r E = \mathcal{E}$, which we have already calculated. In this case, the electric field on the circle might look like Fig. 7.18*a* at a particular instant. But if there are other field sources, it could look quite different. If there happened to be a positive and a negative charge located on the axis as shown in Fig. 7.18*b*, the electric field in the vicinity of the circle would be the superposition of the electrostatic field of the two charges and the induced electric field.

MUTUAL INDUCTANCE

7.6 Two circuits, or loops, C_1 and C_2 are fixed in position relative to one another (Fig. 7.19). By some means, such as a battery and a variable resistance, a controllable current I_1 is caused to flow in circuit C_1. Let $\mathbf{B}_1(x, y, z)$ be the magnetic field that would exist if the current



I_1
C_1
C_2