In the end ...

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J}$$

try it! $\vec{B} = B_\theta \underset{(-\hat{x}\sin\theta + \hat{y}\cos\theta)}{\hat{\theta}}$ $\boxed{r < D/2}$

$$= -\frac{8I}{c}\frac{\sqrt{x^2+y^2}}{D^2}\left(-\hat{x}\frac{y}{\sqrt{x^2+y^2}} + \hat{y}\frac{x}{\sqrt{x^2+y^2}}\right)$$

$$= \frac{-8I}{c}\frac{1}{D^2}\left(-y\hat{x} + x\hat{y}\right)$$

$$\vec{\nabla} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{8I}{c}\frac{1}{D^2}y & -\frac{8I}{c}\frac{1}{D^2}x & 0 \end{vmatrix}$$

$$= \hat{z}\left(-\frac{8I}{cD^2} - \frac{8I}{cD^2}\right) = -\frac{16I}{cD^2}\hat{z}$$

$$= -\frac{4\pi}{c}\frac{I}{\pi(D/2)^2}\hat{z}$$

$$= \frac{4\pi}{c}\vec{J} \qquad \vec{J} = -\frac{I}{\pi(D/2)^2}\hat{z}$$

In general, how do you get $\vec{B}$? Actually several ways, but the "classic" by analogy.

$$\vec{E}$$

$$\downarrow$$

$$\vec{\nabla}\times\vec{E}=0$$

$$\vec{\nabla}\cdot\vec{E}=4\pi\rho$$

$$\downarrow$$

$$\vec{E}=-\vec{\nabla}\phi$$

$q\phi$ is energy

$$\uparrow$$

$$\sim\frac{Q}{r}$$

$$\text{or } \int\frac{\rho(r)\,d^3x}{r}$$

ϕ: scalar potential

$$\vec{B}$$

$$\downarrow$$

$$\vec{\nabla}\times\vec{B}\neq 0\;!!\;=\frac{4\pi}{c}\vec{J}$$

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\downarrow$$

$$\vec{B}=\vec{\nabla}\times\vec{A}$$

$q\vec{A}$ is a momentum(!)

$$\nearrow$$

$$\sim\frac{1}{c}\int\frac{\vec{J}\,d^3x}{r}$$

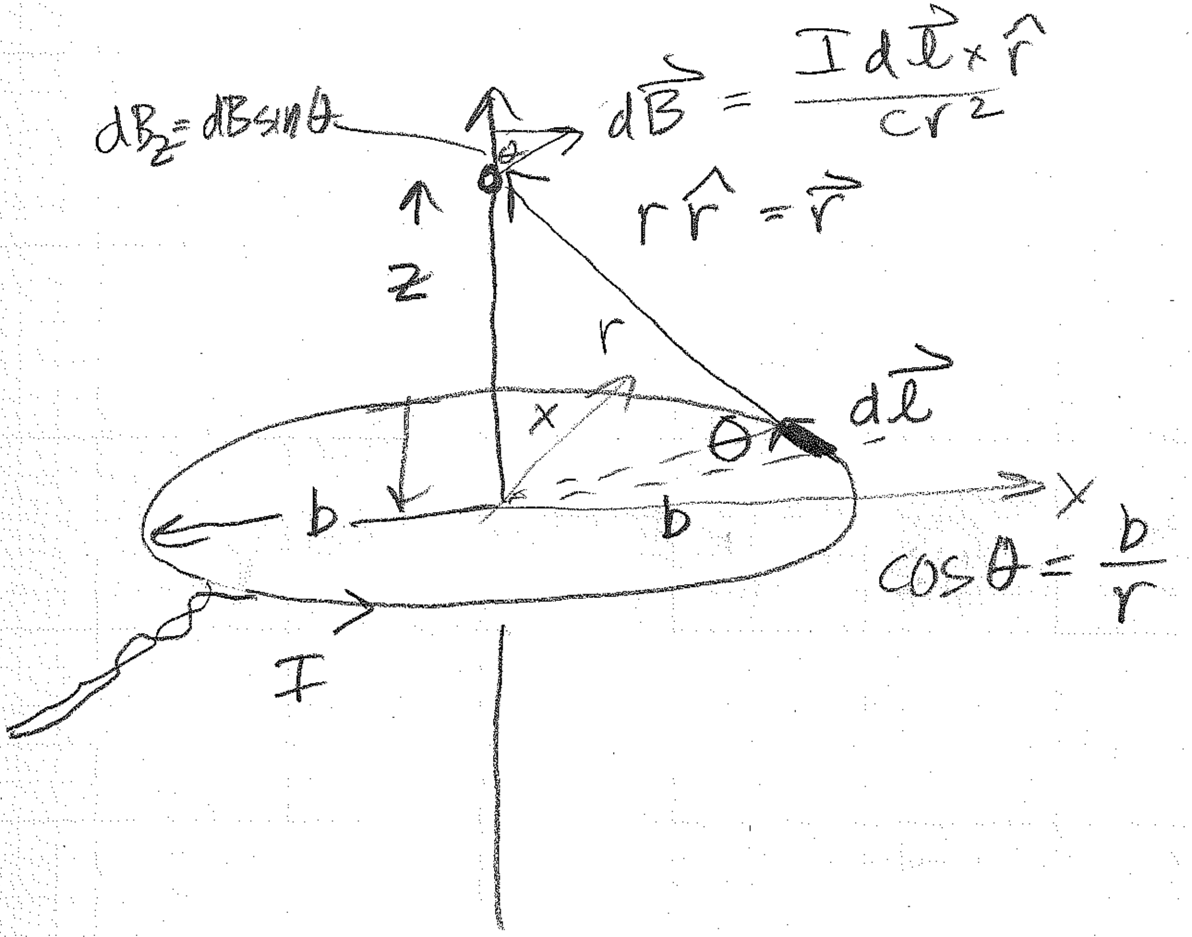
$\vec{A}$: VECTOR POTENTIAL

Leads to Biot-Savart

$$d\vec{B}=\frac{I\,d\vec{\ell}\times\hat{r}}{c\,r^2}$$

Let's go with this!

Ring of Current:



B_x + B_y → average to 0 when one integrates over all $d\vec{l}$

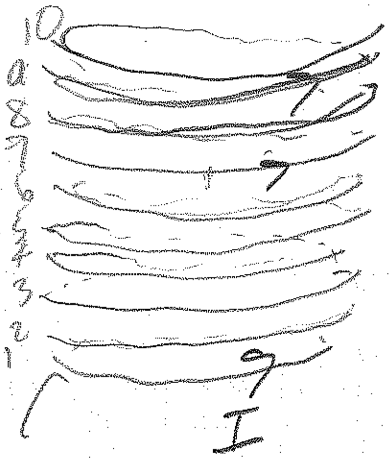
$$B_z = \frac{I \cdot 2\pi b}{cr^2} \times \left(\frac{b}{r} = \cos\theta\right)$$

$$\boxed{B_z = \frac{2\pi b^2 I}{c(b^2+z^2)^{3/2}}}$$

use for

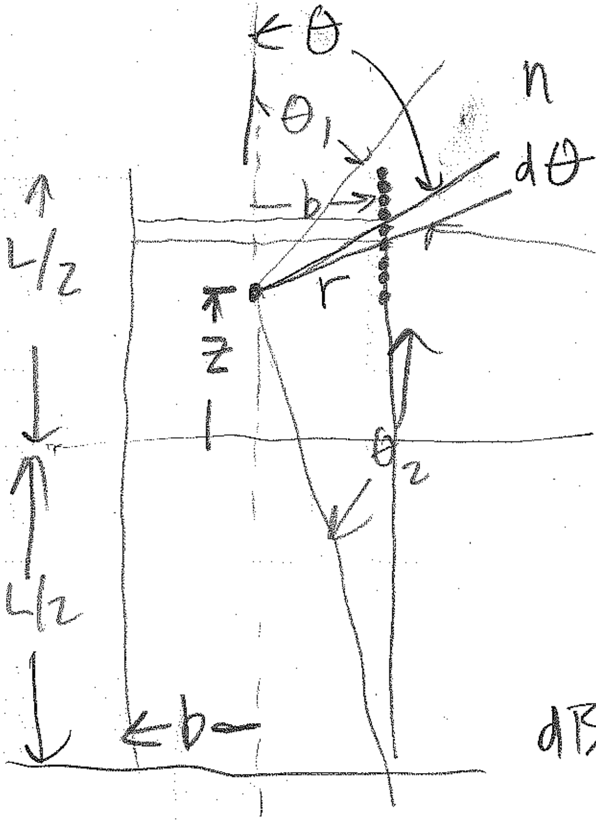
1:30pm Boris

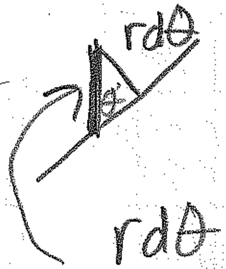
Stack Up Rings



"Solenoid"

"turns/cm"





$$\frac{r\,d\theta}{\sin\theta}$$

$$\sin\theta = \frac{b}{r}$$

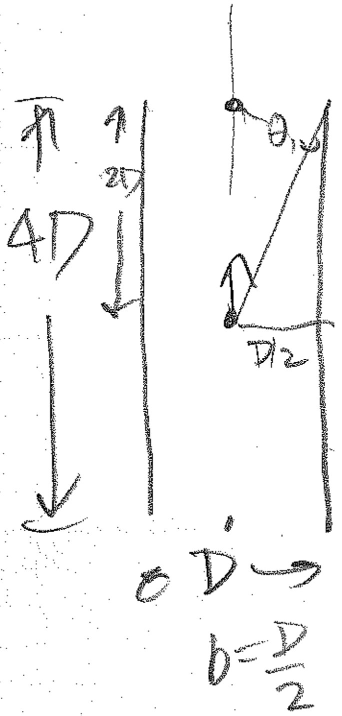
$$dB_z = \frac{2\pi b^2}{c\cancel{r^3}\,r^2}\left(\frac{I n\, r\, d\theta}{\sin\theta}\right)$$

$r \leftarrow b/\sin\theta$

$$= \frac{2\pi I n}{c}\, I n \sin\theta\, d\theta$$

$\frac{\text{esu}}{\text{sec cm}}$

$$B_z = \frac{2\pi I n}{c}\left(\cos\theta_1 - \cos\theta_2\right) \rightarrow \boxed{\frac{4\pi I n}{c}}$$



$$\tan\theta_1 = \frac{D/2}{2D} = \frac{1}{4}$$

$$\theta_1 = 14.0^\circ$$

$$\theta_2 = 180 - \theta_1 = 166^\circ$$

$$B_z = \frac{2\pi I n}{c} \times \left(\underset{\overset{\uparrow}{0.970}}{\cos(14^\circ)} - \underset{\overset{\uparrow}{-0.97}}{\cos(166^\circ)} \right)$$

$$B_z = 0.970 \cdot \frac{4\pi I n}{c}$$

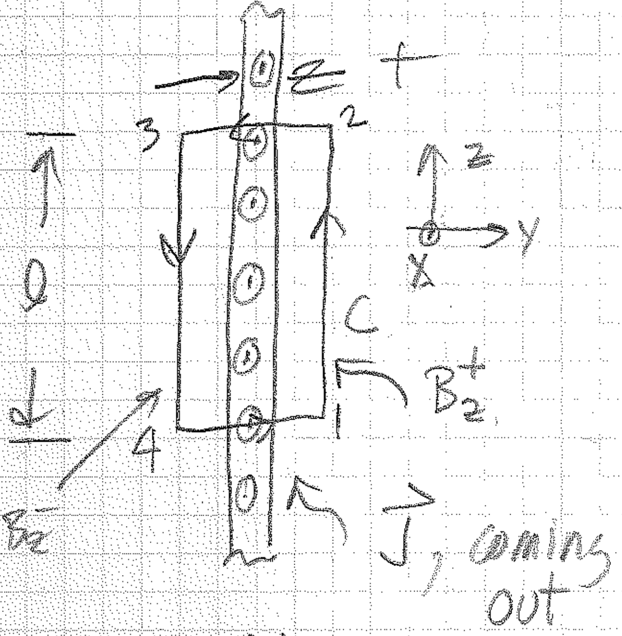
End?, $B_{z\,end} = \frac{2\pi I n}{c} \left(\underbrace{\cos 0}_{0} - (-0.993) \right)$

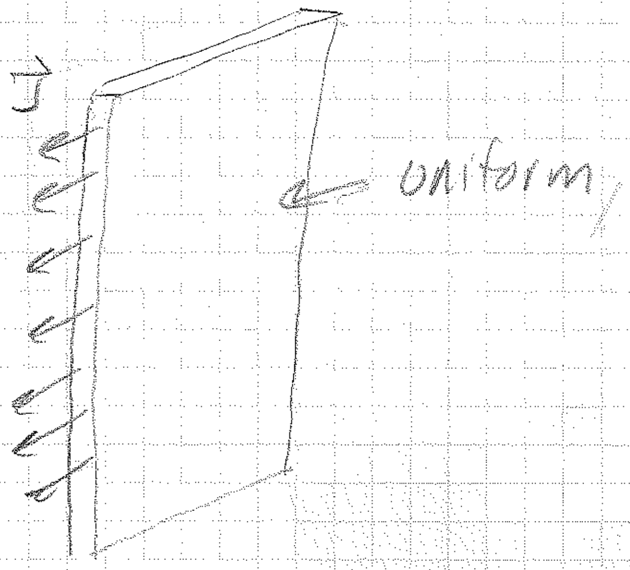
$$= 0.993 \times \frac{2\pi I n}{c}$$

$$\frac{B_{z\,end}}{B_{z\,middle}} \sim \frac{1}{2} \times \frac{0.993}{0.970} = \underline{\frac{1}{2} \times 1.023}$$

$$\to \infty? \to \frac{1}{2}$$

Current Sheets and Magnetic Pressure.





$|\vec J| \to \frac{esu}{sec\ cm^2}$ $|\vec J|\cdot t = j \to \frac{esu}{sec\ cm}$

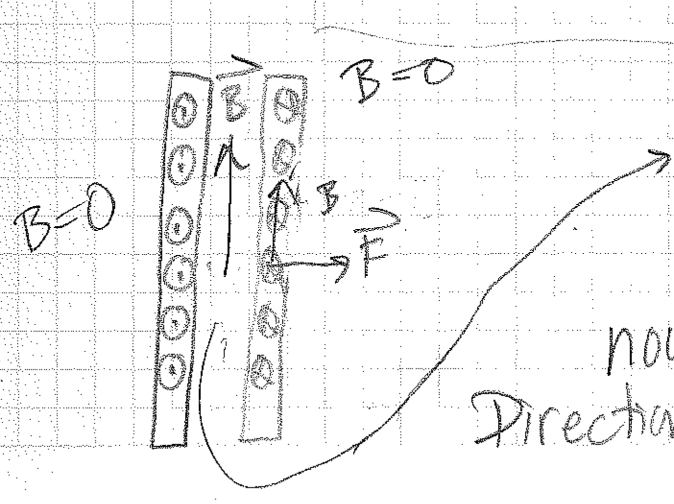
current/length

$$\oint_C \vec B\cdot d\vec l = \left(B_z^+ - B_z^-\right)\cdot l = \frac{4\pi}{c}\underbrace{j\cdot l}_{I}$$

line opposite direction

2→3, 4→1: $\vec B \perp$ line

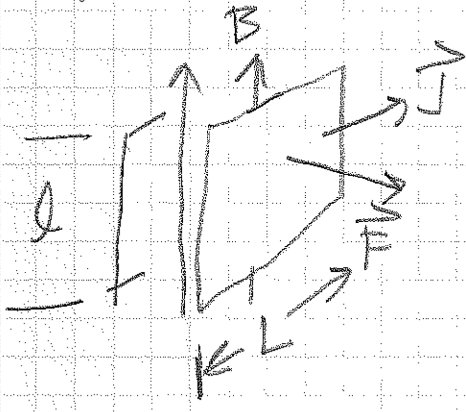
$$\boxed{B_z^+ - B_z^- = \frac{4\pi}{c} j}$$



inside, $B = \frac{4\pi}{c} j$

now work out forces.

Direction: outward!



$$\vec{F} = \underbrace{q\frac{\vec{v}}{c}}\times\vec{B}$$

$\frac{\text{esu}\cdot\text{cm}}{\text{sec}}\cdot\frac{1}{c}$ 0 outside, Full strength Inside ↕

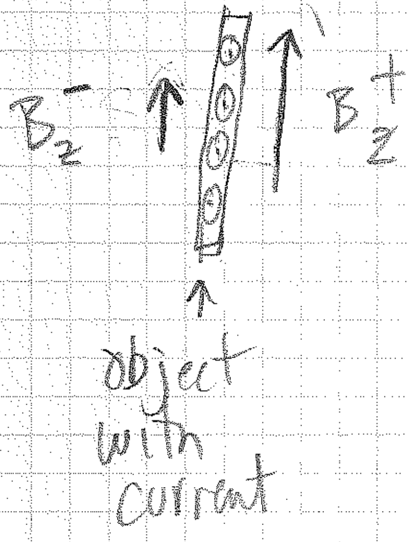
$$\underbrace{J\cdot t}\cdot\underset{\uparrow}{l}\cdot\underset{\uparrow}{L}\cdot\frac{1}{c}\,\frac{1}{2}\cdot\frac{4\pi}{c}J$$

J, $\frac{\text{esu}}{\text{sec cm}}$ height cm length cm

$$\text{Pressure} = \frac{F}{l\cdot L} = \frac{c}{4\pi}B\cdot\frac{1}{c}\cdot\frac{1}{2}B$$

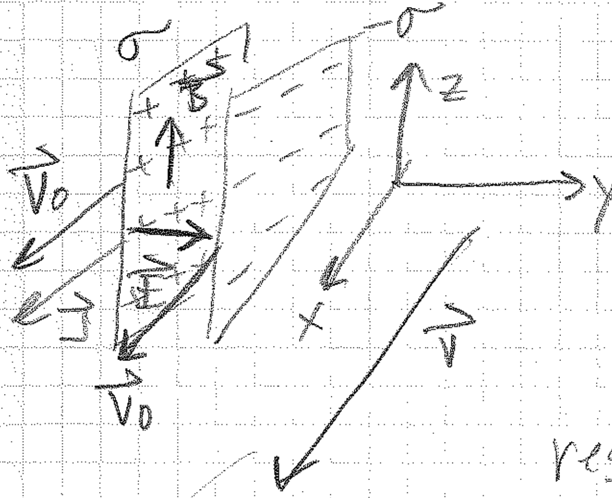
$$\boxed{\text{Pressure} = \frac{1}{8\pi}B^2}$$

more generally,



$$\boxed{\underline{\text{Pressure}} = \frac{1}{8\pi}\left((B_z^+)^2-(B_z^-)^2\right)}$$

away from high field region



charge density σ → in this frame.

$\frac{\sigma}{\gamma_0}$ in its rest frame

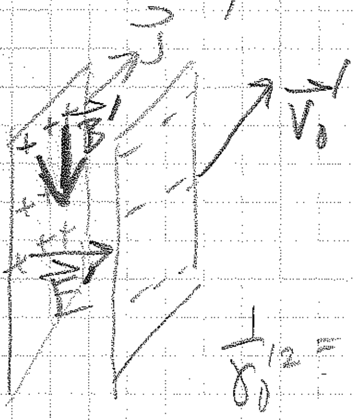
$$\gamma_0 = \frac{1}{\sqrt{1-\left(\frac{v_0}{c}\right)^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

$$J = \sigma v_0 \rightarrow \frac{esu}{cm^2} \cdot \frac{cm}{s}$$

$$E_y = 4\pi\sigma$$

$$B_z = \frac{4\pi}{c} J = \frac{4\pi}{c}\sigma v_0 = 4\pi\sigma\beta_0$$

another frame, primed frame, zooms by with speed $V > \vec{v}_0$ the $\vec{E}', \vec{B}'$ are different, how do they relate?



$$v_0' = \frac{v_0 - V}{1 - \frac{1}{c^2}v_0 V} = c\,\frac{\beta_0 - \beta}{1-\beta_0\beta}$$

$$\frac{1}{\gamma_0'^2} = 1 - \left(\frac{v_0'}{c}\right)^2 = 1 - \frac{(\beta_0-\beta)^2}{(1-\beta_0\beta)^2}$$

$$= \frac{1 \cancel{-2\beta_0\beta} + \beta_0^2\beta^2 - \beta_0^2 \cancel{+2\beta_0\beta} - \beta^2}{(1-\beta_0\beta)^2}$$

$$\frac{1}{\gamma_0'^2} = \frac{(1-\beta_0^2)(1-\beta^2)}{(1-\beta_0\beta)^2} = \frac{1}{\gamma_0^2\gamma^2}\frac{1}{(1-\beta_0\beta)^2}$$

$$\gamma_0' = \gamma_0\gamma\,(1-\beta\beta_0)$$

$$\sigma' = \frac{\gamma_0'}{\gamma_0}\,\sigma = \gamma(1-\beta\beta_0)\,\sigma$$

$$j' = \sigma' v_0' = \sigma\gamma(1-\cancel{\beta\beta_0})\cdot c\cdot\frac{\beta_0-\beta}{\cancel{1-\beta_0\beta}}$$

$$j' = \sigma\gamma(v_0 - v)$$

$$E_y' = 4\pi\sigma' = 4\pi\gamma\sigma(1-\beta_0\beta)$$

$$= \gamma\left[4\pi\sigma - \left(\frac{4\pi\sigma v_0}{c}\right)\left(\frac{v}{c}\right)\right]$$

$$\boxed{E_y' = \gamma(E_y - \beta B_z)}$$

$$B_z' = \frac{4\pi}{c}\,\sigma' v_0' = \frac{4\pi}{c}\,\sigma\gamma(v_0 - v)$$

$$= \gamma\left[4\pi\sigma\frac{v_0}{c} - 4\pi\sigma\frac{v}{c}\right]$$

$$\boxed{B_z' = \gamma(B_z - \beta E_y)}$$

$\vec{v}$ in $\hat{x}$ direction: $\vec{v}\times\vec{B}$ in $-\hat{y}$

$\parallel \rightarrow$ parallel to $\vec{v}$, $\perp \rightarrow$ perp to $\vec{v}$ $\quad\vec{v}\times\vec{E}$ in $+\hat{z}$

$$\vec{E}_\perp' = \gamma(\vec{E}_\perp + \vec{\beta}\times\vec{B}_\perp) \qquad \vec{E}_\parallel' = \vec{E}_\parallel$$

$$\vec{B}_\perp' = \gamma(\vec{B}_\perp - \vec{\beta}\times\vec{E}_\perp) \qquad \vec{B}_\parallel' = \vec{B}_\parallel$$

Special Case

$$\vec{B}=0$$

$$\vec{E}'_{\parallel}=\vec{E}_{\parallel} \quad \vec{E}'_{\perp}=\gamma\vec{E}_{\perp} \quad \vec{B}'_{\parallel}=0$$

$$\vec{B}'_{\perp}=-\gamma\vec{\beta}\times\vec{E}_{\perp}=-\vec{\beta}\times\vec{E}'_{\perp}$$

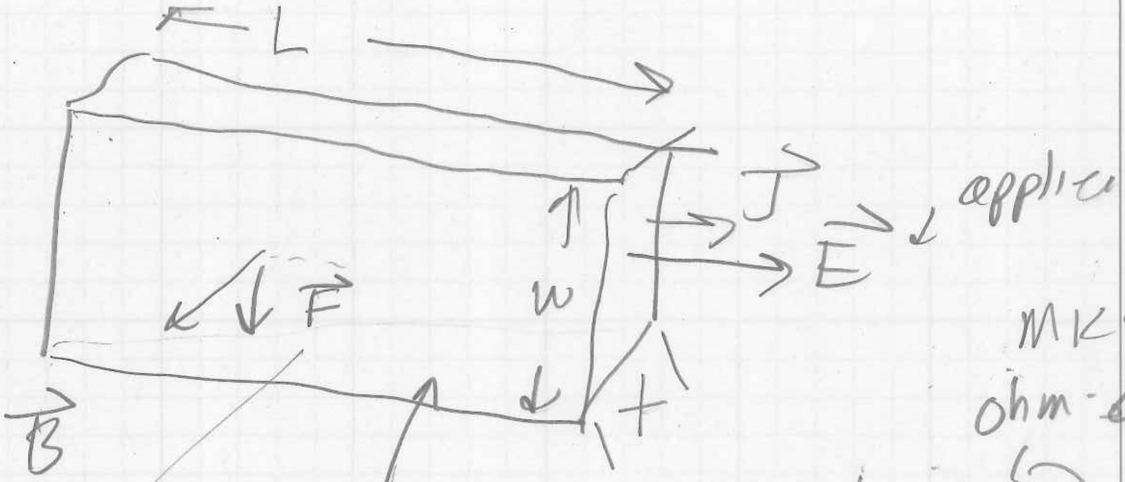
but $\vec{\beta}\times\vec{E}'_{\parallel}=0$ anyhow (they're parallel)

$$\vec{B}'_{\perp}=-\vec{\beta}\times\vec{E}'_{\perp}$$

$$\boxed{\begin{array}{l}\vec{E}=0\\ \quad\vec{E}'_{\perp}=+\vec{\beta}\times\vec{B}'\end{array}}$$

Hall Effect

How semiconductors can measure $\vec{B}$ ...



semiconductor (resistive, ρ)

"Turn on $\vec{B}$"

$$\vec{F} = q\frac{\vec{v}}{c}\times\vec{B}$$

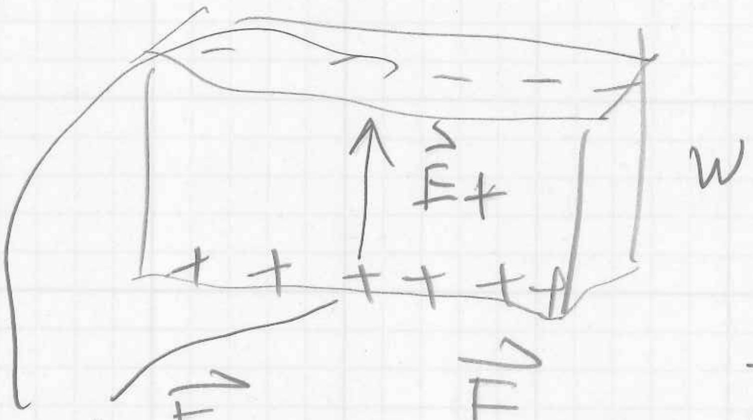
$$\vec{J} = n q \vec{v}$$

$\frac{1}{cm^3}$ ↑ n, charge ↑ q, × cm/c

$$\vec{v} = \frac{\vec{J}}{nq}$$

$$\vec{F} = q\left(\frac{\vec{J}}{nqc}\times\vec{B}\right)$$

$$\vec{B}\simeq 0 \left\{ \begin{array}{l} I = \frac{V}{R} = \frac{E\cdot \cancel{L}}{\rho\cdot\frac{\cancel{L}}{wt}} \\ \quad = \frac{E}{\rho}wt \\ \text{or } J = \frac{I}{wt} = \frac{E}{\rho} = \sigma E \end{array}\right.$$



V! $\vec{E}_t = -\frac{\vec{F}}{q} = -\frac{\vec{J} \times \vec{B}}{nqc}$

$$V = E_t \cdot w$$