Chapter 5 Purcell
Appendix B "

Topics this week:

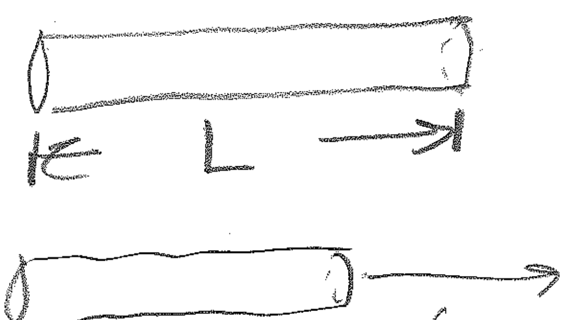
- Simple $\vec{B}$ field:
  Magnetic Dipoles: Earth, Alignment with $\vec{B}$
  $$\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$$
  in cgs!
- How $\vec{E}$ fields transform between inertial reference frames.
- Field of a moving point charge.
- Radiation!!
- Origin of $\vec{B}$ from I...
  RELATIVITY!

Super-Quick Relativity (Appendix A)

Lengths are longest in the rest frame
time intervals are shortest (twin youngest)

$$\beta = \frac{v}{c} < 1 \qquad \vec{\beta} = \frac{\vec{v}}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} > 1 \qquad \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$



rest $\leftarrow L \rightarrow$

v

$\Delta t = 0!$ $\quad L/\gamma \quad$ (L in F' at rest)

$\left.\right\}$ detailed paradoxes Lorentz

$\Delta x' = 0$

$$\Delta t = \gamma \Delta t'$$ $\Leftarrow$ clock, say, at rest in F'

4 vectors: $(\vec{x}, t)$ or $(t, \vec{x})$

$(\vec{p}, E)$ or $(E, \vec{p})$

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

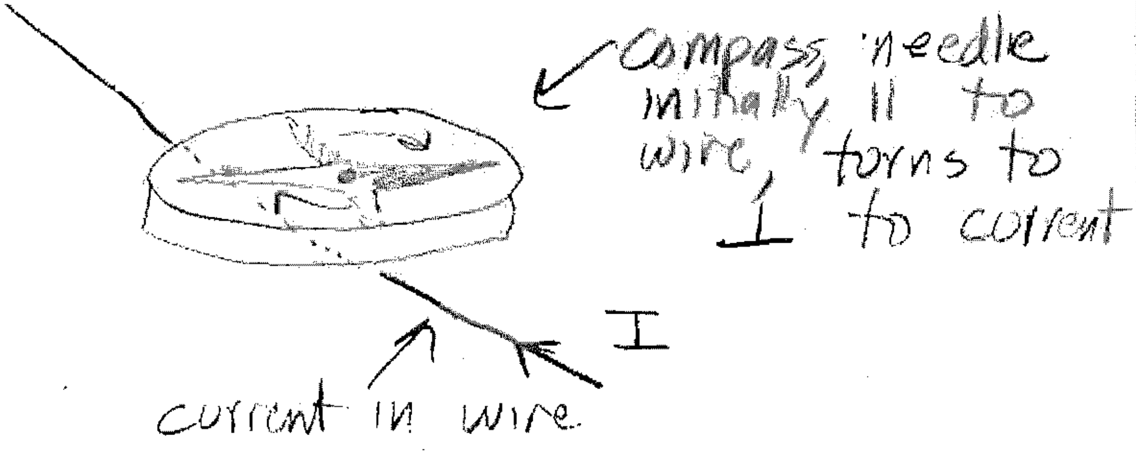
"Rest"

Forces: $$dp_\parallel = \gamma\, dp_\parallel'$$
$$dt = \gamma\, dt'$$

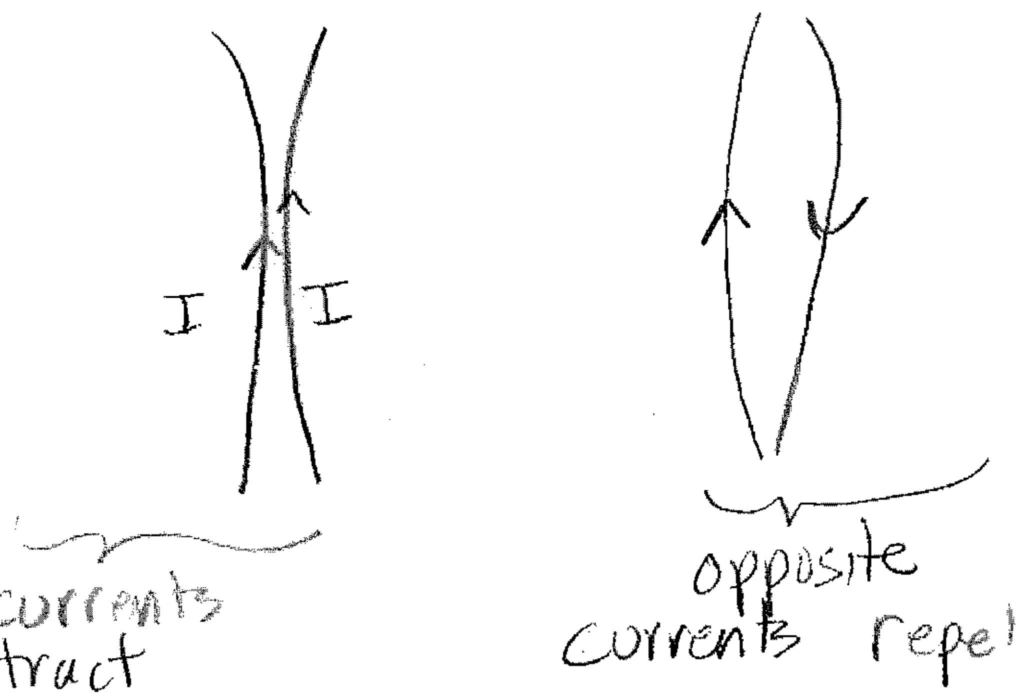
$$F_\parallel = \frac{dp_\parallel}{dt} = \frac{dp_\parallel'}{dt'}$$!!!

but $$F_\perp = \frac{dp_\perp}{dt} = \frac{1}{\gamma}\frac{dp_\perp'}{dt'}$$ like length

Magnetism

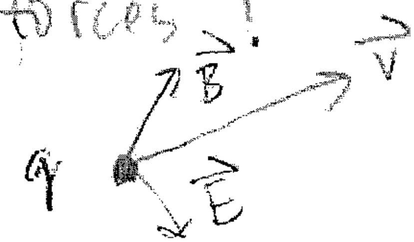


Currents attract



Putting a conducting plate between the wires does not eliminate or reduce the forces!

$\vec{B}$ (magnetic field)



$$\vec{F} = q \cdot \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right)$$

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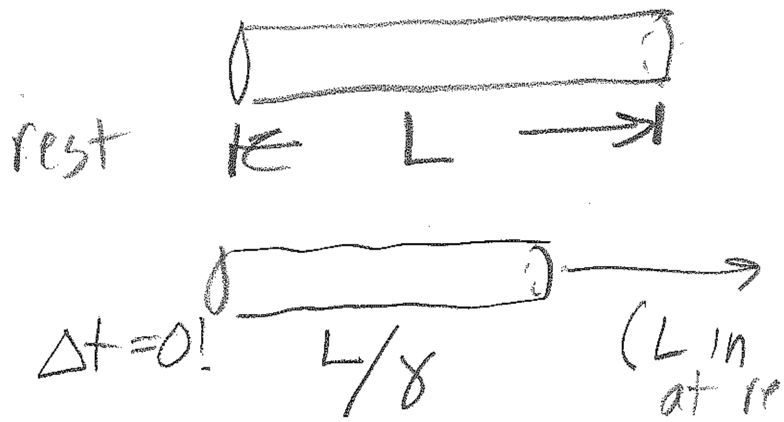


detailed paradoxes Lorentz

$\Delta t = 0!$ (L in F' at rest)

$\Delta x' = 0$

$\Delta t = \gamma \Delta t'$ ← clock, say, at rest in F'

<u>4 vectors</u>: $(\vec{x}, t)$ or $(t, \vec{x})$

$(\vec{p}, E)$ or $(E, \vec{p})$

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

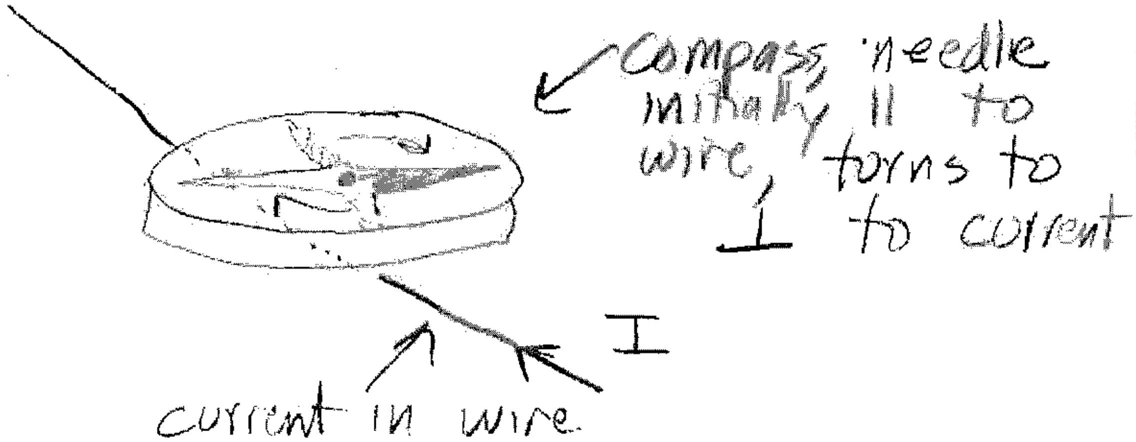
Forces : $dp_{\parallel} = \gamma\, dp_{\parallel}'$ "Rest"

$dt = \gamma\, dt'$

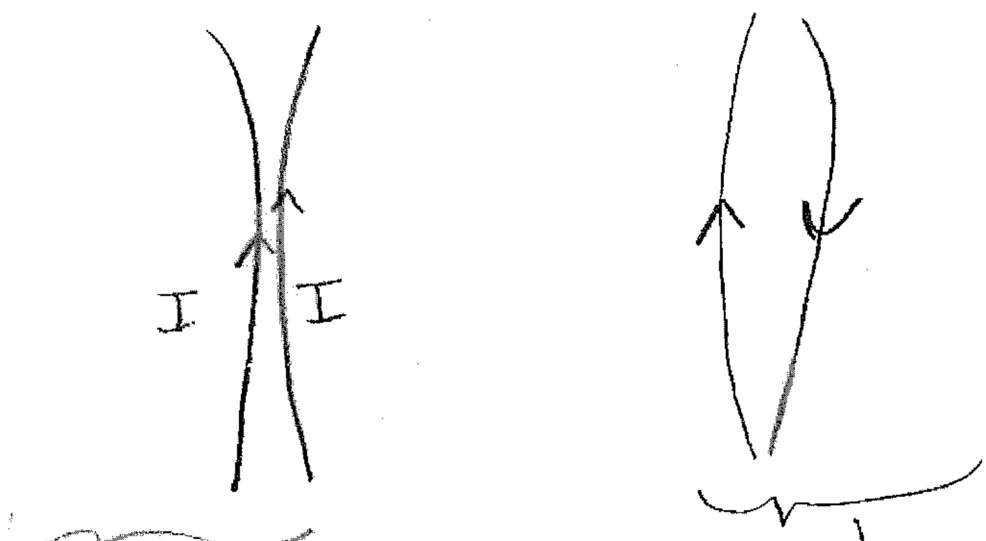
$$F_{\parallel} = \frac{dp_{\parallel}}{dt} = \frac{dp_{\parallel}'}{dt'} \quad \Big] \; !!!$$

$$\text{but } F_{\perp} = \frac{dp_{\perp}}{dt} = \frac{1}{\gamma}\frac{dp_{\perp}'}{dt'} \quad \Big] \; \text{like length}$$

Magnetism



Currents attract

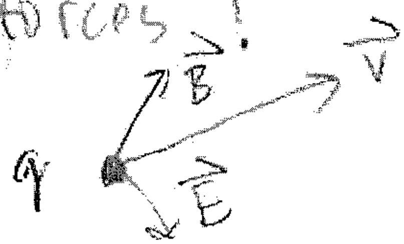


like currents attract

opposite currents repel

Putting a conducting plate between the wires does not eliminate or reduce the forces!

$\vec{B}$ (magnetic field)

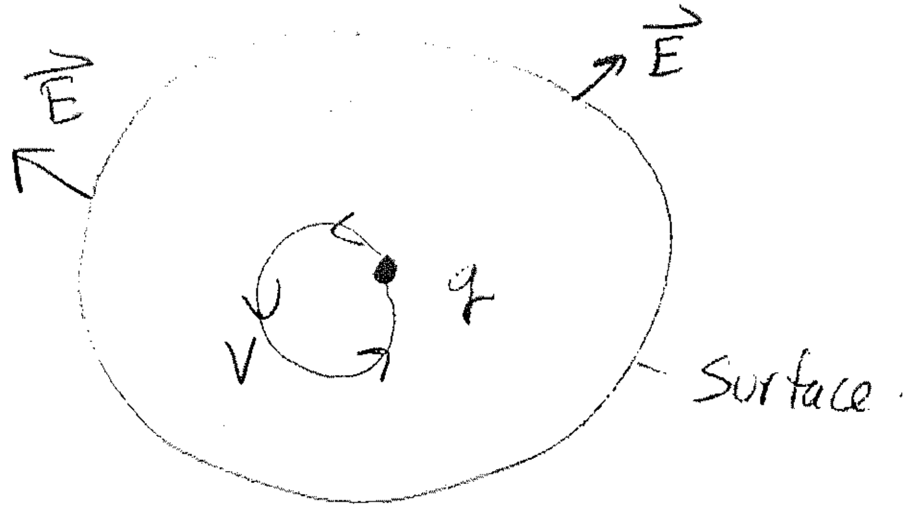


$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

Electric Charge Independent of $\vec{v}$ (!)

mass was not: $\dfrac{m_0}{\sqrt{1-(\frac{v}{c})^2}}$

Use Gauss' Law:



$$\int \vec{E} \cdot d\vec{A} = 4\pi q$$

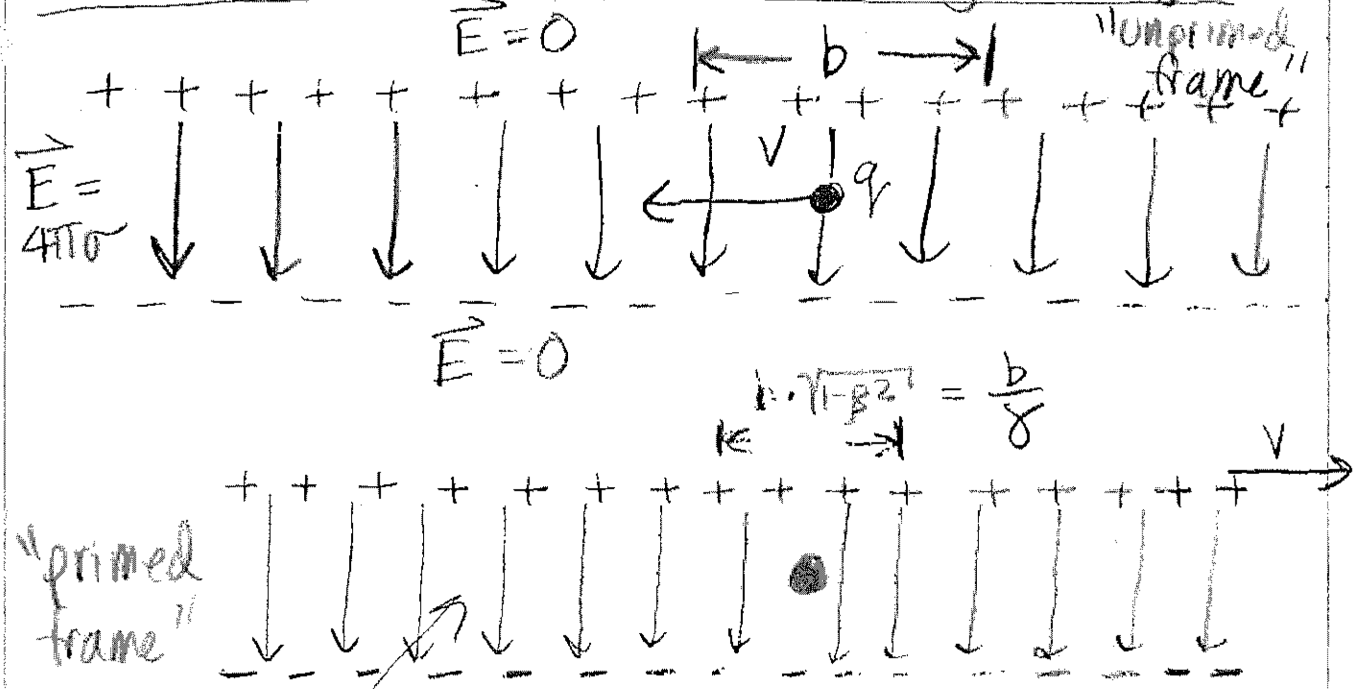
even if charge moving at speed of light!

Note: (x,t) of surface is actually different as viewed in different frame: (x',t'). But

$$\int_S \vec{E} \cdot d\vec{A} = \int_{S'} \vec{E}' \cdot d\vec{A}' \quad (!)$$

since charge is enclosed in both cases

Field Viewed From a Moving Charge

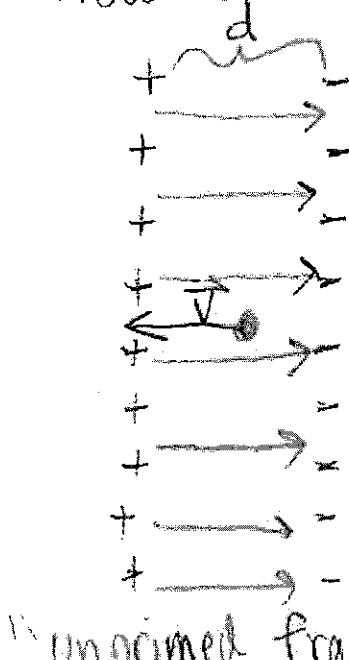


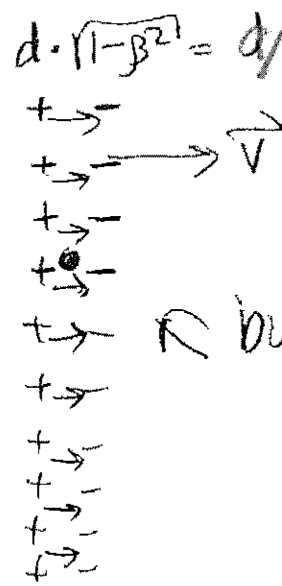
stronger: $\sigma' = \gamma\sigma$

$$\vec{E}' = \gamma \vec{E}$$

more generally, $\underbrace{\vec{E}'_{\perp} = \gamma \vec{E}_{\perp}}$ → $\perp$ to direction of motion

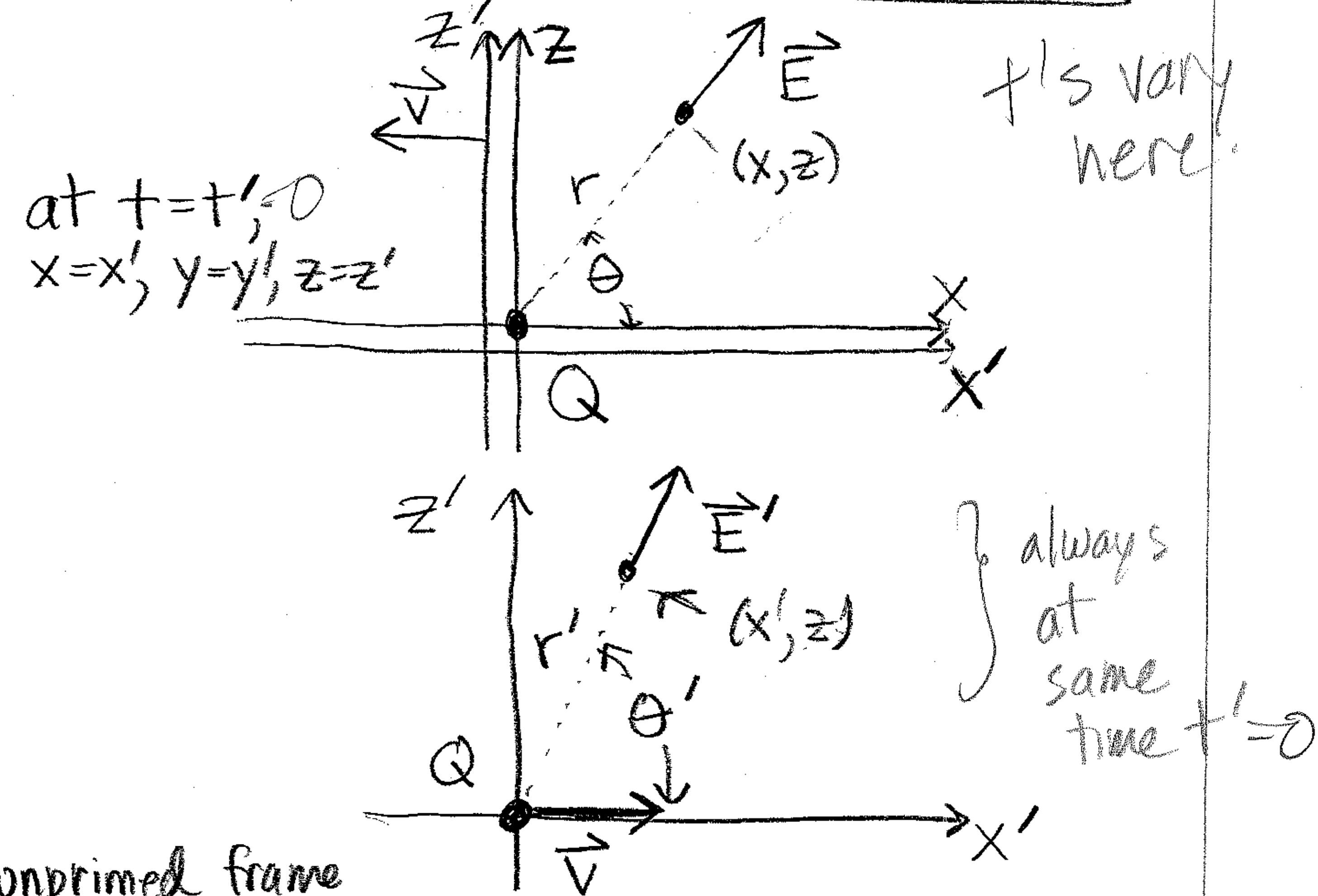
How about $\parallel$ to direction of motion





but, $\vec{E}'_{\parallel} = \vec{E}_{\parallel}$

Field of a Point Charge That is Moving



unprimed frame

$$E_x = \frac{Q}{r^2} \times \cos\theta = \frac{Q}{r^2} \cdot \frac{x}{r} = \frac{Qx}{r^3}$$

$$= \frac{Qx}{(x^2+z^2)^{3/2}}$$

$$E_z = \frac{Q}{r^2} \cdot \sin\theta = \frac{Q}{r^2} \cdot \frac{z}{r} = \frac{Qz}{r^3}$$

$$= \frac{Qz}{(x^2+z^2)^{3/2}}$$

$$x = \gamma(x' - \beta c t') = \gamma x' \quad \text{at } t = t' = 0$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' - \beta \frac{x'}{c}\right) = -\gamma\beta\frac{x'}{c} \quad \text{at } t = t' = 0$$

$$E_x' = E_x = \frac{Qx}{(x^2+z^2)^{3/2}} = \frac{\gamma Q x'}{((\gamma x')^2 + z'^2)^{3/2}}$$

$\parallel$ to direction of motion

$$E_z' = \gamma E_z = \frac{\gamma Q z}{(x^2+z^2)^{3/2}} = \frac{\gamma Q z'}{((\gamma x')^2 + z'^2)^{3/2}}$$

$\perp$ to direction of motion

note: $$\frac{E_x'}{E_z'} = \frac{x'}{z'} \quad \Big\} \ \vec{E}' \text{ still points along radii!}$$

How strong is it?

$$E_x'^2 + E_z'^2 = \frac{\gamma^2 Q^2 (x'^2 + z'^2)}{(\gamma^2 x'^2 + z'^2)^3}$$

$$= \frac{\gamma^2}{\gamma^6} \frac{Q^2 (x'^2 + z'^2)}{\left(x'^2 + \frac{1}{\gamma^2} z'^2\right)^3} \qquad \frac{1}{\gamma^2} = 1 - \beta^2$$

$$= \frac{1}{\gamma^4} \frac{(x'^2 + z'^2)}{(x'^2 + z'^2)^3} \cdot \frac{Q^2}{\left(1 - \beta^2 \frac{z'^2}{x'^2 + z'^2}\right)^3}$$

$$E'^2 = \frac{1}{r'^4} \cdot \frac{Q^2(1-\beta^2)^2}{(1-\beta^2\sin^2\theta')^3}$$

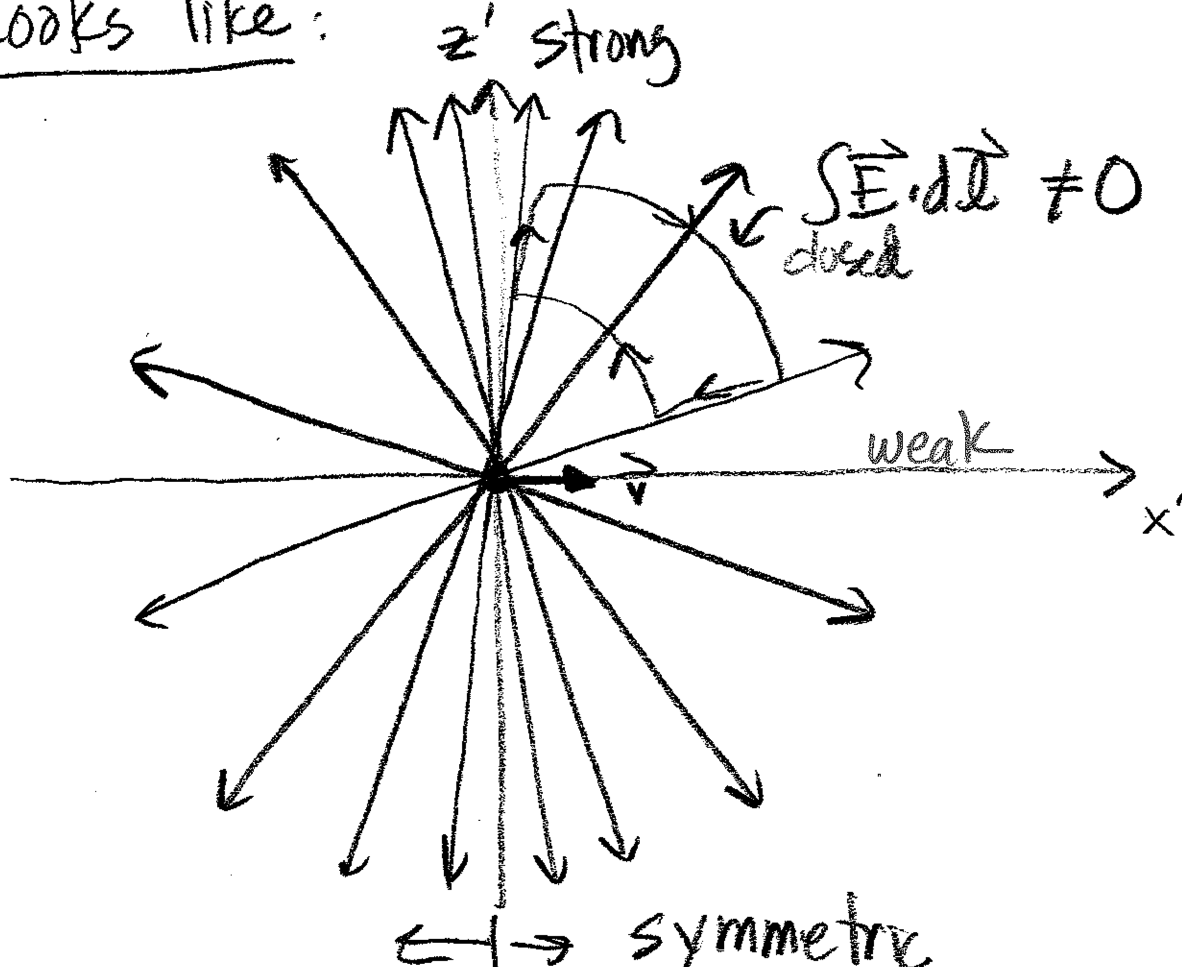
$$\boxed{E' = \frac{Q}{r'^2} \cdot \frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}}$$

1) $\theta' = 0 \rightarrow E' = \frac{Q}{r'^2}(1-\beta^2)$

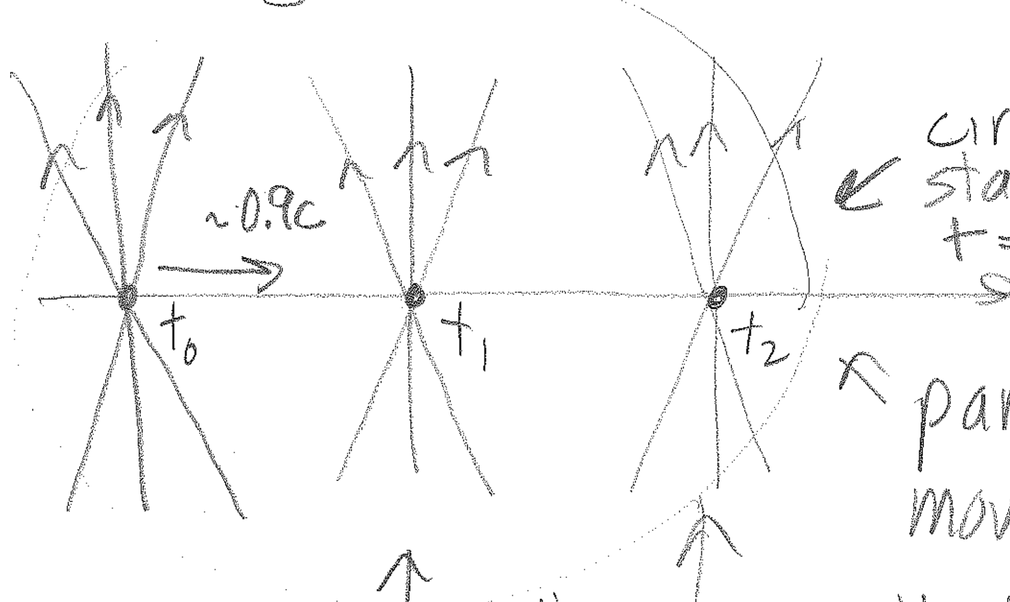
2) $\theta' = \frac{\pi}{2} \rightarrow E' = \frac{Q}{r'^2} \cdot \frac{1-\beta^2}{(1-\beta^2)^{3/2}} = \frac{Q}{r'^2} \cdot \frac{1}{\sqrt{1-\beta^2}}$

$$= \gamma \cdot \frac{Q}{r'^2}$$

Looks like:



Now imagine motion - - -

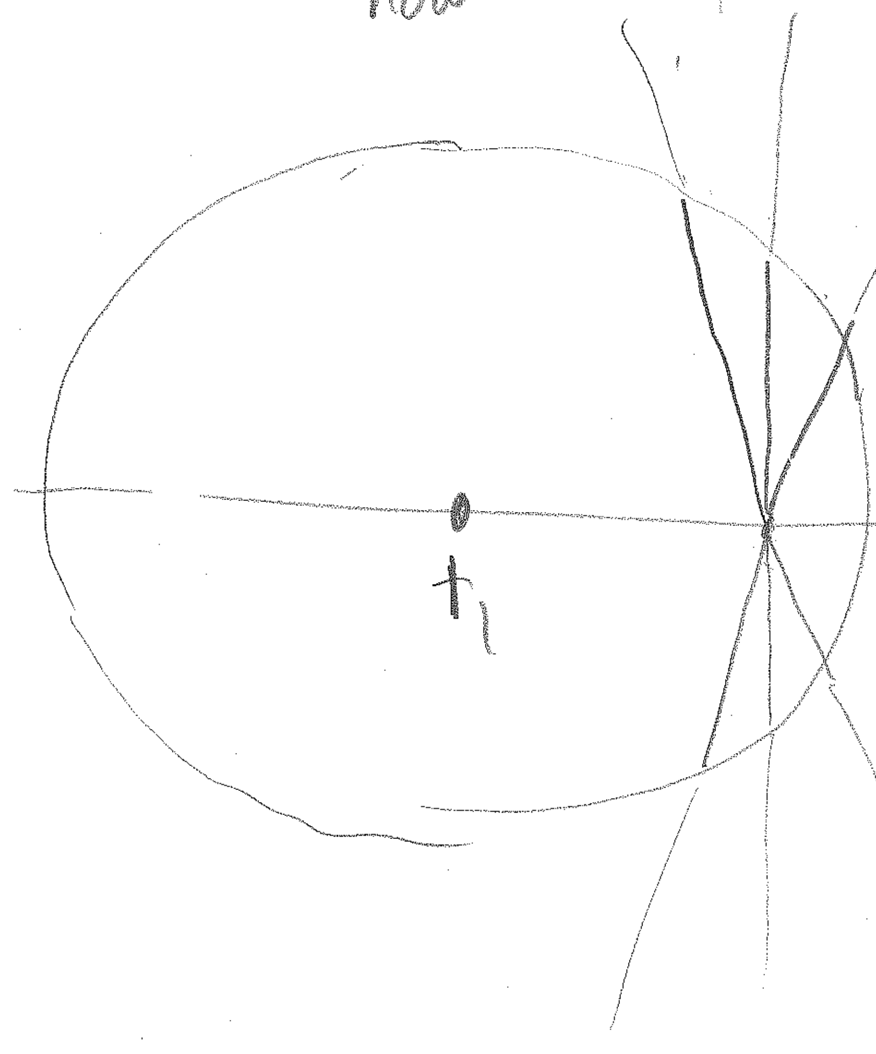


circle started $t = t_1$, by $t = t_2$

particle mov

imagine now

"light emitted"



CAN'T Be Influenced by what happened at t_1!

Suppose at t_1 charge sped up!

What if charge stops!

or speeds up!!!

$\boxed{\text{ACCELERATION OF CHARGES CAUSES RADIATION}}$

5.11 (Scan figure)

stopped charge:

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$\int_0^{2\pi} d\phi \int_0^{\theta_0} \frac{\cancel{r^2}\,\sin\theta\, d\theta\; q}{\cancel{r^2}}$$

$$= 2\pi q \int_{\cos\theta_0}^{1} du = 2\pi q\,(1-\cos\theta_0)$$

$$\int_0^{2\pi} d\phi \int_0^{\phi_0} \frac{\cancel{r^2}\,\sin\phi\, d\phi \cdot q(1-\beta^2)}{\cancel{r^2}\,(1-\beta^2\sin^2\phi)^{3/2}}$$

$\sin^2\phi = 1-\cos^2\phi$

$$= 2\pi q(1-\beta^2)\int_{\cos\phi_0}^{1} \frac{du}{((1-\beta^2)+\beta^2 u^2)^{3/2}}$$

$x = u\beta$

$$2\pi q\left(\frac{1-\beta^2}{\beta}\right)\int_{\beta\cos\phi_0}^{\beta}\frac{dx}{\left((1-\beta^2)+x^2\right)^{3/2}}$$

$$=\frac{2\pi q}{\beta}\frac{\cancel{(1-\beta^2)}}{\cancel{(1-\beta^2)}}\left.\frac{x}{\left((1-\beta^2)+x^2\right)^{1/2}}\right|_{\beta\cos\phi_0}^{\beta}$$

$$=\frac{2\pi q}{\beta}\left[\frac{\beta}{(1-\beta^2+\beta^2)^{1/2}}-\frac{\beta\cos\phi_0}{(1-\beta^2(1-\cos^2\phi_0))^{1/2}}\right]$$

$$=2\pi q\left[1-\frac{\cos\phi_0}{(1-\beta^2\sin^2\phi_0)^{1/2}}\right]$$

set equal

$$\cancel{2\pi q}(1-\cos\theta_0)=\cancel{2\pi q}\left(\cancel{1}-\frac{\cos\phi_0}{(1-\beta^2\sin^2\phi_0)^{1/2}}\right)$$

$$\cos\theta_0=\frac{\cos\phi_0}{(1-\beta^2\sin^2\phi_0)^{1/2}}$$

$$\sin\theta_0=\left[1-\cos^2\theta_0\right]^{1/2}=\left[1-\frac{\cos^2\phi_0}{1-\beta^2\sin^2\phi_0}\right]^{1/2}$$

$$\sin\theta_0=\left[\frac{1-\beta^2\sin^2\phi_0-\cos^2\phi_0}{1-\beta^2\sin^2\phi_0}\right]^{1/2}$$

$$\sin\theta_0 = \left[\frac{\sin^2\phi_0(1-\beta^2)}{1-\beta^2\sin^2\phi_0}\right]^{1/2} = \frac{\sin\phi_0}{\gamma(1-\beta^2\sin^2\phi_0)^{1/2}}$$

so

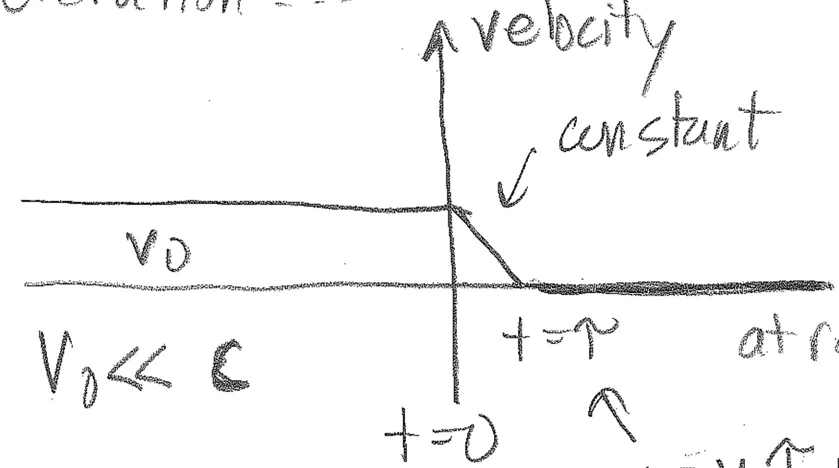
$$\frac{\sin\theta_0}{\cos\theta_0} = \frac{\dfrac{\sin\phi_0}{\gamma\,\cancel{(1-\beta^2\sin^2\phi_0)^{1/2}}}}{\dfrac{\cos\phi_0}{\cancel{(1-\beta^2\sin^2\phi_0)^{1/2}}}}$$

or

$$\boxed{\tan\phi_0 = \gamma\tan\theta_0}$$

Radiation & Acceleration (App B)

Easiest to reason from deceleration ...



$v_0 \ll c$

$a < 0$

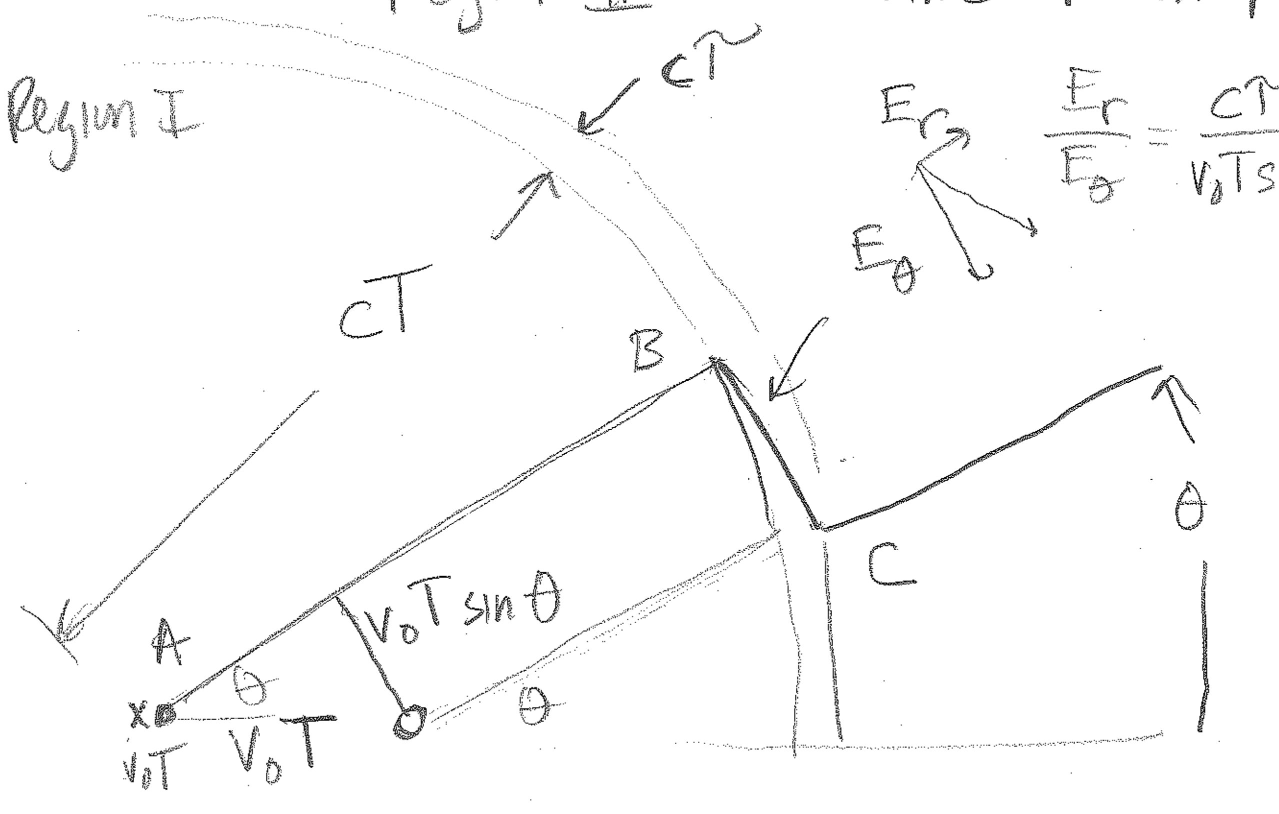
$v_0 + a\tau = 0$

$a\tau = -v_0$

$a = \frac{-v_0}{\tau}$

$$x = v_0\tau + \frac{1}{2}a\tau^2$$
$$= v_0\tau - \frac{1}{2}v_0\tau$$
$$= \frac{1}{2}v_0\tau$$

Region II time T later



$$\frac{E_r}{E_\theta} = \frac{c\tau}{v_0 T \sin\theta}$$

$$E_\theta = \frac{v_0 T \sin\theta}{c\tau}\, E_r = \frac{v_0 \overbrace{T}^{R/c} \sin\theta}{c\tau}\,\frac{q}{R^2}$$

$$\frac{v_0}{\tau} = |a|$$

$$\boxed{E_\theta = \frac{q\,|a| \sin\theta}{c^2 R}}$$

falls ... ONLY AS $1/R$

not $1/R^2$!!

$$\frac{\text{Energy}}{\text{Volume}} = \frac{E_\theta^2}{8\pi} = \frac{q^2 a^2 \sin^2\theta}{8\pi c^4 R^2}$$

Do volume integral...

$$\text{Energy} = \frac{q^2 a^2 \cdot \frac{2}{3}}{8\pi c^4 R^2} \cdot 4\pi R^2 c\tau$$

$$\frac{\text{Energy}}{\tau} = \text{Power} = \frac{1}{3}\frac{q^2 a^2}{c^3}$$

E field

~~B field~~ DOUBLES

$$\boxed{P_{rad} = \frac{2}{3}\frac{q^2 a^2}{c^3}}$$

LARMOR POWER FORMULA