$\bullet\ q$ (at rest)

$\Rightarrow$ no radiation

$q \bullet\!\xrightarrow{\vec{v}}$ constant $\vec{v}$

$\Rightarrow$ no radiation

$\bullet\!\xrightarrow{\vec{v}(t_1)}$ RADIATION

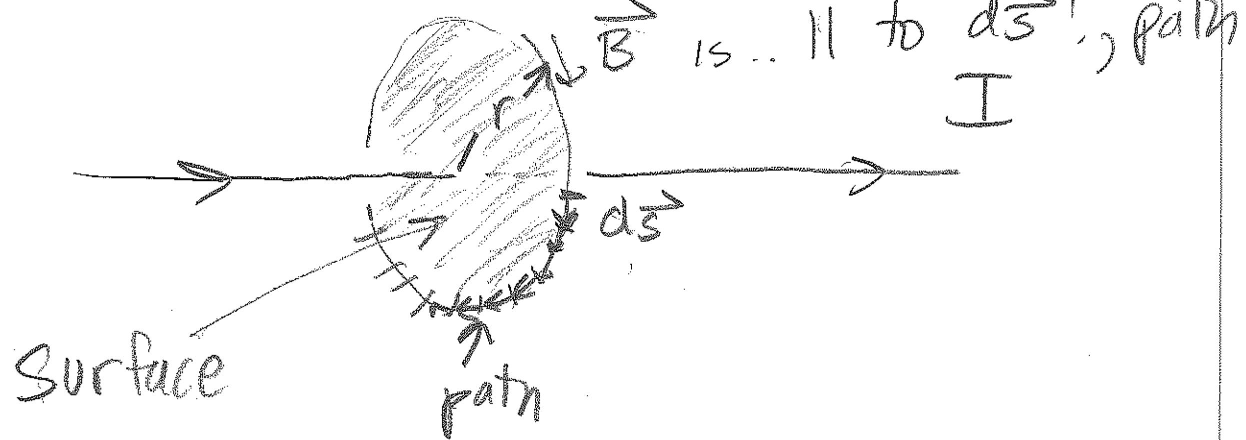
$\vec{v}(t_2) \neq \vec{v}(t_1)$

<u>Larmor</u> : $P_{rad} = \frac{2}{3} \frac{q^2 |\vec{a}|^2}{c^3}$ ← instantaneous a

- non-relativistic
- $\frac{2}{3} \Rightarrow$ peaks $\perp$ to direction
- Lets the Universe know ... question ...
  electrons orbiting nuclei

<u>$\vec{B}$ fields</u> : caused by charges in motion

5) Ampere's Law



$$\int \vec{B} \cdot \vec{ds} = \frac{4\pi}{c} I \text{ (that penetrates surface)}$$

check $\frac{2I}{rc} \cdot 2\pi r = \frac{4\pi}{c} I$

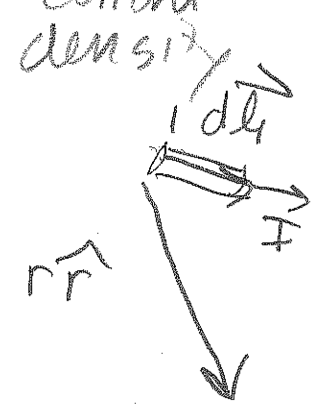
<u>point</u>: path need not be perfect

Analogous to Gaussian Surface
"Amperean Loop"

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J} \qquad \vec{J} = \text{current density}$$

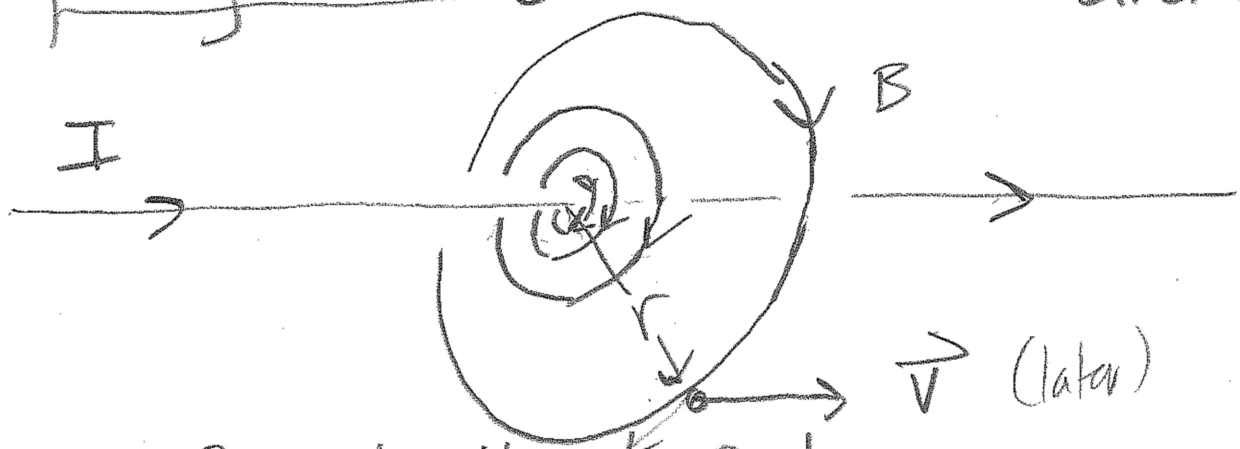
6) Biot Savart

$$\vec{dB} = \frac{I \vec{dl} \times \hat{r}}{c r^2}$$



QUALITATIVE: paradigm

$\boxed{\text{Long straight wire}}$ $\vec{B}$ around in circles



1) Right Hand Rule
   thumb along current
   fingers along B

2) $\vec{B}$: no start, no end!
   $\vec{E}$: starts on charge

$$\vec{\nabla} \cdot \vec{E} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot \left(E_x\hat{x} + E_y\hat{y} + E_z\hat{z}\right)$$

$$= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 4\pi\rho$$

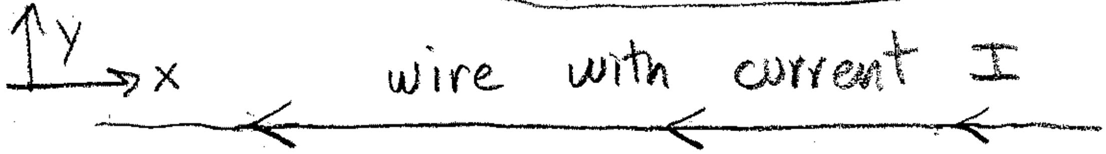
$$\vec{\nabla} \cdot \vec{B} = 0$$ (no magnetic charge)

3) In this case, $|\vec{B}| = \frac{2I}{rc}$

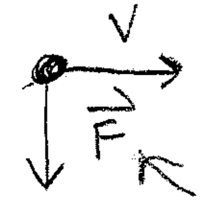
4) $\vec{F} = q\frac{\vec{V}}{c} \times \vec{B}$ • Cross product important !!!! (• wires • rotate)

$F/\ell = -\frac{2I_1I_2}{c^2 r}$

Origin of Magnetic Force



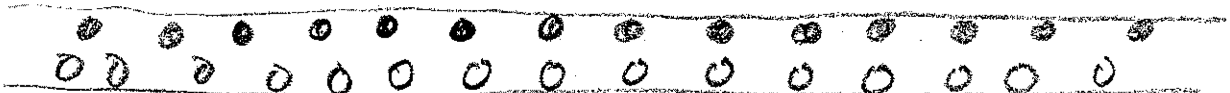
"Lab" frame.



(like a current ∥ to I)

feels repulsive force.
(opposite currents repel)

Lab Frame: ions, + charge, immobile, λ_0 esu/cm

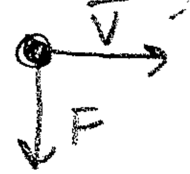


Blow up wire

$-\lambda_0$ esu/cm $\longrightarrow \vec{v}_0$

$\beta_0 = \frac{v_0}{c}$

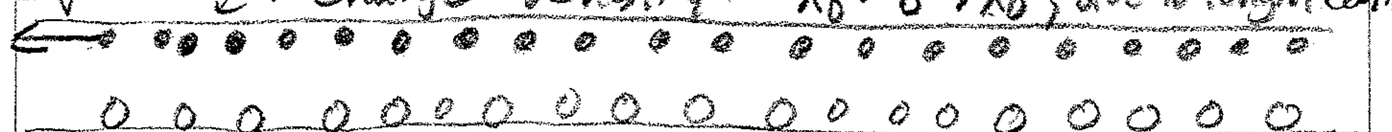
electrons moving, left to right, velocity $\vec{v}_0$
negative charge
current from right to left.



NO $\vec{E}$ field!! $\lambda_0 - \lambda_0 = 0$.

Boost to charge particle rest frame.

$-\vec{v}$ + charge density: $\lambda_0 \cdot \gamma > \lambda_0$, due to length contraction



− charge.

$\beta = \frac{v}{c}$

• q at rest

$\longrightarrow v_0' \rightarrow$ what is γ_0'?

$$\beta_0' = \frac{\beta_0 - \beta}{1 - \beta\beta_0}$$

$$1 - \beta_0'^2 = 1 - \frac{(\beta_0 - \beta)^2}{(1-\beta\beta_0)^2} = \frac{(1-\beta\beta_0)^2 - (\beta_0-\beta)^2}{(1-\beta\beta_0)^2}$$

$$\frac{1}{\gamma_0'^2} = 1-\beta_0'^2 = \frac{1-2\beta\beta_0+\beta^2\beta_0^2-\beta_0^2+2\beta\beta_0-\beta^2}{(1-\beta\beta_0)^2}$$

$$\frac{1}{\gamma_0'^2} = \frac{1-\beta_0^2-\beta^2+\beta^2\beta_0^2}{(1-\beta\beta_0)^2} = \frac{(1-\beta^2)(1-\beta_0^2)}{(1-\beta\beta_0)^2}$$

$$\boxed{\gamma_0' = \frac{1-\beta\beta_0}{\sqrt{1-\beta^2}\sqrt{1-\beta_0^2}} = \gamma\gamma_0(1-\beta\beta_0)}$$

want the charge density of the negative charge in q's rest frame.

Route: (2 steps)

$$\underbrace{-\lambda_0\,\frac{\text{esu}}{\text{cm}}}_{\boxed{\text{wire rest frame already length contracted}}} \Rightarrow \underbrace{\frac{-\lambda_0}{\gamma_0}\,\frac{\text{esu}}{\text{cm}}}_{-\text{ charge rest frame}} \Rightarrow \underbrace{-\frac{\lambda_0\gamma_0'}{\gamma_0}\,\frac{\text{esu}}{\text{cm}}}_{\text{q's rest frame!}}$$

length contracted again

$$\left.\begin{array}{l}-\text{charge}\\ \text{density}\end{array}\right\}\ \begin{array}{l}q\text{ rest}\\ \text{frame}\end{array} = -\frac{\lambda_0}{\gamma_0}\cdot\gamma\gamma_0(1-\beta\beta_0)$$

$$= -\lambda_0\gamma(1-\beta\beta_0)$$

Net charge density:

$$\underbrace{+\ +\ -}_{\text{for } q \text{ at rest frame}} = \gamma\lambda_0 - \lambda_0\gamma(1-\beta\beta_0)$$

$$\boxed{\lambda' = +\gamma\beta\beta_0\lambda_0}$$

note, linear in velocity of q!

In q's rest frame, there is a net charge density!

radial $E_r' = \frac{2\lambda'}{r'} = \frac{2\gamma\beta\beta_0\lambda_0}{r'}$

boost back to the lab frame.

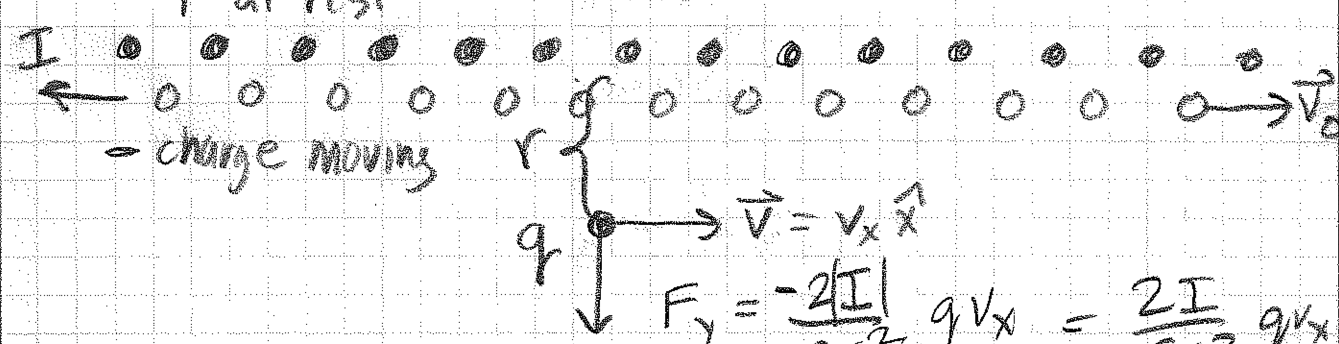
$$F_r = \frac{dp_\perp}{dt} = \frac{1}{\gamma}\frac{dp_\perp'}{dt'} = \frac{1}{\gamma} q E_r' \quad , \; r = r' \; (\perp \text{ to boost})$$

$$F_r = \frac{2\beta\beta_0\lambda_0 q}{r}$$

$$F_y = -F_r = -\frac{2\beta\beta_0\lambda_0 q}{r}$$

$$I = -\lambda_0 \cdot v_0 = -\lambda_0\beta_0 c \qquad \beta = v_x\, c$$

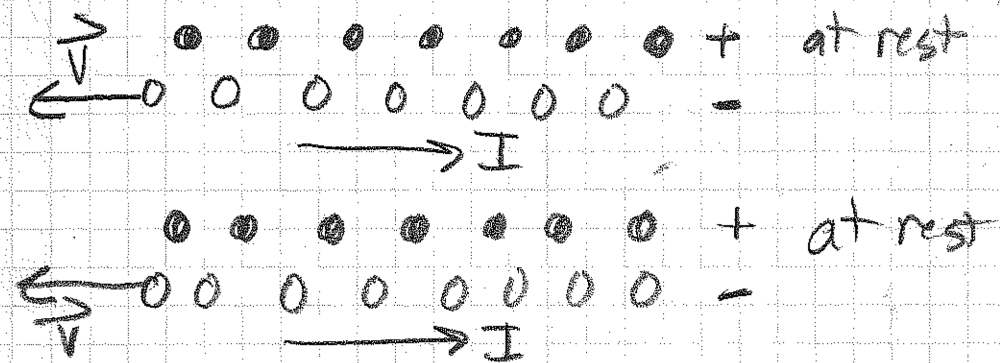
$$\boxed{F_y = \frac{2I}{rc^2} q v_x}$$



$$F_y = \frac{-2|I|}{rc^2} q v_x = \frac{2I}{rc^2} q v_x$$

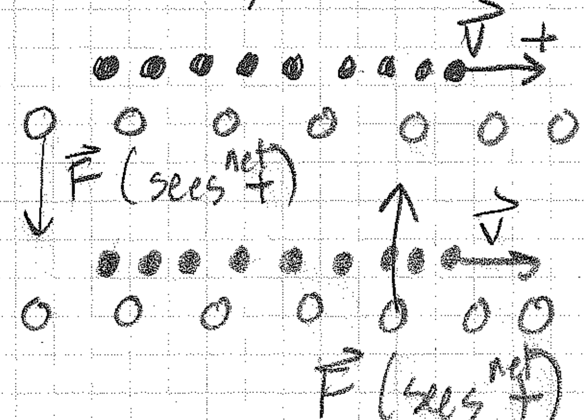
↑ I negative

<u>Pair of wires</u>: reason through antractions.



note: now in + rest frame, in that frame, no force on + charge!

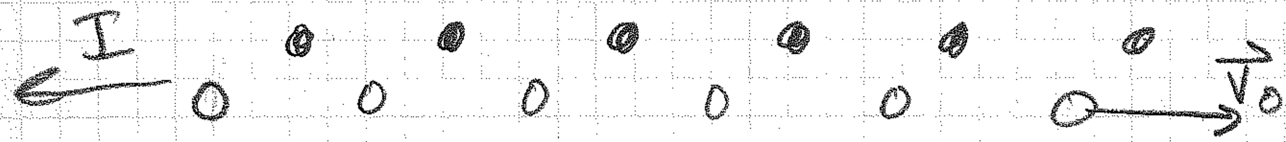
"boost" or "jump" to - charge rest frame, to analyze that

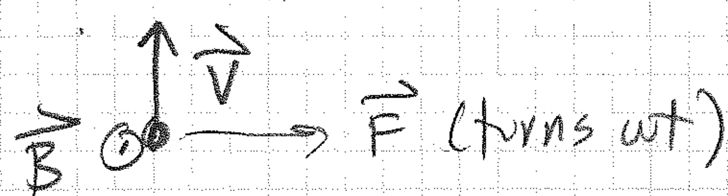


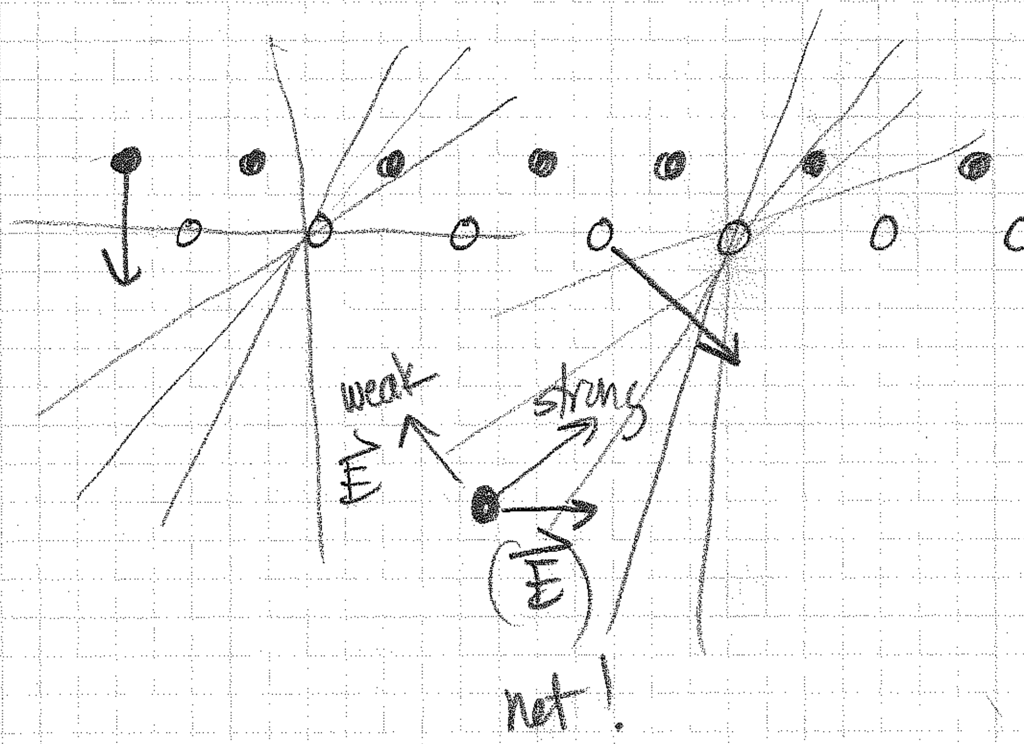
-ls in <u>their</u> rest frame feel ATTRACTION

net attraction, when we boost back to original frame

What about charge heading toward wire?







Definition of $\vec{E}$: put charge q at rest at some point in space. Measure $\vec{F}$.

$$\vec{E} = \vec{F}/q$$

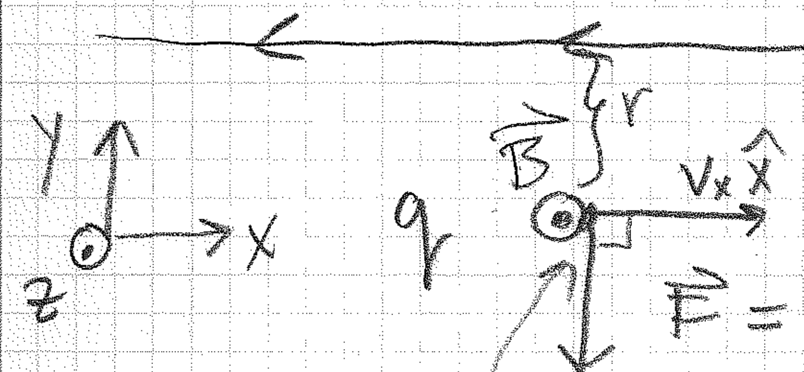
Definition of $\vec{B}$: (magnetic field) go to that same spot in space, Give q a velocity $\vec{v}$, measure $\vec{F}'$

$$\text{"magnetic force"} \equiv \vec{F}' - \frac{\vec{F}}{q}$$

$$\equiv \underbrace{q\frac{\vec{v}}{c} \times \vec{B}}_{} \quad \text{(by definition)}$$

presence of cross product makes solving for $\vec{B}$ a challenge!

$I < 0$



$$\vec{F} = \frac{2I}{rc^2} q v_x \hat{y} \quad \left(\text{note } I < 0\right)$$

deduce $\vec{B}$ in $\hat{z}$ direction.

since right angles

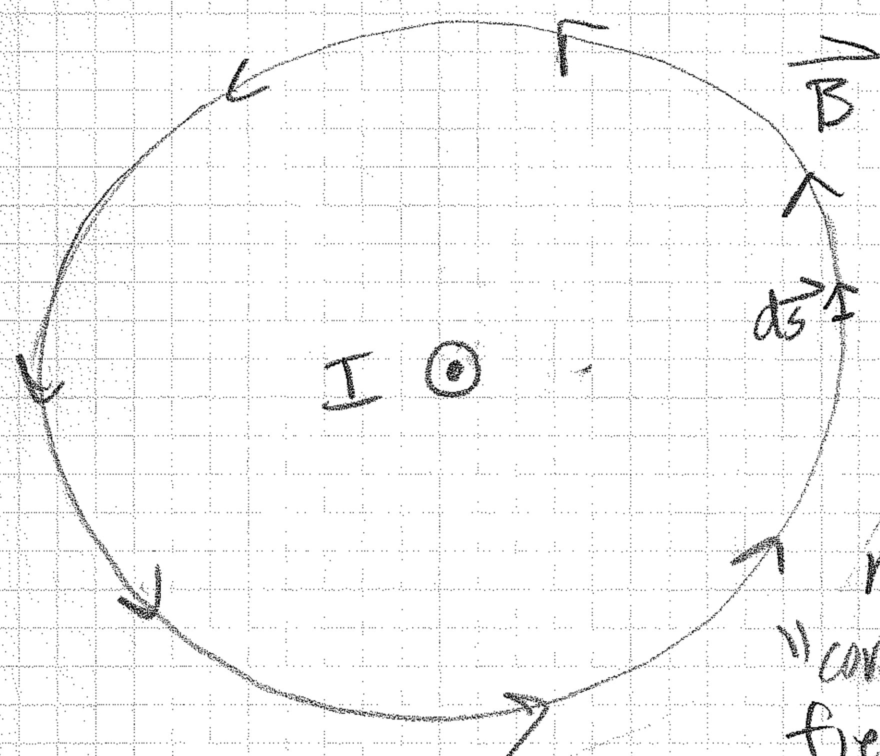
$$\cancel{\frac{q v_x}{c}} B = \frac{2I}{rc^2} \cancel{q} v_x$$

comes out in Gauss, input esu/sec, cm, cm/s ...

$$\boxed{B = \frac{2I}{rc}}$$

NOT radial!

remember E_r around wire? $2 \cdot \frac{\lambda}{r}$



$\oint \vec{B} \cdot d\vec{s} \neq 0$!!!

not a "conservative" field

MKS:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Newtons

Coulombs

no c

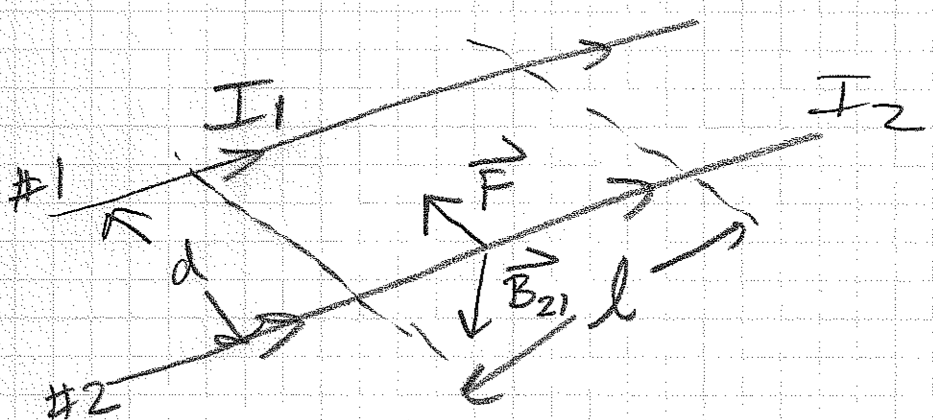
in Tesla ...

The weird thing here is computing $\vec{B}$ from I.

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \qquad \mu_0 = 4\pi \cdot 10^{-7}$$

$$1 \text{ Tesla} = 10^4 \text{ Gauss}$$

Force/Length on Wires



$$\vec{B}_{21} = \frac{2I_1}{dc}$$

$$I_2 = n_2 q_2 v_2$$

n_2 = # charges/length
q_2 = charge of carriers
v_2 = velocity of carriers.

force on one carrier $= B_{21} \dfrac{q_2 v_2}{c} \quad (\perp)$

force on n_2 carriers/length $= n_2 B_{21} q_2 \dfrac{v_2}{c} = \dfrac{2 I_1 I_2}{dc^2}$

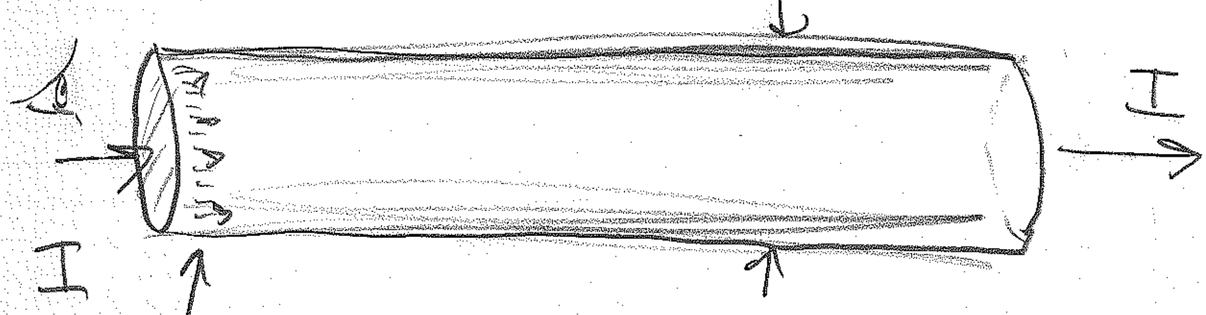
$$\frac{\text{dynes}}{\text{cm}} \quad \boxed{\frac{F}{\ell} = 2\,\frac{I_1 I_2}{d\,c^2}} \quad \frac{\left(\frac{esu}{sec}\right)^2}{cm\left(\frac{cm}{s}\right)^2}$$

used to define $1\ \frac{esu}{sec}$ of current

$$\frac{\text{Newtons}}{m} \quad \boxed{\frac{F}{\ell} = \frac{\mu_0}{2\pi}\,\frac{I_1 I_2}{d}} \quad \frac{(\text{Amp})^2}{m}$$

$4\pi \cdot 10^{-7}$

Ampere's Law + Rod



Current
Spread
Out... "Uniformly"

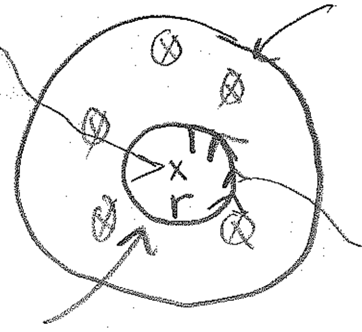
Current / (Area → ⊥ to direction of flow...)

$$J = \frac{I}{\left(\frac{\pi D^2}{4}\right)} = \frac{4I}{\pi D^2} \text{ everywhere!}$$

Find $\vec{B}$ everywhere! Look End on, use Ampere's Law!

$\vec{J}$... $|\vec{J}| = \frac{4I}{\pi D^2}$

axis of symmetry



path

Cylindrical Symmetry

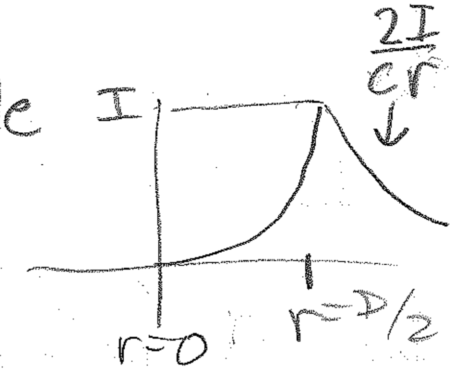
Imagine loop, radius r

Current "enclosed" (± .. Right Hand Rule)

$= - J \cdot \pi r^2$

↑ opposite to RH Rule

$= -4I\left(\frac{r}{D}\right)^2$



$= \oint \vec{B} \cdot d\vec{s}$

↑
- symmetry
- no radial component .. $\vec{\nabla} \cdot \vec{B} = 0$

$= 2\pi r B_\theta = \frac{4\pi}{c} \times \left(-4I\left(\frac{r}{D}\right)^2\right)$

$$\boxed{B_\theta = -\frac{8I}{c}\frac{r}{D^2}}$$

Imagine tiny, tiny loop...

circumference ↓ as size

↑ I ↓ as $\frac{1}{\text{Area}} \sim (\text{size})^2 \cdot J$

$B \cdot \text{size} \sim (\text{size})^2 \cdot J$

$J \sim \frac{1}{\text{size}} \cdot B$ ← derivative of B